MPC DESIGN FOR CONSTRAINED MULTIVARIABLE SYSTEMS UNDER ACTUATOR BACKLASH

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Abstract: A new approach to the design of Model Predictive Controller (MPC) that simultaneously addresses the actuator saturation and backlash is proposed in this paper. The discrete characteristics of the actuator backlash allows one to reformulate the input constraints as a set of mixed integer linear inequalities. As a result, the MPC is designed by solving a *Mixed-integer Quadratic Programming* problem. Simulation results are presented to show how this new approach performs as compared to the existing techniques of the backlash compensation when they are applied to an industrial case study.

Keywords: actuator backlash, constrained systems, model predictive control, mixed integer quadratic programming

1. INTRODUCTION

Actuators, e.g. control valves in most industrial processes, play an important role in automatic control systems. As commonly the weakest link of the control loop, the control valve accounts for about 32% of "poor" or "fail" control loops [Miller and Desborough, 2001]. A recent survey in Yang and Clarke [1999] indicated that 30% of all control loops in Canadian paper mills were oscillating because of valve problems. Also, [Ruel, 2000] observed a similar problem in refinery plants. Poor control valve performance is therefore the single biggest source of controller-induced variability.

Control valves contain static (e.g. saturation) or dynamic (e.g. backlash) non-linearities as well as other non-linear characteristics such as stiction, hysteresis, etc [Corradini and Orlando, 2002]. Model Predictive Control (MPC) has had a significant and widespread impact on industrial process control because of its ability to explicitly handle actuator saturation. The performance of MPC, however, is sensitive to model uncertainty [Kothare et al., 1996]. Since the dynamics of actuator backlash is not accommodated in the MPC design formulation, the existence of backlash results in uncertainty in the plant inputs. The dynamics of actuator backlash introduces a limit cycle in the proximity of the steady state operating points even if the controlled system behaves linearly. As a result, the MPC performance would significantly degrade if the actuator backlash is present in the feedback loop. Various approaches have been adopted for compensating the backlash effect, including nonlinear inverse strategy [Corradini and Orlando, 2002; Tao and Kokotovic, 1993; Nordin and Gutman, 2002; Chow and Clarke, 1994], or re-tuning strategy Ghazzawi et al. [2001].

In this paper, we propose a tactical approach that simultaneously addresses the actuator backlash and saturation within the MPC design framework. This approach is adopted to alleviate the drawbacks of the re-tuning strategy (i.e. a sluggish

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response as well as the trial-and-error nature), and the nonlinear-inverse strategy (i.e. in the case of encountering actuator saturation limits).

2. MPC DESIGN FRAMEWORK

Assume that the system is described by the following linear time-invariant model:

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

with $x(k)$ is the state vector $\in \mathbb{R}^n$, $u(k)$ is the input vector $\in \mathbb{R}^m$, and $y(k)$ is the output vector $\in \mathbb{R}^m$. The matrices $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ are associated with the state, input and output matrices, respectively, and are assumed to be stabilizable and detectable.

Consider the problem of tracking a constant set-point y_s and rejecting a time-varying output disturbance $\{d(k)\}$: it is wished to regulate the error $e(k)$ to zero: $e(k) = y(k) + (d(k) - y_s)$. Then, let us define d_e as the external signal: $d_e(k) = d(k) - y_s$. Given knowledge of the external signal d_e and the current state measurement $x(0)$ (or its on-line estimates), the MPC design is aimed at finding the M-move control sequence $\{u(0), u(1), \ldots, u(M-1)\}$ that minimizes the finite-horizon performance index:

$$\begin{aligned} J_0 = {} & [x(P) - x_s]^T \mathbf{\Psi}_f [x(P) - x_s] \\ & + \sum_{k=0}^{P-1} e^T(k) \mathbf{\Psi} e(k) \\ & + \sum_{k=0}^{M-1} [u(k) - u_s]^T \mathbf{\Phi} [u(k) - u_s] \end{aligned} \tag{3}$$

where $\mathbf{\Psi} \geq 0$, $\mathbf{\Phi} \geq 0$, $\mathbf{\Psi}_f \geq 0$. In (3), P is the prediction horizon, $M \leq P$ is the control horizon. This optimization can be translated into a convex QP problem as:

$$U^{OPT} = arg \min_{\substack{U \\ LU \leq K}} U^T \mathbf{W} U + 2U^T \mathbf{V} \tag{4}$$

where $\mathbf{W}$ and $\mathbf{V}$ are the dynamic matrices obtained from the linear model for the P prediction horizon of the outputs, and the M-move of input sequence.

