Design of Robust Controller for the Active Noise Control Systems

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Abstract- *This paper presents a robust controller design for the active noise control system using feed forward strategy. Robust designs for uncertain systems assures robust stability and robust performance considering uncertainty. There are several approaches for robust controller design like* H_∞, μ *,…*
In this paper our design is based on μ *synthesis. Simulation results shows that robust controller excels ordinary adaptive controller.*

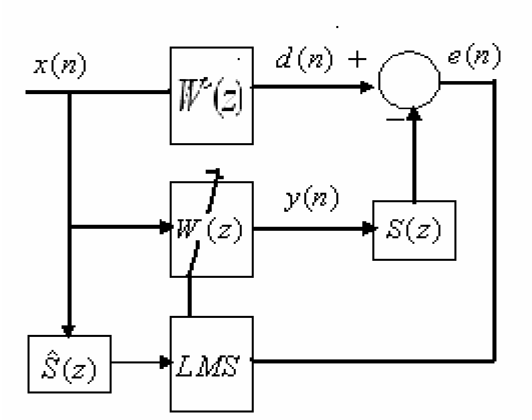
Index Terms-*Robust control, Adaptive,* μ *synthesis ,Broadband control*

1. Introduction

In Active Noise Control (ANC) systems, acoustical noise is tried to be cancelled (in a desired point or zone) by the anti noise signal generated by controlled loudspeakers [1], [2].
In ANC applications a common approach is to use adaptive FIR filters trained with the filtered X_ LMS algorithm [1].This algorithm is a modification of the well known LMS algorithm. Here, the reference signal is filtered to compensate for a filtering operation which is inherent in acoustic adaptation loop. Another way to control and cancel the acoustical noise is using fix controller based on optimal and robust design [3],[4]. In this papers the controller is designed based on LQG theory.
In [5] the ANC problem is solved using H_2,H_∞ model matching. A problem with H_∞ design is that the robust performance might not be meet because in H_∞ approach, the controller is designed according to nominal system without considering uncertainty[6].To solve this problem μ synthesis is proposed in [7] that considers uncertainty therefore achieves better robust performance.

Using robust controller for ANC systems, enables us to reduce the effect of unwanted noise in a

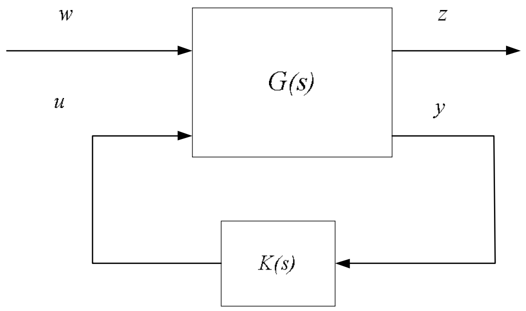
suitable frequency range while adaptive controllers [1] hardly could converge in the wideband frequency range.
Figure (1) describes the ordinary ANC system using adaptive filters to reduce the primary noise effect in microphone location. In this paper adaptive filter is replaced by a robust controller to produce control signals. In 3 the μ synthesis is introduced.
In section 4 the ANC problem is solved by robust controller and in section 5 simulation results presented. Simulation results confirm that robust controllers could achieve better performance comparing with the adaptive filters.



Fig(1) FX-LMS block diagram

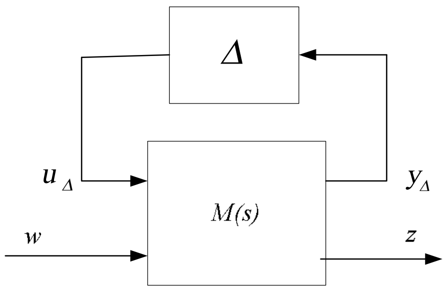
2. Robust Design

The goal of robust control design is to achieve acceptable performance for all models within a uncertainty.
all control structure can be represents as Fig(2) as LFT (linear fractional description)



Fig(2) General loop structure

In Fig (2) w is a vector including noise, disturbance, reference signals, z is a vector of controlled signals, y is a vector of measurement outputs and u is control signal vector. Considering uncertainty blocks in loop structure Fig(2) converts to Fig(3).



