SUBSPACE IDENTIFICATION USING THE PARITY SPACE

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Abstract: It is known that most subspace identification algorithms give biased estimates for closed-loop data due to a projection performed in the algorithms. In this work, consistency analysis of SIMPCA is given and the exact input requirement is formulated. The effect of column weighting in subspace identification algorithms is discussed and the column weighting for SIMPCA is designed which gives consistent estimates of state-space models from both open loop and closed-loop data. A simulation example is given to demonstrate the performance of the proposed algorithm.

Keywords: subspace identification, instrumental variables, closed-loop identification, principal component analysis, consistency analysis, parity space

1. INTRODUCTION

Subspace identification algorithms have been developed in the past two decades. Based on numerically robust singular value decomposition (SVD), these algorithms are simple and completely bypass the need for the estimation of structure index, which causes no additional difficulty when handling MIMO systems (Gevers, 2003). The asymptotic properties of these subspace algorithms also have been investigated in the past decade and consistency conditions of the estimates have been identified (Deistler *et al.*, 1995; Peternell *et al.*, 1996; Jansson and Wahlberg, 1998; Bauer *et al.*, 1999; Knudsen, 2001).

Because most subspace identification algorithms perform a projection of the future output onto the orthogonal complement of future input, which requires future input to be uncorrelated to past noise, the application of these algorithms to closed-loop data typically gives biased estimates even though the data satisfy identifiability conditions for prediction error methods. To address this aspect, algorithms for closed-loop identification have been developed in the last a few years (Chou and Verhaegen, 1997; Ljung and McKelvey, 1996; Forssell and Ljung, 1999; Wang and Qin, 2002; Huang *et al.*, 2003).

The aim of this paper is to present a modification of our recently developed algorithm — SIMPCA (subspace identification method via principal component analysis), which gives consistent estimates of state-space models from both open loop and closed-loop data, under some input excitation conditions. The remaining part of the paper is organized as follows. Section 2 gives the problem formulation and assumptions. Section 3 reviews the original SIMPCA algorithm. The exact input requirement and consistency analysis are

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given in this section. Section 4 presents the modified algorithm, SIMPCA with column weighting (SIMPCA-Wc), and discusses the effect of column weighting. Section 5 gives the simulation example. The final section gives conclusions to the paper.

2. PROBLEM FORMULATION AND ASSUMPTIONS

The original SIMPCA algorithm (Wang and Qin, 2002) was developed to address errors-in-variables case, and it is applicable to innovation formulation. Consider the linear time-invariant system in its innovation representation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{K}\mathbf{e}(k) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{e}(k) \quad (2)$$

Here $\mathbf{x}(k) \in \Re^n$ is the state vector, $\mathbf{u}(k) \in \Re^l$ and $\mathbf{y}(k) \in \Re^m$ are the measured input and output signals. $\mathbf{e}(k) \in \Re^m$ is the innovation process.

2.1 Assumptions

We introduce the following assumptions:

A1: The system is asymptotically stable.

A2: $(\mathbf{A}, \ \ \mathbf{C})$ is observable.

A3: $(\mathbf{A}, \ \ [\mathbf{B} \ \ \mathbf{K}])$ is controllable.

A4: The input $\mathbf{u}$ and innovation $\mathbf{e}$ are jointly stationary and one-way uncorrelated, i.e.,

$$\bar{\mathbf{E}}[\mathbf{e}(k)\mathbf{e}(l)^T] = \mathbf{R}_e \delta_{kl} \quad (3)$$

$$\bar{\mathbf{E}}[\mathbf{e}(k)\mathbf{u}(l)^T] = \mathbf{0}, \quad k > l \quad (4)$$

where $\bar{\mathbf{E}}$ is defined as (Ljung, 1999)

$$\bar{\mathbf{E}}\{\bullet\} = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \mathbf{E}\{\bullet\} \quad (5)$$

Eqn (4) allows us to include closed-loop identification in the proposed method. In order to get consistent identification results, a persistent excitation condition on the input is introduced later.

