MODEL-BASED AUTOTUNING SYSTEM USING ANN AND RELAY FEEDBACK TEST

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Abstract: A model-based autotuning system is presented for PI/PID controllers. A conventional relay feedback test is used to generate the response for estimations of parameters such as: process gain, ultimate gain, ultimate frequency, and apparent deadtime. Two ANNs are built based on the data generated from standard SOPDT processes to enhance the estimation of apparent deadtime. Upon having the estimated results, the dynamics of the process is classified into one of two groups. In one group, PI/PID controller will be tuned based on the FOPDT parameterization. In the other group, only PID controller will be considered for dynamic control, and the controller will be tuned according to the SOPDT parameterization. The tuning rules are given in terms of ultimate gain, ultimate frequency, and the apparent deadtime. Since the tuning rules are derived from model-based control, they are superior to the Z-N rules in performance.

Keywords: relay feedback, autotuning, ANN, FOPDT, SOPDT

1. INTRODUCTION

In 1984, Åström and Hägglund (1984) presented a relay feedback system to generate sustained oscillation as an alternative to the conventional continuous cycling technique for controller tuning. This relay feedback test was soon referred as autotune variation (ATV) (Luyben, 1987). As shown in Fig. 1(a) is the block diagram of the ATV loop. Figure 1(b) illustrates the typical response curves from the ATV system. The test provides ultimate gain (k_{cu}) and ultimate period (P_u) of the following to apply Ziegler-Nichols method for controller tuning.

$$k_{cu} = \frac{4h}{\pi A}\ ; \qquad \omega_u = \frac{2\pi}{P_u} \tag{1}$$

Controller tuning via the above mentioned ATV test is attractive, because it is operated under closed-loop and no *a priori* knowledge of system is need. But, the control performance thus obtained is, in general, inferior to those tuned by model-based, such as IMC and other related methods. This fact is most obvious when the processes have significantly underdamped and second-order dynamics. To employ model-based tuning, an adequate and reduced order dynamic model (usually, FOPDT or SOPDT) is required. In literature, there are reported efforts (Luyben, 1987; Li *et al.*, 1991; Chang *et al.*, 1992; Wang *et al.*, 1997; Huang *et al.*, 2000; Kaya and Atherton, 2001; Huang and Jeng, 2003) trying to develop such models from the ATV tests. Nevertheless, according to those reports in literature, two major difficulties are encountered. One of difficulties is that estimation of ultimate gain and frequency according to Eq.(1) is subject to errors, sometimes as high as 20 percent for ultimate gain (Li *et al.*, 1991). The second difficulty is due to the fact that one simple ATV test, in general, does not provide sufficient data for identifying a parametric model aforementioned. Improvements to give better accuracy and efficiency by using saturation relay (Shen *et al.*, 1996) or by reducing high-order harmonic terms using the Fourier analysis (Lee *et al.*, 1995; Sung and Lee, 1997; Wang *et al.*, 1997) have also been reported. But, these does not alter the encountered second difficulty above mentioned. To overcome the second difficulty, more than one ATV test is usually required (Li *et al.*, 1991; Scali *et al.*, 1999). In order to have the superior performance that the model-based controller used to have, while having the simple experiment like the conventional autotuning via ATV, a new autotuning method is presented.

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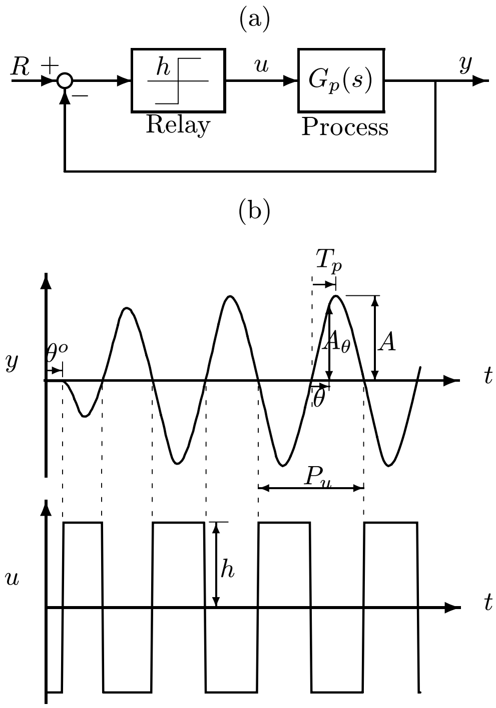