3. MODEL OF BACKLASH AND ITS INVERSE

Fig. 1 describes an input-output map of the actuator backlash. Let us define two quantities to represent two situations; (i) if $u_I(t)$ is inside the backlash and on the positive side of the backlash, the distance from the positive boundary is defined as d_p (see Fig. 2(a)), and (ii) if $u_I(t)$ is inside the backlash and on negative side, the distance from its negative boundary is defined as d_n (see Fig. 2(b)). Without loss of generality, we assume that $m_b = 1$. Then, we have:

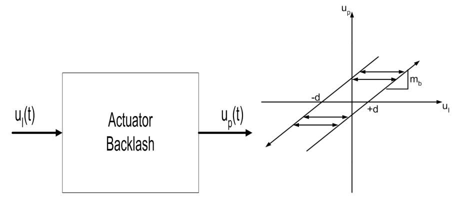


Fig. 1. An input-output map of backlash

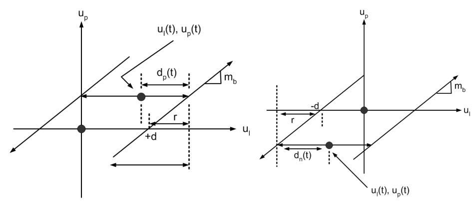

(a) $d_p(t)$ (b) $d_n(t)$

Fig. 2. Computation of $d_p(t)$ and $d_n(t)$

$$d_p(t) = [u_p(t) + d] - u_I(t)$$
$$d_n(t) = [u_p(t) + (-d)] - u_I(t)$$

Hence, we obtain the following backlash model:

$$u_p(t) = \begin{cases} u_I(t) - d & : \quad \Delta u_I(t) > d_p(t-1) \\ u_I(t) - (-d) & : \quad \Delta u_I(t) < d_n(t-1) \\ u_p(t-1) & : \quad otherwise \end{cases}$$

The inverse of the backlash model is then given as:

$$u_I(t) = \begin{cases} u(t) + d & : \quad \Delta u(t) > 0 \\ u(t) - d & : \quad \Delta u(t) < 0 \\ u_I(t-1) & : \quad \Delta u(t) = 0 \end{cases}$$

Clearly, the nonlinear inverse strategy of compensating the backlash effect is, in principle, to bump up the controller outputs without changing the input direction. Based on this property, we integrate the backlash inverse model within the MPC design framework.

4. MIQP-BASED DESIGN OF MPC

4.1 Reformulation of Input Constraints

In the QP-based MPC design, the input (saturation) constraint is given by:

$$u_{min} \leq u(k) \leq u_{max} \tag{5}$$

To incorporate the backlash dynamics within the MPC computational framework, we reformulate the input constraints so that the inverse model of the actuator backlash is integrated within the MPC design framework as in Fig. 3.

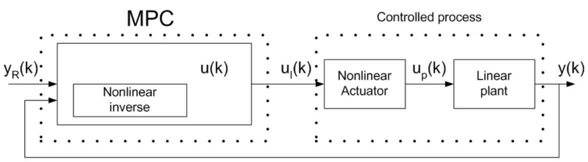


Fig. 3. An MPC scheme integrated with the backlash nonlinear inverse

By applying this integration strategy, $u_I(k)$ can be represented in terms of $u(k)$ with three different conditions depending upon the value of $\Delta u(k)$. The inverse model of the actuator backlash dictates that, for example, if the input change is positive (or $\Delta u(k) > 0$), the MPC should produce its output $u_I(t) = u(k) + d$. Similarly, for a negative input change (or $\Delta u(k) < 0$), the resulting MPC output is reduced by d. For a zero input change, the previous value of u_I should be used. Therefore, the inclusion of the backlash within the MPC computational framework is, in essence, to produce optimal input signals that are outside the dead-band region.