2.2 Preliminaries

An extended state-space model can be formulated,

$$\mathbf{Y}_f = \mathbf{\Gamma}_f \mathbf{X}_k + \mathbf{H}_f \mathbf{U}_f + \mathbf{G}_f \mathbf{E}_f \quad (6)$$

$$\mathbf{Y}_p = \mathbf{\Gamma}_p \mathbf{X}_{k-p} + \mathbf{H}_p \mathbf{U}_p + \mathbf{G}_p \mathbf{E}_p \quad (7)$$

where $\mathbf{\Gamma}_f$ is the extended observability matrix with rank n, $\mathbf{H}_f$ and $\mathbf{G}_f$ are two triangular Toeplitz matrices as follows,

$$\mathbf{\Gamma}_f = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{f-1} \end{bmatrix} \in \Re^{mf\times n} \quad (8)$$

$$\mathbf{H}_f = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{f-2}\mathbf{B} & \mathbf{CA}^{f-3}\mathbf{B} & \cdots & \mathbf{D} \end{bmatrix} \in \Re^{mf\times lf} \quad (9)$$

$$\mathbf{G}_f = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CK} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{f-2}\mathbf{K} & \mathbf{CA}^{f-3}\mathbf{K} & \cdots & \mathbf{I} \end{bmatrix} \in \Re^{mf\times mf} \quad (10)$$

The future and past data are arranged in the following Hankel form:

$$\mathbf{Y}_f = \begin{bmatrix} \mathbf{y}(k) & \mathbf{y}(k+1) & \cdots & \mathbf{y}(k+N-1) \\ \mathbf{y}(k+1) & \mathbf{y}(k+2) & \cdots & \mathbf{y}(k+N) \\ \mathbf{y}(k+f-1) & \mathbf{y}(k+f) & \cdots & \mathbf{y}(k+f+N-2) \end{bmatrix}$$
$$\equiv \begin{bmatrix} \mathbf{y}_f(k) & \mathbf{y}_f(k+1) & \cdots & \mathbf{y}_f(k+N-1) \end{bmatrix} \in \Re^{mf\times N}$$

$$\mathbf{Y}_p = \begin{bmatrix} \mathbf{y}(k-p) & \mathbf{y}(k-p+1) & \cdots & \mathbf{y}(k-p+N-1) \\ \mathbf{y}(k-p+1) & \mathbf{y}(k-p+2) & \cdots & \mathbf{y}(k-p+N) \\ \mathbf{y}(k-1) & \mathbf{y}(k) & \cdots & \mathbf{y}(k+N-2) \end{bmatrix}$$
$$\equiv \begin{bmatrix} \mathbf{y}_p(k) & \mathbf{y}_p f(k+1) & \cdots & \mathbf{y}_p(k+N-1) \end{bmatrix} \in \Re^{mp\times N}$$

$\mathbf{U}_f$, $\mathbf{U}_p$, $\mathbf{E}_f$ and $\mathbf{E}_p$ are formulated similarly.

Eqn (6) is the basis for many subspace identification methods (SIM).

3. THE ORIGINAL SIMPCA ALGORITHM

SIMPCA includes two steps: i) identifying the extended observability matrix $\mathbf{\Gamma}_f$ and the block triangular Toeplitz matrix $\mathbf{H}_f$; ii) estimating system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$.

We begin with the extended state-space model of Eqn (6). Instead of focusing on the observable subspace, SIMPCA algorithm focuses on the parity subspace, which is commonly used in fault detection (Chow and Willsky, 1984). Pre-multiplying Eqn (6) with $(\mathbf{\Gamma}_f^{\perp})^T$, the orthogonal complement of $\mathbf{\Gamma}_f$ with *full* column rank, and moving the input term to the left hand side, Eqn (6) becomes

$$\left(\mathbf{\Gamma}_f^{\perp}\right)^T [\mathbf{I} \mid -\mathbf{H}_f]\, \mathbf{Z}_f = \left(\mathbf{\Gamma}_f^{\perp}\right)^T \mathbf{G}_f \mathbf{E}_f \quad (11)$$

where $\mathbf{Z}_f \equiv \begin{bmatrix} \mathbf{Y}_f \\ \mathbf{U}_f \end{bmatrix}$.