Fig. 1. (a) Block diagram of a relay feedback system (b) Response curves in relay feedback test

In the following, it is assumed that zero offset is a common specification of control. For zero offset, in general, PI or PID controller is considered for control. Usage of either type of the controllers mentioned depends not only on the application occasions, but also on their dynamic characteristics. Thus, an autotuning system should provide the best flexibility to cope with the application demand and the dynamics. For this purpose, model-based tuning rules including PI and PID controllers are considered, and, classification based on the dynamics of the processes into two groups for controller tuning is presented. The first group of processes are suitable for either PI or PID controllers for dynamic compensations, and, the tuning formula are derived based on an FOPDT model. The second group of processes, on the other hand, are considered good only for PID controllers which are derived based on an underdamped SOPDT model. Identification for classification of these two groups is thus presented. Since the apparent deadtime of process is crucial for applying the proposed classification, a novel approach enhanced by artificial neural network for estimation of the apparent deadtime is presented. The autotuning procedure starts with an ATV test. The steady-state gain, k_p, and k_{cu} are estimated along with the experiment until constant cycling occurs. The constant cycles at the output are then used to estimate the apparent deadtime, and the process is classified for tuning. After classifying to either of the two groups, model-based tuning rules are assigned and the controller parameters are computed accordingly. Several examples are used to illustrate this proposed method.

2. MODEL-BASED CONTROLLERS DESIGN

In general, models with dynamics up to second order plus deadtime are adequate for designing PI/PID controllers. For higher order processes, dynamics are usually represented by reduced order models of the following:

- FOPDT $\quad G_p(s) = \dfrac{k_p\ e^{-\theta s}}{\tau s + 1} \qquad (2)$
- SOPDT $\quad G_p(s) = \dfrac{k_p\ e^{-\theta s}}{\tau^2 s^2 + 2\tau\zeta s + 1} \qquad (3)$

2.1 Tuning PI/PID based on FOPDT model

A direct synthesis approach is used to synthesize PI/PID controller using an FOPDT parameterization. In case of demanding for a PI controller, the controller is devised so as to compensate the process dynamics to have a loop transfer function (LTF) of the following standard form:

$$G_{loop}(s) = \frac{k_o}{\theta}\frac{e^{-\theta s}}{s} \qquad (4)$$

In those cases that demand for a PID controller, LTF is chosen to be compensated the process to become:

$$G_{loop}(s) = \frac{k_o}{\theta}\frac{(1 + a\theta s)}{s\ (1 + \tau_f s)}\ e^{-\theta s} \qquad (5)$$

The use of loop transfer function in the form of Eq.(4) or Eq.(5) has been explained in a late work of Huang and Jeng (2002). According to the standard forms, parameters of the PI/PID controllers can be derived as following:

$$k_c^{'} = \frac{k_o\ \tau}{k_p\ \theta}\ ;\quad \tau_R^{'} = \tau\ ;\quad \tau_D^{'} = a\ \theta \qquad (6)$$

Notice that the PID controller used here is the actual, or series, form. In this tuning, k_o and a are parameters for performance or robustness specification, and the filter time constant, τ_f, is taken arbitrarily small (e.g. $0.05\tau_D^{'}$). The defaulted values of k_o and a are suggested as 0.65 and 0.4, respectively, for the PID controller. On the other hand, the defaulted values of k_o for the PI controller are 0.5. These defaulted values will provide about 3 and 60° as gain and phase margin of the system, respectively.