That basic idea of incorporating the actuator backlash is systematically implemented by reformulating the input constraints for the MPC design. Let us introduce a set of logical variables δ_{ij}, for $j = 1, 2, 3$, for representing the input change conditions (e.g. positive, negative, or zero). Then, we impose the following propositional logics on the inputs $u(k)$:

$$\delta_{i1} = 1 \longleftrightarrow u_i(k) \geq d_i \tag{6}$$

$$\delta_{i2} = 1 \longleftrightarrow u_i(k) \leq -d_i \tag{7}$$

$$\delta_{i3} = 1 \longleftrightarrow u_i(k) = 0 \tag{8}$$

$$\sum_{j=1}^{3} \delta_{ij} = 1 \tag{9}$$

for $i = 1, 2, ..., m$, with m is the number of manipulated inputs in the system. The propositional logics specified in (6), (7), and (8) can be transformed into a set of *mixed-integer linear inequalities*, i.e. linear inequalities involving both *continuous variables* $u \in \mathbb{R}^m$ and *logical variables* $\delta \in \{0, 1\}$, by adopting the framework described in Bemporad and Morari [1999]. The set of mixed-integer linear inequalities constraints for the i^{th} input is then given by:

$$\begin{aligned}
u_i(k) - d_i &\geq (u_{i_{min}} - d_i)(1 - \delta_{i1}) \\
u_i(k) - d_i &\leq (u_{i_{max}} - d_i)\delta_{i1} \\
u_i(k) + d_i &\geq (u_{i_{min}} + d_i)\delta_{i2} \\
u_i(k) + d_i &\leq (u_{i_{max}} + d_i)(1 - \delta_{i2}) \\
u_i(k) &\geq u_{i_{min}}(1 - \delta_{i3}) \\
u_i(k) &\leq u_{i_{max}}(1 - \delta_{i3})
\end{aligned} \tag{10}$$

for $i = 1, 2, ..., m$, with m is the number of manipulated inputs in the system.

Note that if $d_i = 0$, (10) reduces to $u_{i_{min}} \leq u_i(k) \leq u_{i_{max}}$. This implies that if the backlash is not active, we should assign $d_i = 0$. Otherwise, the MPC controller would have a sluggish response (or even infeasible solution) because of the shrinkage of feasible input constraints.

4.2 MIQP formulation

The MPC design is now aimed at determining its optimal $u(k)$ at each sampling time subject to the reconfigured input constraints in (10). At every time step, the dead-band region is completely avoided by suppressing a further movement of the input to avoid its direction change. This is done by activating *one and only one* logical variable at a time, and at the proximity of steady states where the backlash is active.

We show that this formulation belongs to an MIQP optimization problem. Let us consider, for illustration only, the case of m inputs and control horizon, $M = 1$. The linear inequality constraints in (10) is then re-arranged as:

$$\begin{bmatrix} -\underline{\mathbf{1}} & \theta & \mathbf{0} & \mathbf{0} \\ -\underline{\mathbf{1}} & \mathbf{0} & \mathbf{\Omega} & \mathbf{0} \\ -\underline{\mathbf{1}} & \mathbf{0} & \mathbf{0} & \mu \\ \underline{\mathbf{1}} & \alpha & \mathbf{0} & \mathbf{0} \\ \underline{\mathbf{1}} & \mathbf{0} & \beta & \mathbf{0} \\ \underline{\mathbf{1}} & \mathbf{0} & \mathbf{0} & \tau \\ \mathbf{0} & \underline{\mathbf{1}} & \underline{\mathbf{1}} & \underline{\mathbf{1}} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_m \\ \delta_{11} \\ \vdots \\ \delta_{m1} \\ \delta_{12} \\ \vdots \\ \delta_{m2} \\ \delta_{13} \\ \vdots \\ \delta_{m3} \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{U}}_{min} \\ -\underline{\mathbf{D}} \\ \underline{\mathbf{U}}_{min} \\ -\underline{\mathbf{D}} \\ -\underline{\mathbf{U}}_{max} \\ -\underline{\mathbf{U}}_{max} \\ -\underline{\mathbf{I}} \end{bmatrix} \leq \underline{\mathbf{0}} \tag{11}$$