In order to achieve the consistent estimates of $\mathbf{\Gamma}_f$ and $\mathbf{H}_f$ under the noise-corrupted condition, we use instrumental variables to remove the noise. An important property of the instrumental variables is that they should be uncorrelated with the noise, and sufficiently correlated with the informative part of the data. Other work (Verhaegen and Dewilde, 1992; Verhaegen, 1994) shows that past input and past output together is a good choice for instrumental variables. It is easy to see that

$$\lim_{N\to\infty}\frac{1}{N}\mathbf{E}_f\mathbf{Z}_p^T = \lim_{N\to\infty}\frac{1}{N}\mathbf{E}_f[\mathbf{Y}_p^T \quad \mathbf{U}_p^T] = \mathbf{0} \quad (12)$$

because of the Assumption A4.

In order to obtain consistent estimation under the noise-corrupted condition, the input excitation condition is needed, which is stated in the following lemma.

Lemma 1. Given general Assumptions A1 $\sim$ A4 and $p \geq f > n$, $\frac{1}{N}\mathbf{Z}_f\mathbf{Z}_p^T$ has $mf - n$ zero singular values when $N \to \infty$, if $\bar{\mathbf{E}}\left\{\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{u}_f(k)\end{bmatrix}\mathbf{z}_p(k)^T\right\}$ has full row rank.

Proof. See Appendix A.

A similar condition ($\bar{\mathbf{E}}\left\{\begin{bmatrix}\mathbf{u}_f(k)\\ \mathbf{x}(k)\end{bmatrix}\begin{bmatrix}\mathbf{u}_f(k)\\ \mathbf{z}_p(k)\end{bmatrix}^T\right\}$ has full row rank) was derived in Viberg *et al.* (1997) for the IV-4SID method.

Remark 1 Although regarded as fully excited for other subspace identification methods, white noise input is not a suitable input for SIMPCA. If the input is white noise, $\lim_{N\to\infty}\frac{1}{N}\mathbf{U}_f\mathbf{Z}_p^T = 0$ and the input excitation condition is not satisfied for SIMPCA. Similarly, for closed-loop identification, the input perturbation needs to be correlated in order to meet the input excitation condition.

Based on lemma 1, we formulate the following theorem for the combined determinate-stochastic realization.

Theorem 1. Given the general assumptions A1 $\sim$ A4, the persistent excitation condition in Lemma 1, and the following PCA decomposition,

$$\frac{1}{N}\mathbf{Z}_f\mathbf{Z}_p^T = \mathbf{P}\mathbf{T}^T + \tilde{\mathbf{P}}\tilde{\mathbf{T}}^T \quad (13)$$

when $N \to \infty$, we have

$$\begin{bmatrix}\mathbf{\Gamma}_f^{\perp}\\ -\mathbf{H}_f^T\mathbf{\Gamma}_f^{\perp}\end{bmatrix} = \tilde{\mathbf{P}}\mathbf{M} \quad (14)$$

where $\mathbf{M} \in \Re^{(mf-n)\times(mf-n)}$ is a non-singular matrix.

Proof. See Appendix B.

Remark 2 Because projecting out the future input is avoided in SIMPCA, we only require that the future noise is independent of past input, which is true for closed-loop operation. Therefore, SIMPCA is applicable to closed-loop identification provided that the input excitation condition is satisfied.

Remark 3 SIMPCA is applicable to errors-in-variables situation, which has been discussed in Wang and Qin (2002). Besides, SIMPCA is applicable to the colored noise that is finitely correlated. We only need to separate $\mathbf{Z}_f$ and $\mathbf{Z}_p$ farther beyond the correlation window. Colored noise is also treated in Stoica *et al.* (1995) and Li and Qin (2001).

After the parity space is identified, $\mathbf{\Gamma}_f$, $\mathbf{H}_f$ and matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ can be estimated following Wang and Qin (2002).

4. SIMPCA WITH COLUMN WEIGHTING

Like other subspace-based system identification methods, SIMPCA also obtains an estimate of the extended observability matrix first, that is, the column space of the extended observability matrix. It is thus of great interest to optimize the estimate of the observability matrix. One special direction is to apply certain weighting matrix to improve the estimate. Several contributions in the literature have appeared in this area (Jansson and Wahlberg, 1996; Viberg *et al.*, 1997; Gustafsson, 2002). Although different approaches of analysis were adopted, it is interesting to note that the column weightings in Jansson and Wahlberg (1996) and Verhaegen (1994) all correspond to the CVA weighting of Larimore (1983).