Using ultimate gain and ultimate frequency, the tuning formula of Eq.(6) that are originally in terms of the FOPDT parameterization can be re-written as:

$$\begin{aligned} k_c^{'} &= \frac{k_o\sqrt{{K_u}^2 - 1}}{k_p[\pi - \tan^{-1}(\sqrt{{K_u}^2 - 1})]} \\ \tau_R^{'} &= \frac{\sqrt{{K_u}^2 - 1}}{\omega_u} \\ \tau_D^{'} &= a\ \left[\frac{\pi - \tan^{-1}(\sqrt{{K_u}^2 - 1})}{\omega_u}\right] \end{aligned} \qquad (7)$$

where $K_u = k_p k_{cu}$.

2.2 Tuning PID based on SOPDT model

The PID controller in terms of SOPDT parameterization is derived in the similar way to give a loop transfer function of Eq.(4). The resulting controller parameters are given as:

$$k_c = \frac{k_o\ (2\tau\zeta)}{k_p\ \theta}\ ;\quad \tau_R = 2\tau\zeta\ ;\quad \tau_D = \frac{\tau}{2\zeta} \tag{8}$$

In this case, the controller is an ideal PID. As has been mentioned, the defaulted value of k_o is taken as 0.5. As a result, in terms of the ultimate gain and ultimate frequency, the PID parameters of Eq.(8) can be written as:

$$\begin{aligned} k_c &= \frac{k_o\ K_u \sin(\omega_u\theta)}{k_p\ \omega_u\ \theta} \\ \tau_R &= \frac{K_u \sin(\omega_u\theta)}{\omega_u} \\ \tau_D &= \frac{1 + K_u\cos(\omega_u\theta)}{\omega_u K_u \sin(\omega_u\theta)} \end{aligned} \tag{9}$$

If k_{cu} and ω_u of the given process are obtained from ATV test, the parameters of the PI or PID controllers can be computed based on either Eq.(7) or Eq.(9). However, as one can seen in Eq.(9), the tuning formula need to know the values of k_p and the apparent deadtime θ in advance. The estimation for k_p and θ will be given in the next section.

3. PROCESS IDENTIFICATION USING ATV TEST

To apply the model-based controllers derived in the previous section, reduced order dynamic models parameterized as FOPDT of Eq.(2), and, as SOPDT of Eq.(3) are required. An ATV response is thus used to develop such parametric models. As shown in Fig. 1(a) is a ATV system which consists of process, G_p, and a relay controller. The relay controller provides output at $+h$ or $-h$ only as on-off control. The controlled process output consists of transient oscillations after a pure deadtime, θ^o, and develops constant cycling with magnitude, A, and period, P_u. Notice that A and P_u are used in the autotuning system of Åström and Hägglund (1984) to apply Z-N method to compute the PI/PID controller parameters. In the meantime, T_p, θ and an associated height, A_θ, within one of the constant cycles are also indicated in Fig. 1(b). Here, θ is used to designate an apparent deadtime in the FOPDT or the SOPDT model, which is used to represent the dynamics of higher order processes. All these quantities measured from an ATV response are governed by the dynamic model and the relay. In other words, they can be expressed as:

$$\frac{A}{k_p h} = f_1(\bar{\tau}) \quad \text{or} \quad f_1(\bar{\tau}, \zeta) \tag{10}$$

$$\frac{P_u}{\theta} = f_2(\bar{\tau}) \quad \text{or} \quad f_2(\bar{\tau}, \zeta) \tag{11}$$

$$\frac{A_\theta}{A} = f_3(\bar{\tau}) \quad \text{or} \quad f_3(\bar{\tau}, \zeta) \tag{12}$$

$$\frac{\theta}{T_p} = f_4(\bar{\tau}) \quad \text{or} \quad f_4(\bar{\tau}, \zeta) \tag{13}$$

where $\bar{\tau}$ represents τ/θ. The equations given above describe the dynamic features of the ATV response in time domain. Two equations which correspond to Eqs.(10) and (11) derived from frequency domain are:

$$\omega_u\theta + \tan^{-1}\left(\frac{2\ \omega_u\tau\zeta}{1 - {\omega_u}^2\tau^2}\right) = \pi \tag{14}$$

$$\frac{k_p}{\sqrt{(1 - {\omega_u}^2\tau^2)^2 + 4\ {\omega_u}^2\tau^2\zeta^2}} = \frac{1}{k_{cu}} \tag{15}$$

where ω_u is taken as $2\pi/P_u$.