and

$$\underline{\mathbf{1}} = diag[1_1, \ldots, 1_m]$$
$$\theta = diag[-(u_{1_{min}} - d_1), \ldots, -(u_{m_{min}} - d_m)]$$
$$\mathbf{\Omega} = diag[(u_{1_{min}} + d_1), \ldots, (u_{m_{min}} + d_m)]$$
$$\alpha = diag[-(u_{1_{max}} - d_1), \ldots, -(u_{m_{max}} - d_m)]$$
$$\beta = diag[(u_{1_{max}} + d_1), \ldots, (u_{m_{max}} + d_m)]$$
$$\mu = diag[u_{1_{min}}, \ldots, u_{m_{min}}]$$
$$\tau = diag[u_{1_{max}}, \ldots, u_{m_{max}}]$$
$$\underline{\mathbf{U}}_{min} = [u_{1_{min}}, \ldots, u_{m_{min}}]^T$$
$$\underline{\mathbf{U}}_{max} = [u_{1_{max}}, \ldots, u_{m_{max}}]^T$$
$$\underline{\mathbf{D}} = [d_1, \ldots, d_m]^T; \underline{\mathbf{I}} = [1_1, \ldots, 1_m]^T$$

Define a matrix $\mathbf{Q}_{4m\times 4m}$ and a vector $\mathbf{b}_{4m\times 1}$ as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{W} & 0 & \ldots & 0 \\ 0 & \vdots & \ddots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \ldots & 0 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 2\mathbf{V} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (12)$$

By defining the mixed linear inequalities of (11) as $Cz + \wp \leq 0$ with $z = [z_c, z_d]^T$, where z_c are the continuous variables u, and z_d are logical variables δ_{ij}, the MPC optimization problem can be expressed as:

$$\min_z \; J_{QP} \triangleq z^T \mathbf{Q} z + \mathbf{b}^T z \quad (13)$$
$$\text{s.t.}$$
$$Cz + \wp \leq 0$$
$$z = \begin{bmatrix} z_c \\ z_d \end{bmatrix}$$
$$z_c \in \mathbb{R}^{n_c}$$
$$z_d \in \{0,1\}^{n_d}$$

This is a *Mixed Integer Quadratic Programming* (MIQP) problem. Note that the matrix $\mathbf{Q}$ in (13) is positive semi-definite - an important criteria that is necessary if such an optimization needs to be solved *on-line* for a global optimality.

Since the proposed technique considers both the logical conditions and saturation simultaneously during the determination of the MPC optimal inputs, we avoid the problem of producing the infinite input for compensating the backlash due to the input saturation as is the case when the nonlinear inverse strategy is applied.

4.3 Stability Analysis of MIQP-based MPC

In this section, we address the stability condition for the MIQP-based MPC. For this purpose, we follow the stability analysis of Bemporad and Morari [1999] for the general system of Mixed Logical Dynamical. Define the states and inputs equilibrium as (x_s, u_s), and let the logical variables be definitely admissible, meaning that δ_s corresponds to the desired steady-state values for the logical variables. Let t be the current time t, and $x(t)$ the current state. The MPC optimization problem (3) can be equivalently expressed in the general form of Mixed Logical Dynamical system (MLD) as the following,

$$\min_u \; J \triangleq \sum_{k=0}^{M-1} \| u(k) - u_s \|^2_{\mathbf{\Phi}} + \| \delta(k|t) - \delta_s \|^2_R + \| x(k|t) - x_s \|^2_{\mathbf{\Psi}} \quad (14)$$
$$\text{s.t.}$$
$$x(M|t) = x_s$$
$$x(k+1|t) = Ax(k|t) + Bu(k)$$
$$y(k|t) = Cx(k|t)$$
$$E_2\delta(k|t) \leq E_1 u(k) + E_3 \quad (15)$$

where $\mathbf{\Phi} > 0; R \geq 0; \mathbf{\Psi} > 0$ and E_1, E_2 and E_3 can be straightforwardly defined from (11). Assume that the optimal solution is given by $U^* = \{u^*(0), u^*(1), \ldots, u^*(M-1)\}$, and the *receding horizon* strategy is applied as $u(t) = u^*(0)$ before repeating the whole optimization procedure at time $t+1$.

Theorem 1. Let (x_s, u_s) be an equilibrium pair and δ_s definitely admissible. Assume that the initial states $x(0)$ is such that a feasible solution of problem (14) exists at time $t = 0$. Then, the optimal inputs obtained by solving (14)-(15) stabilizes the system in the following sense:

$$lim_{t\to\infty} x(t) = x_s; \quad lim_{t\to\infty} u(t) = u_s;$$
$$lim_{t\to\infty} \| \delta(k|t) - \delta_s \|^2_R = 0$$

while fulfilling the constraints (15).