In Gustafsson (2002), an asymptotic analysis of the estimated observability matrix is presented, and an optimal column weighting matrix $\mathbf{W}_c^o$ is derived. It is also interesting to note that an approximation of $\mathbf{W}_c^o$ given in Gustafsson (2002) is the same as the column weighting employed by CVA and MOESP, as pointed out in Viberg *et al.* (1997).

Define $\mathbf{R}_{\mathbf{xy}}(\tau)$ as the cross-covariance matrix of $\mathbf{x}$ and $\mathbf{y}$

$$\mathbf{R}_{\mathbf{xy}}(\tau) = \bar{\mathbf{E}}\{\mathbf{x}(t+\tau)\mathbf{y}^T(t)\} \quad (15)$$

and let $\mathbf{R}_{\mathbf{xy}} = \mathbf{R}_{\mathbf{xy}}(0)$, the approximate optimal weighting in Gustufsson (2002) is

$$\begin{aligned}\mathbf{W}_c^a &= (\mathbf{R}_{\mathbf{z}_p\mathbf{z}_p} - \mathbf{R}_{\mathbf{u}_f\mathbf{z}_p}^T\mathbf{R}_{\mathbf{u}_f\mathbf{u}_f}^{-1}\mathbf{R}_{\mathbf{u}_f\mathbf{z}_p})^{-\frac{1}{2}}\\ &= (\frac{1}{N}\mathbf{Z}_p\mathbf{\Pi}_{\mathbf{U}_f}^{\perp}\mathbf{Z}_p^T)^{-\frac{1}{2}} \quad (16)\end{aligned}$$

where $\mathbf{\Pi}_{\mathbf{U}_f}^{\perp} = \mathbf{I} - \mathbf{U}_f^T(\mathbf{U}_f\mathbf{U}_f^T)^{-1}\mathbf{U}_f$.

In the framework studied in Gustafsson (2002), an instrumental variable approach reduces Eqn (6) to the following relation,

$$\lim_{N\to\infty}\frac{1}{N}\mathbf{Y}_f\mathbf{\Pi}_{\mathbf{U}_f}^{\perp}\mathbf{Z}_p^T = \lim_{N\to\infty}\frac{1}{N}\mathbf{\Gamma}_f\mathbf{X}_k\mathbf{\Pi}_{\mathbf{U}_f}^{\perp}\mathbf{Z}_p^T \quad (17)$$

Post-multiplying the column weighting in Eqn (16) leads to:

$$\lim_{N\to\infty}\frac{1}{N}\mathbf{Y}_f\mathbf{\Pi}_{\mathbf{U}_f}^{\perp}\mathbf{Z}_p^T(\mathbf{Z}_p\mathbf{\Pi}_{\mathbf{U}_f}^{\perp}\mathbf{Z}_p^T)^{-\frac{1}{2}}$$

$$= \lim_{N\to\infty} \frac{1}{N} \mathbf{\Gamma}_f \mathbf{X}_k \mathbf{\Pi}^{\perp}_{\mathbf{U}_f} \mathbf{Z}_p^T (\mathbf{Z}_p \mathbf{\Pi}^{\perp}_{\mathbf{U}_f} \mathbf{Z}_p^T)^{-\frac{1}{2}} \quad (18)$$

Denoting $\mathbf{Z}_{\mathbf{U}^\perp} \equiv \mathbf{Z}_p \mathbf{\Pi}^{\perp}_{\mathbf{U}_f}$ as the generalized instrumental variables, the left hand side of the above equation reduces to

$$\mathbf{Y}_f \mathbf{Z}^T_{\mathbf{U}^\perp} (\mathbf{Z}_{\mathbf{U}^\perp} \mathbf{Z}^T_{\mathbf{U}^\perp})^{-\frac{1}{2}} = \mathbf{Y}_f \mathbf{Z}^T_{\mathbf{U}^\perp} \mathbf{W}^a_c \quad (19)$$

Therefore, the approximate optimal column weighting can be interpreted as scaling the generalized instrumental variable to unit variance since

$$(\mathbf{Z}^T_{\mathbf{U}^\perp} \mathbf{W}^a_c)^T (\mathbf{Z}^T_{\mathbf{U}^\perp} \mathbf{W}^a_c) = \mathbf{W}^a_c \mathbf{Z}_{\mathbf{U}^\perp} \mathbf{Z}^T_{\mathbf{U}^\perp} \mathbf{W}^a_c = \mathbf{I} \quad (20)$$

Because SIMPCA also picks the subspace by performing SVD (PCA) on process data and instrumental variables are applied to remove noise, it would be beneficial to scale the instrumental variables to unit variance. In SIMPCA, since projecting out of future input is not needed, the generalized instrumental variables is just past data $\mathbf{Z}_p$. In this case, $\mathbf{W}^a_c$ is simply $\left(\mathbf{Z}_p \mathbf{Z}_p^T\right)^{-\frac{1}{2}}$, which is the same weighting applied in IV-4SID (Viberg, 1995).