Theoretically, provided that k_p and k_{cu} are given, the identification problem to find an reduced order SOPDT model can be solved by finding τ, ζ, and θ that fit Eqs.(10)-(13) or Eqs.(12)-(15) in the sense of least-squares. The explicit functional forms for Eqs.(10) -(13) are not available, and numerical method to solve the above equations will not be convenient. Since estimation of apparent deadtime θ as well as k_p and k_{cu} is required for this proposed autotuning method, in the following, a simplified algorithm will be presented to estimate these required data from ATV test.

3.1 Estimation of k_p and k_{cu}

To estimate k_p, the experimental ATV test is started with a temporal disturbance to either the set-point or the process input (i.e. u) for a short period of time and restored back to its origin. The disturbance introduced has two main purposes. One is to initialize the relay feedback control, the other one is to generate data for computing the steady-state process gain, k_p. The estimation is made along the ATV test in one run as the following.

Let y^I and u^I designate the integrations of y and u from the very beginning of the experiment in one run. That is:

$$y^I(t) = \int_0^t y(\tau)\ d\tau\ ;\quad u^I(t) = \int_0^t u(\tau)\ d\tau \tag{16}$$

For some $t > T$ when y in an ATV test starts to oscillate with constant period and amplitude, y^I and u^I will have similar cycling responses. Let y^I_{av} and u^I_{av} designate the average heights of constant cycles of y^I and u^I, respectively. The value of k_p can be estimated as:

$$k_p = \frac{y^I_{av}}{u^I_{av}} \tag{17}$$

On the other hand, due to the use of describing function for estimation, the ultimate gain computed from

Eq.(1) is subjected to error, which may, sometimes, be as high as 20%. For tuning purpose, estimation of the ultimate gain k_{cu} with improved accuracy is desirable. Since $u(t)$ and $y(t)$ are periodic with period P_u, they can be expanded into Fourier series. If the first harmonics are extracted, their coefficients give one point of process frequency response at ultimate frequency ω_u via the following equation (Wang *et al.*, 1997):

$$G_p(j\omega_u) = \frac{\int_{t_0}^{t_0+P_u} y(t)\ e^{-j\omega_u t}\ dt}{\int_{t_0}^{t_0+P_u} u(t)\ e^{-j\omega_u t}\ dt} \qquad (18)$$

where t_0 is taken as any time instant in a constant cycle. Then, the ultimate gain can be computed exactly as:

$$k_{cu} = \frac{1}{|G_p(j\omega_u)|} \qquad (19)$$

3.2 Estimation of apparent deadtime

The apparent deadtime is the deadtime appearing in an FOPDT or SOPDT model that approximates best the higher order process. As a result, this apparent deadtime differs, in general, from its true deadtime which is designated as θ^o and can be detected at the very beginning of the test. Features in the cycling response can be used to distinguish the FOPDT from SOPDT dynamics. For example, in case of FOPDT process, T_p equals θ^o or θ. The same equality does not apply to the SOPDT case. In an ATV test, two measured quantities, A_θ/A and θ/T_p, are used to characterize the effect of the apparent deadtime. These two quantities, as mentioned earlier, are functions of $\bar{\tau}$ and ζ. To explore their functional relations, simulations of ATV tests on the standard SOPDT processes covering wide range of $\bar{\tau}$ (in dimensionless form) and ζ are carried out. Results of A_θ/A and θ/T_p for underdamped SOPDT processes are plotted as a graph as shown in Fig. 2(a), using $\bar{\tau}$ and ζ as parameters. Each pair of the two measured quantities corresponding to a point in the graph, where a specific pair of values for $\bar{\tau}$ and ζ can be found. As shown in the figure, it is difficult to read $\bar{\tau}$ and ζ from the figure. In order to make each curve in Fig. 2(a) more readable, the coordinate is rotated through the following transformation.