Proof. The proof follows from standard Lyapunov arguments. Let U^* denote the optimal control sequence $\{u^*(0), \ldots, u^*(M-1)\}$. Let

$$V(t) \triangleq J(U^*, x(t))$$

denote the corresponding value attained by the performance index. Also, let $U_1 = \{u^*(1), \ldots, u^*(M-2), u_s\}$. Then U_1 is feasible (but not optimal) at time $t+1$, along with the vectors $\delta(k|t+1) = \delta(k+1|t)$ for $k = 0, \ldots, M-1$. Also $\delta(M-1|t+1) = \delta_s$ since $x(M-1|t-1) = x(M|t) = x_s$ and δ_s definitely admissible. Hence,

$$V(t+1) \leq J(U_1, x(t+1)) = V(t) - \| u(k) - u_s \|^2_{\mathbf{\Phi}} - \| \delta(k|t) - \delta_s \|^2_R - \| x(k|t) - x_s \|^2_{\mathbf{\Psi}}$$

and $V(t)$ is decreasing. Since $V(t)$ is lower-bounded by zero, there exists $V_\infty = lim_{t\to\infty} V(t)$, which implies $V(t+1) - V(t) \to 0$. Therefore, each term of the sum

$$\| u(k) - u_s \|^2_{\mathbf{\Phi}} + \| \delta(k|t) - \delta_s \|^2_R + \| x(k|t) - x_s \|^2_{\mathbf{\Psi}} \leq V(t) - V(t-1)$$

converges to zero as well, which proves the theorem.

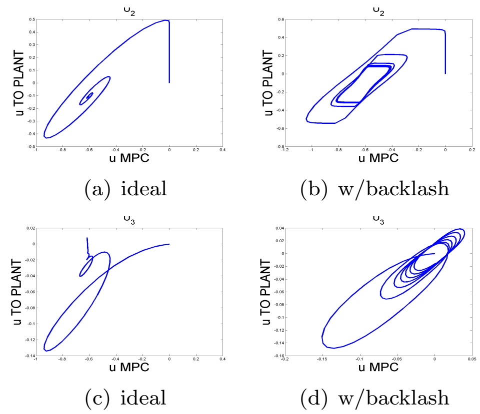
Note that our result on the stability analysis of the MIQP-based MPC is a subset of the stability result of Bemporad and Morari [1999], where in their case, the integer variable may appear in the state and input variables.

5. APPLICATION TO AN INDUSTRIAL FCCU PROCESS

The FCCU case study was taken from [Grosdidier et al., 1993]. At the MPC level, we control seven variables, denoted as $y_1, ..., y_7$ by manipulating six variables $u_1, ..., u_6$, see [Grosdidier et al., 1993] for details. Three proportional-integral (PI) controllers were applied in the flows of the combustion air, the hot gas oils and the combined cold gas and recycle oils (u_1, u_2, u_3). Two PI controllers were to control the feed preheat (u_4) and the riser outlet temperature (u_5). Recycle flow (u_6) is regulated by adjusting the output of a hand controller. Each variable is constrained by their associated high and low limits. All manipulated variables are at their ideal resting values (IRV), which is assumed to be their steady-state, except for combustion air flow (u_1) which is allowed to move freely. In this study, we consider the following conditions: (1) Backlash is known to be active only in the hot feed flow (u_2) valve; (2) Prediction horizon, P is fixed at 20; Control horizon, M is fixed at 1; and (3) The proposed MIQP MPC is then compared to the Nonlinear Inverse strategy ([Chow and Clarke, 1994]) and to the strategy of re-tuning λ, where λ is the element of Φ.

5.1 Effect of Backlash

In response to a disturbance change of +2.5% in u_2, Fig. 4 shows the existence of backlash in u_2, where its effect propagates to other input channels because of the interacting nature of the FCCU process. This backlash result in a significant degradation in the performance of MPC designed using the standard approach, producing oscillations in the manipulated inputs and the controlled variables. In the presence of backlash of 2.5%, a small oscillation in the controller outputs makes the backlash active. Furthermore, we should note that the backlash in the u_2 valve is not active when the change in the controller outputs is large; however, as the movement nearing the steady-state and the controller outputs are still oscillating, the backlash is then active.