In summary, SIMPCA with column weighting (SIMPCA-Wc) is based on the following equation,

$$\frac{1}{N} \left(\mathbf{\Gamma}^{\perp}_f\right)^T [\mathbf{I} \mid -\mathbf{H}_f]\, \mathbf{Z}_f \mathbf{Z}_p^T \left(\mathbf{Z}_p \mathbf{Z}_p^T\right)^{-\frac{1}{2}}$$
$$= \frac{1}{N} \left(\mathbf{\Gamma}^{\perp}_f\right)^T \mathbf{G}_f \mathbf{E}_f \mathbf{Z}_p^T \left(\mathbf{Z}_p \mathbf{Z}_p^T\right)^{-\frac{1}{2}} \quad (21)$$

The right hand side of Eqn (21) goes to zero as $N \to \infty$ due to Assumption A4. Therefore, performing PCA on $\mathbf{Z}_f \mathbf{Z}_p^T \left(\mathbf{Z}_p \mathbf{Z}_p^T\right)^{-\frac{1}{2}}$ can extract the residual subspace which contains the system information. The remaining steps in SIMPCA-Wc to estimate $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are the same as in SIMPCA. Simulation results show that the column weighting matrix $\left(\mathbf{Z}_p \mathbf{Z}_p^T\right)^{-\frac{1}{2}}$ can significantly improve the variance of estimates compared to SIMPCA, as shown in the next section. It is interesting that applying the CVA column weighting can achieve the similar performance as the SIMPCA column weighting.

In Huang et al. (2003) a subspace orthogonal projection identification method (SOPIM) for closed-loop identification is proposed based on the SIMPCA structure. In the proposed SOPIM, the parity space is extracted through an orthogonal projection of the future input and output on the row space of past input and output, i.e.,

$$\left(\mathbf{\Gamma}^{\perp}_f\right)^T \left[\, \mathbf{I} \;\; -\mathbf{H}_f \,\right] \mathbf{Z}_f \mathbf{\Pi}_{\mathbf{Z}_p} = \left(\mathbf{\Gamma}^{\perp}_f\right)^T \mathbf{G}_f \mathbf{E}_f \mathbf{\Pi}_{\mathbf{Z}_p} \quad (22)$$

where the right hand side goes to zero when the number of observations goes to infinity. It is interesting to notice that the projection of future data on the row space of past data is

$$\mathbf{Z}_f \mathbf{\Pi}_{\mathbf{Z}_p} = \mathbf{Z}_f \mathbf{Z}_p^T (\mathbf{Z}_p \mathbf{Z}_p^T)^{-1} \mathbf{Z}_p \quad (23)$$

which is closely related to SIMPCA-Wc, as shown in the following,

$$\mathbf{Z}_f \mathbf{Z}_p^T \mathbf{W}_c = \mathbf{Z}_f \mathbf{Z}_p^T (\mathbf{Z}_p \mathbf{Z}_p^T)^{-\frac{1}{2}} \quad (24)$$

it is clear that $\mathbf{\Pi}_{\mathbf{Z}_p} = \left(\mathbf{Z}_p^T \mathbf{W}_c\right) \left(\mathbf{Z}_p^T \mathbf{W}_c\right)^T$ also has unit variance.

5. SIMULATION STUDY

In this section a simulation study is presented to demonstrate the performance of SIMPCA algorithm. Results from SIMPCA, N4SID with CVA weighting in the Matlab System Identification Toolbox (version 5.0) (CVA via N4SID), SIMPCA with column weighting (SIMPCA-Wc) and MOESP-PO are presented for comparison. The simulation example is a first order SISO system, where both open-loop and closed-loop operations are examined. The system order is given.