$$\begin{aligned} X &= \frac{\theta}{T_p}\cos(\pi/3) - \frac{A_\theta}{A}\sin(\pi/3) \\ Y &= \frac{\theta}{T_p}\sin(\pi/3) + \frac{A_\theta}{A}\cos(\pi/3) \end{aligned} \qquad (20)$$

The results are plotted in Fig. 2(b). Moreover, for applying these data efficiently, two artificial neural networks (ANN) are constructed. The network architecture is as shown in Fig. 3. These two neural networks are fed with X and Y, and compute $\bar{\tau}$ and ζ, respectively. Each network consists of feedforward net with one input layer, one hidden layer, and one

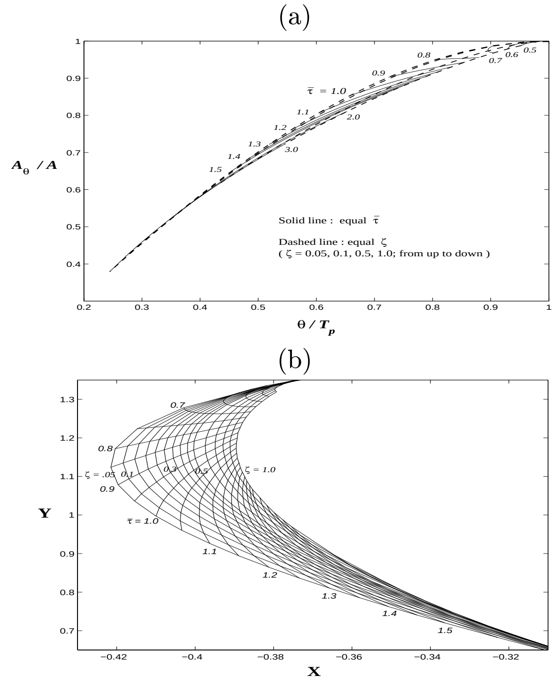

Fig. 2. Results of A_θ/A and θ/T_p in the ATV test for SOPDT processes (a) original (b) transformed

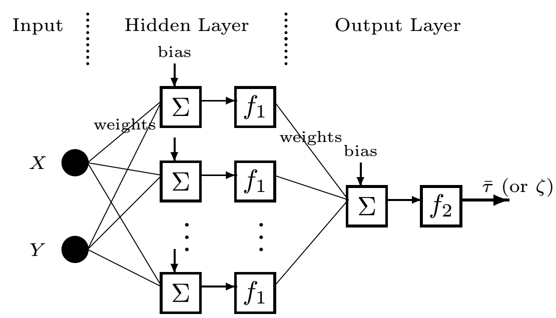

Fig. 3. Neural network architecture

output layer. The sigmoid function f is used in an error backpropagation technique to minimize the error between prediction and target values.

With these two networks, estimation of the apparent deadtime can be proceeded. The estimation makes uses of the phase criterion in Eq.(14). Notice that three unknowns are required to satisfy at least four functional relations (i.e. Eqs.(10)-(13) or Eqs.(12)-(15)), and iterative check is thus necessary to find the solution in a least-squares sense. By solving Eqs.(12)-(14), a unique solution for τ, ζ, and θ can be found. But, the resulting solution may not necessarily satisfy Eq.(15). The final solution needs further iterative procedures. To satisfy the extra Eq.(15), it is found that manifold in the space of τ and ζ results from the relation of the following:

$$\tan^{-1}\left(\frac{2\omega_u\tau\zeta}{1-{\omega_u}^2\tau^2}\right) = \pi - \omega_u\theta \qquad (21)$$

With ω_u and θ being fixed, there are many pairs of τ and ζ that satisfy Eq.(14), and one of these pairs would make Eq.(15) satisfied. However, in this proposed autotuning system, only the apparent deadtime

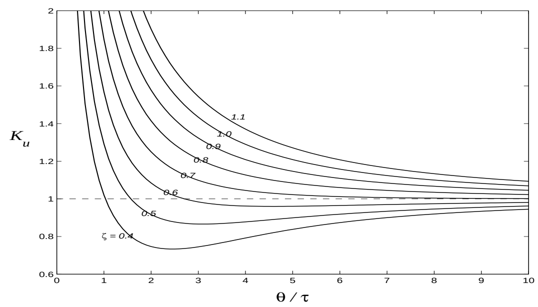

Fig. 4. K_u for different SOPDT processes

of the process is required, estimation of the exact values of τ and ζ can be skipped except a complete process model is desired.