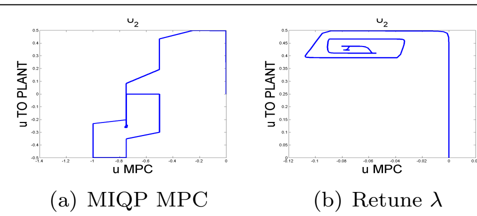


(a) ideal (b) w/backlash (c) ideal (d) w/backlash

Fig. 4. u_2 and u_3 response with and without backlash.

5.2 MIQP vs. Re-tuning Strategies

Figs. 5 and 6 show the comparison of MIQP-based MPC with the strategy of re-tuning λ. The input moves u_2 for the different compensation methods are shown in Fig. 5. Fig. 6 shows the corresponding responses of the outputs for: the standard MPC with no backlash (i.e. *"ideal"*), MIQP-based MPC design (i.e. *"MIQP MPC"*) and the re-tuning of λ.



(a) MIQP MPC (b) Retune λ

Fig. 5. Input signal u_2 for different approaches in the presence of backlash

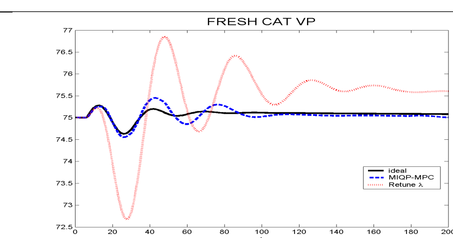


Fig. 6. Fresh cat VP in the presence of backlash

The re-tuning strategy, by increasing the move suppression factor λ, produces a more sluggish response in the manipulated and controlled variables (see Fig. 5(b)). This compensation effect is similar to the case when we apply the MIQP approach; however, note that the movement of u_2 in the transient is different. This is due to the fact that the MIQP technique suppresses the movement on u_2 only at the proximity of steady state in order to eliminate the oscillations caused by the backlash. On the other hand, the re-tuning strategy avoid the backlash by imposing a sluggish response of the controller from the beginning. This re-tuning strategy, however, results in a poor disturbance rejection as shown in Fig. 6.

5.3 MIQP vs. Nonlinear Inverse Strategies

We study the case when the valve of u_2 is operating near the saturation limit by considering the disturbance of -2.5%, with 5% backlash. The nonlinear inverse method fails to eliminate the backlash since the saturation limit eclipses the total amount of energy required to compensate the backlash (see Figs. 7 and 8(d)). The MIQP MPC method, on the other hand, produces a better performance than the nonlinear inverse strategy as it anticipates the saturation limits in u_2 when determining the optimal input moves.

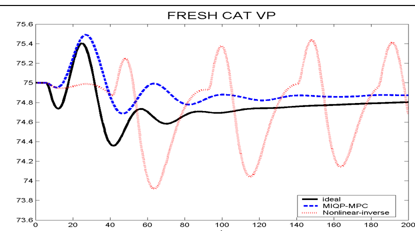
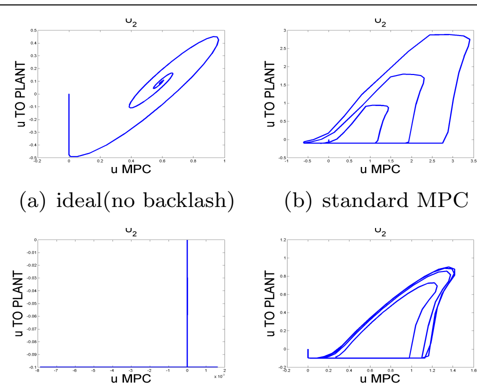


Fig. 7. Fresh cat VP; valve u_2 near saturation under disturbance change and 5% backlash



(a) ideal(no backlash) (b) standard MPC

(c) MIQP MPC (d) Nonlinear inverse

Fig. 8. Signals in u_2 for disturbance change with valve u_2 near saturation in the presence of backlash

6. CONCLUSIONS

In this paper, we have extended the MPC design methodology for addressing simultaneously the actuator backlash and saturation. We have shown that a set of logical constraints for the inputs has been produced so that the MPC design has been formulated as a mixed integer programming problem. Simulation studies using the FCCU process has been presented to compare the developed approach with other techniques i.e. the nonlinear inverse and re-tuning strategies. The MIQP-based MPC design has outperformed other compensation methods in the presence of both actuator saturation and backlash.

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