The system under test is given by the following difference equation:

$$y(k) - 0.9y(k-1) = u(k-1) + e(k) + 0.9e(k-1) \quad (25)$$

The input to the system has the following structure

$$u(k) = -\lambda y(k) + r(k) \quad (26)$$

where λ can be set to different values to achieve both open loop and closed-loop operations. The open loop input excitation signal $\mathbf{r}(k)$ is a moving average process:

$$r(k) = (1 + 0.8\, q^{-1} + 0.6\, q^{-2}) r_0(k) \quad (27)$$

where $r_0(k)$ is zero mean white noise sequence with unit variance.

5.1 Open-loop case

In this case $\lambda = 0$. Process noise $\mathbf{e}(k)$ is added to the system, where $\mathbf{e}(k)$ is zero mean white noise sequence with standard deviation $\sigma_e = 1.2$. Monte-Carlo experiments are conducted and 50 runs are performed. For each run, 7000 data points are collected for identification. Fig. 1 shows the Bode plot of the identified system from different algorithms. In order to compare the relative efficiency of different methods, the asymptotical performance is shown in Fig. 2, where the mean squared error of the pole estimation given different number of samples is plotted. It shows that SIMPCA-Wc performs similarly to CVA(via N4SID) and MOESP, while SIMPCA is worse than the other three methods.

5.2 Closed-loop case

In this case $\lambda = -0.6$. Fig. 3 shows the Bode plot under combined deterministic-stochastic case where the number of data point is 10^5. Apparently CVA(via N4SID) and MOESP give biased estimates, while estimates from SIMPCA and SIMPCA-Wc are unbiased. We also plot the asymptotic performance of different methods under closed-loop in Fig. 4, which confirms that SIMPCA and SIMPCA-Wc give unbiased estimates. Compared to more efficient SIMPCA-Wc, SIMPCA requires more data to achieve same level of estimate variance.

As we discussed in Remark 1, white input perturbation is not ideal for SIMPCA because it does not satisfy the input excitation condition. Huang et al. (2003) point out that if the input perturbation is white noise, the controller dynamic will also fall into the left null space of $\mathbf{Z}_f \mathbf{Z}_p^T$, which will make the estimate from SIMPCA biased. A remedy is also proposed in Huang et al. (2003). By including the input perturbation as part of the instrumental variable, it is guaranteed that the left null space of $\mathbf{Z}_f \mathbf{Z}_p^T$ contains only the process dynamics.

6. CONCLUSION

In this paper, consistency analysis of SIMPCA with focus on the deterministic part is given. Because SIMPCA makes use of parity space to estimate system model and avoids projecting out the future input, it is applicable to closed-loop data, provided that the input excitation condition is satisfied. A modification of original SIMPCA (SIMPCA-Wc) is presented. By scaling the instrumental variables to unit variance through column weighting, SIMPCA-Wc can significantly improve the estimate efficiency. A simulation example shows that the designed column weighting can significantly reduce the variance of the estimates.

REFERENCES

Bauer, D., M. Deistler and W. Scherrer (1999). Consistency and asymptotic normality of some subspace algorithms for systems without observed inputs. *Automatica* **35**, 1243–1254.

Chou, C.T. and M. Verhaegen (1997). Subspace algorithms for the identification of multivariable dynamic errors-in-variables models. *Automatica* **33**(10), 1857–1869.

Chow, E.Y. and A.S. Willsky (1984). Analytical redundancy and the design of robust failure detection systems. *IEEE Trans. Auto. Cont.* **AC-29**(7), 603–614.

Deistler, M., K. Peternell and Scherrer (1995). Consistency and relative efficiency of subspace methods. *Automatica* **31**, 1865–1875.

Forssell, U. and L. Ljung (1999). Closed-loop identification revisited. *Automatica* **35**, 1215–1241.

Gevers, M. (2003). A personal view on the development of system identification. In: *Proceedings of 13th IFAC symposium on System Identification*. Rotterdam, Netherlands. pp. 773–784.

Gustafsson, T. (2002). Suspace-based system identification: weighting and pre-filtering of instruments. *Automatica* **38**, 433–443.

Huang, B., S. Ding and S.J. Qin (2003). Closed-loop subspace identificaiton: an orthogonal projection approach. Submitted to Journal of Process Control.

Jansson, M. and B. Wahlberg (1996). A linear regression approach to state-space subspace system identification. *Signal Processing* **52**, 103–129.