With the theory presented above, the algorithm for the estimation of apparent deadtime can be made in a simpler way as the following:

1) Starting from a guessed value of θ, which is initially taken as θ^o.
2) The values of X and Y are calculated and fed into two networks. Then, parameters $\bar{\tau}$ and ζ are computed, and check if Eq.(14) holds. The procedure proceeds iteratively by increasing the guess value of θ until θ equals T_p.
3) When, at certain value of guessed θ, Eq.(14) holds true, the resulting guess value θ is taken as the estimated value of apparent deadtime.
4) If, until the guessed value of θ exceeds T_p and no candidate solution is found, the underdamped SOPDT model is not good for describing the dynamics of the process. As a result, model of FOPDT or overdamped SOPDT should be used.

3.3 Classification for controller tuning

In general, processes with SOPDT dynamics, which are overdamped or slightly underdamped, are sometimes identified with FOPDT models for controller design without significant difference in performance. It is thus curious to know under what condition can an SOPDT process has controller parameters in terms of an FOPDT parameterization. The result in Eq.(7) indicates that it is necessary with $K_u > 1$. As shown in Fig. 4, K_u of SOPDT processes is plotted against θ/τ using ζ as a parameter. It is found that $K_u > 1$ happens when $\zeta \geq 0.7$. Thus, the uses of Eq.(7) and of Eq.(9) for controller tuning are discriminated using $\zeta = 0.7$ as a boundary. As mentioned earlier, from the ATV test, it provides $A/(k_p h)$ and P_u/θ which are functions of $\bar{\tau}$ and ζ. As shown in Fig. 5, there are two curves that correspond to SOPDT processes with $\zeta = 0.7$ (curve A) and FOPDT processes (curve B). Curve A and curve B are found to be represented, respectively, by the following equations:

$$\Omega = 0.465\Lambda^3 - 2.002\Lambda^2 + 0.958\Lambda - 0.007 \quad (22)$$

$$\Omega = 30.93\Lambda^3 - 56.78\Lambda^2 + 29.03\Lambda - 4.51 \quad (23)$$

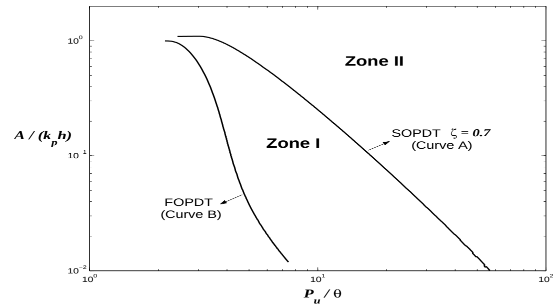

Fig. 5. Curves for process classification

where $\Omega = \log(A/k_p h)$ and $\Lambda = \log(P_u/\theta)$. These two curves divide the graph of Fig. 5 into two zones. One is between the curve A and curve B (i.e. Zone I) that represents the feasible region for using parameterization of FOPDT due to $\zeta \geq 0.7$, and the other region above curve A (i.e. Zone II) that represents the feasible region to use parameterization of underdamped SOPDT. Thus, once k_p and the apparent deadtime θ are obtained, the normalized value of $A/(k_p h)$ and P_u/θ can be calculated. Then, compare the resulting value of $\log(A/k_p h)$ with the one calculated from Eq.(22). If the former is smaller, then this process is classified into the group (*Group I*) where Eq.(7) applies to tune PI/PID controllers. Otherwise, the process is classified into the other group (*Group II*) where Eq.(9) will be used for tuning PID controllers.