Jansson, M. and B. Wahlberg (1998). On consistency of subspace methods for system identification. *Automatica* **34**(12), 1507–1519.

Knudsen, T. (2001). Consistency analysis of subspace idenfification methods based on a linear regression approach. *Automatica* **37**, 81–89.

Larimore, W.E. (1983). System identification, reduced-order filtering and modeling via canonical variate analysis. In: *Proc. 1983 American Control Conf., H.S. Rao and T. Dorato (Ed), IEEE*. New York.

Li, W. and S.J. Qin (2001). Consistent dynamic PCA based on errors-in-variables subspace identification. *J. Process Control* **11(6)**, 661–678.

Ljung, L. (1999). *System Identification: Theory for the User*. Prentice-Hall, Inc.. Englewood Cliffs, New Jersey.

Ljung, L. and T. McKelvey (1996). Subspace identification from closed loop data. *Signal Processing* **52**, 209–215.

Peternell, K., W. Scherrer and M. Deistler (1996). Statistical analysis of novel subspace identification methods. *Signal Processing* **52**, 161–177.

Stoica, P., M. Cedervall and A. Eriksson (1995). Combined instrumental variable and subspace fitting approach to parameter estimation of noisy input-output systems. *IEEE Transactions on Signal Processing* **43(10)**, 2386–2397.

Verhaegen, M. (1994). Identification of the deterministic part of MIMO state space models given in innovations form from input-output data. *Automatica* **30**, 61–74.

Verhaegen, M. and P. Dewilde (1992). Subspace model identification. part i: the output-error state-space model identification class of algorithms. *International Journal of Control* **56**, 1187–1210.

Viberg, M. (1995). Subspaced-based methods for the identification of linear time-invariant systems. *Automatica* **31(12)**, 1835–1851.

Viberg, M., B. Wahlberg and B. Ottersten (1997). Analysis of state space system identification methods based on instrumental variables and subspace fitting. *Automatica* **33**, 1603–1616.

Wang, J. and S.J. Qin (2002). A new subspace identification approach based on principal component analysis. *Journal of Process Control* **12(8)**, 841–855.

Appendix A. PROOF OF LEMMA 1

From the extended state-space model Eqn (6), we have

$$\mathbf{Z}_f = \begin{bmatrix} \mathbf{Y}_f \\ \mathbf{U}_f \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_f & \mathbf{H}_f \\ \mathbf{0} & \mathbf{I}_{lf} \end{bmatrix} \begin{bmatrix} \mathbf{X}_k \\ \mathbf{U}_f \end{bmatrix} + \begin{bmatrix} \mathbf{G}_f \\ \mathbf{0} \end{bmatrix} \mathbf{E}_f \tag{A.1}$$

Post-multiply the instrumental variable $\mathbf{Z}_p^T$ to remove noise,

$$\lim_{N\to\infty} \frac{1}{N} \mathbf{Z}_f \mathbf{Z}_p^T = \begin{bmatrix} \mathbf{\Gamma}_f & \mathbf{H}_f \\ \mathbf{0} & \mathbf{I}_{lf} \end{bmatrix} \begin{bmatrix} \mathbf{X}_k \\ \mathbf{U}_f \end{bmatrix} \mathbf{Z}_p^T \tag{A.2}$$

Because $f > n$, $rank(\mathbf{\Gamma}_f) = n$, and $\begin{bmatrix} \mathbf{\Gamma}_f & \mathbf{H}_f \\ \mathbf{0} & \mathbf{I}_{lf} \end{bmatrix}$ has full column rank, which is $lf + n$. Therefore, if $\bar{\mathbf{E}}\left(\begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}_f(k) \end{bmatrix} \mathbf{z}_p(k)^T \right)$ has full row rank, $rank\left(\frac{1}{N}\mathbf{Z}_f\mathbf{Z}_p^T\right) = lf + n$ as $N \to \infty$.

Appendix B. PROOF OF THEOREM 1

Eqn (6) can be rearranged as

$$[\,\mathbf{I} \mid -\mathbf{H}_f\,]\,\mathbf{Z}_f = \mathbf{\Gamma}_f \mathbf{X}_k + \mathbf{G}_f \mathbf{E}_f \tag{B.1}$$

In order to remove the noise term, multiply $\frac{1}{N}\mathbf{Z}_p^T$ to both sides of the above equation,

$$\frac{1}{N}[\,\mathbf{I} \mid -\mathbf{H}_f\,]\,\mathbf{Z}_f\mathbf{Z}_p^T = \frac{1}{N}\mathbf{\Gamma}_f \mathbf{X}_k\mathbf{Z}_p^T + \frac{1}{N}\mathbf{G}_f \mathbf{E}_f\mathbf{Z}_p^T \tag{B.2}$$

while

$$\lim_{N\to\infty} \frac{1}{N}\mathbf{E}_f\mathbf{Z}_p^T = 0 \tag{B.3}$$

when $N \to \infty$

$$\frac{1}{N}[\,\mathbf{I} \mid -\mathbf{H}_f\,]\,\mathbf{Z}_f\mathbf{Z}_p^T = \frac{1}{N}\mathbf{\Gamma}_f \mathbf{X}_k\mathbf{Z}_p^T \tag{B.4}$$

pre-multiply the above equation with $\left(\mathbf{\Gamma}_f^{\perp}\right)^T$ to remove state,

$$\frac{1}{N}\left(\mathbf{\Gamma}_f^{\perp}\right)^T [\,\mathbf{I} \mid -\mathbf{H}_f\,]\,\mathbf{Z}_f\mathbf{Z}_p^T = 0 \tag{B.5}$$

Following the same procedure as in proof of theorem 1, it is straight forward to show that

$$\begin{bmatrix} \mathbf{\Gamma}_f^{\perp} \\ -\mathbf{H}_f^T\mathbf{\Gamma}_f^{\perp} \end{bmatrix} = \tilde{\mathbf{P}}\mathbf{M} \tag{B.6}$$

where $\mathbf{M} \in \Re^{(mf-n)\times(mf-n)}$ is a non-singular matrix.

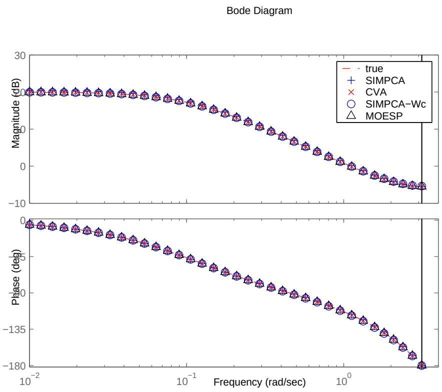

Fig. 1. Bode plot of the identified system for the open loop case

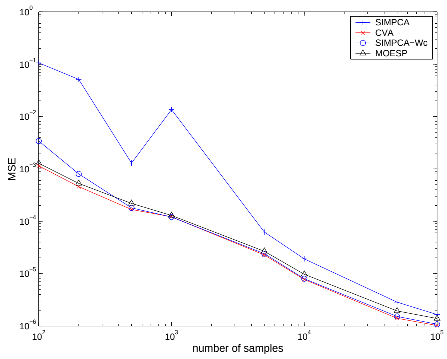

Fig. 2. Mean squared error of pole estimate versus number of samples for the open loop case

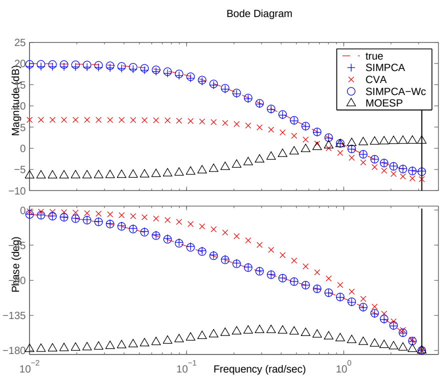

Fig. 3. Bode plot of the identified system for the closed-loop case

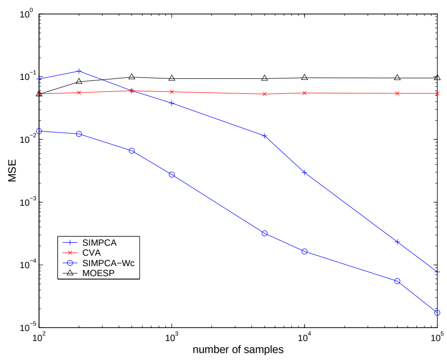

Fig. 4. Mean squared error of pole estimate versus number of samples for the closed-loop case