4. AUTOTUNING PROCEDURES WITH ILLUSTRATED EXAMPLES

To perform the autotuning, the relay feedback of the ATV test is initialized by a short period of manual disturbance. After the response has constant cycling, A and P_u are recorded, and the following procedure steps are taken:

1) Compute k_p from Eq.(17) and k_{cu} from Eq.(19).
2) Estimate the apparent deadtime θ.
3) Having the values of k_p and θ, the process is classified by comparing the value of $A/(k_p h)$ from ATV test with the one calculated by Eq.(22).
4) If the process belongs to *Group I*, Eq.(7) is used for tuning PI or PID controllers. Otherwise, Eq.(9) is used to tune a PID controller.

Notice that, in case of significantly overdamped dynamics, the trajectory of the $(\theta/T_p, A_\theta/A)$ will be crowded at the lower edge of the graph in Fig. 2(a). In that case, the process can be classified into *Group I* directly without estimating the apparent deadtime.

For practical application, the issue to be considered is the measurement noise. When the measured signals are blurred with noise, data used in the proposed method are taken as the average values of measurements from several constant cycles to ensure the success of this autotuning.

In order to illustrate the above autotuning procedures, a few examples are used for simulation. The results of applying proposed autotuning method are

Table 1. Results of autotuning for simulated processes

Example	Process	ATV test		Estimated parameters			Classification	Controller tuning		
		A	P_u	k_p	k_{cu}	θ		k_c'	τ_R'	τ_D'
1	$\frac{e^{-1.5s}}{(s+1)^5}$	0.72	12.14	1.01	1.81	2.91	*Group I*	0.46	2.96	1.66
2	$\frac{(0.5s+1)\ e^{-s}}{(s+1)^2(2s+1)}$	0.35	6.56	1.0	3.74	1.32	*Group I*	1.29	3.78	0.77
								k_c	τ_R	τ_D
3	$\frac{e^{-2s}}{(9s^2+2.4s+1)(s+1)}$	1.17	15.90	1.0	1.11	2.72	*Group II*	0.45	2.47	3.96

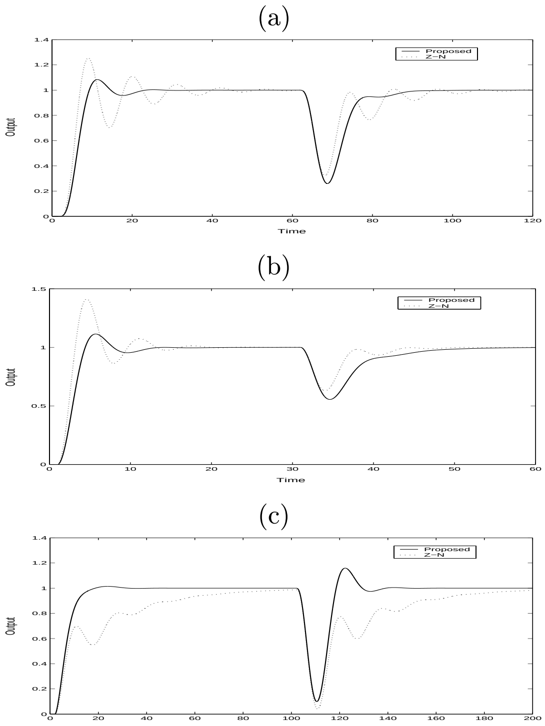

Fig. 6. Control responses of (a) Example 1 (b) Example 2 (c) Example 3

summarized in Table 1. The control results using proposed model-based method and Z-N method are shown in Fig. 6. Notice that the Z-N settings are obtained from the same set of experiment data. These results show that the proposed method can give more satisfactory control performance than conventional autotuning system.

5. CONCLUSIONS

In this paper, a systematic procedure is used to perform the autotuning of PI/PID controllers using one ATV experiment. The usage of PI or PID controller depends on the control application and on the dynamic characteristics of the process. The autotuning system considered the effective damping factor for classifying the process into one of two groups. In either case, besides parameters k_p and θ, the tuning formula are given in terms of ultimate gain and ultimate frequency obtained from a constant cycle. These parameters can be estimated within one ATV experiment. Two ANNs are constructed to enhance the estimation of apparent deadtime. The simulation results have shown that this proposed autotuning system is efficient and self-contained.

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