Fig(3) LFT with uncertainty

In Fig(3) $M(s)$ is combination of controller and plant without uncertainty and Δ is general uncertainty block.

There are two important definition for robust systems[6];

Definition 1: the feedback loop shown in Fig(3) achieves robust performance if and only if for all members of the family of plants describe by the Fig(3)the energy of the output z due to disturbance with energy bounded by 1 is also bounded by 1.

$$\|z(s)\| \le 1, \qquad \forall\{w \in L_2, \|w(s)\| \le 1\} \tag{1}$$

Definition 2:the interconnection shown in Fig(3) is robust stable if and only if for all members of the family of plants be stable.

consider again Fig(3), the transfer matrix relating the input and output signals is given by LFT between the nominal and uncertainty blocks that is;

$$\begin{aligned} z(s) &= F_u[M(s),\Delta(s)]w(s) \\ &= \{M_{22}(s) + M_{21}(s)\Delta(s)[I - M_{11}(s)\Delta(s)]^{-1}M_{12}(s)\} \end{aligned} \tag{2}$$

Lemma : the interconnection of Fig(3) is robustly stable if and only if $\|M_{11}\|_\infty \le 1$

Proof :[6]

above condition is a well known result of minimizing H_∞ norm of suitable matrix and is solvable by H_∞ design.

In addition in [6] it is shown that robust performance in systems that suffer from senor and actuator uncertainty is dependent to the uncertainty block and cannot be solved using H_∞ approach. the solution of this problem is addressed in [7] as μ synthesis.

3. μ Synthesis

Consider again Fig(3) , the closed loop transfer function M is derived as a lower LFT :

$$M = F_l(G,K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{3}$$

and the transfer function from $w(t)$ to $z(t)$ is represented as an upper LFT

$$z = F_u(M,\Delta)w \tag{4}$$

The uncertainty model Δ is assumed to belong to the set[8]

$$\Delta = \{diag(\delta_1 I_{k_1}, \ldots \delta_r I_{k_r}, \Delta_1, \ldots \Delta_s) : \delta_i \in C, \Delta_j \in C^{m_j \times m_j}\} \tag{5}$$

Definition 3: the structured singular value μ_Δ is define as[9]

$$\mu_\delta[M(j\omega)] \equiv \left[\inf_{\delta \in \Delta}\{\bar{\sigma}(\delta) | det[I - M(j\omega)\delta] = 0\}\right] \tag{6}$$

From the above definitions, the new uncertainty block is defined as follows:

$$\tilde{\Delta}' = \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \tilde{\Delta} \end{bmatrix} \middle| \begin{matrix} \Delta \in \Delta_d \\ \tilde{\Delta} \in C^{r \times r} \end{matrix} \right\} \tag{7}$$

where

$$\Delta_d = \left\{ \begin{bmatrix} \Delta_1 & & 0 \\ & \ddots & \\ 0 & & \Delta_M \end{bmatrix} \middle| \begin{matrix} \Delta_i \in C^{r_i \times r_i} \\ i = 1, \ldots, m \end{matrix} \right\} \tag{8}$$

Now the main result of μ-synthesis is the below theorem

Theorem: the set of models $M(s), \Delta(s)$ of Fig(3) has robust performance if and only if

$$\mu_{\tilde{\Delta}}[M(j\omega)] \le 1, \qquad \forall \omega \in R \tag{9}$$

The objective of μ synthesis is to design a controller $K(s)$ such that the transfer function between w to z has norm less than 1. This problem is an important point of robust control because its minimization isn't in convex space so it may have more than on solutions [10].

There is no specific way to obtain controllers that satisfy (9) but there are some suboptimal solutions.

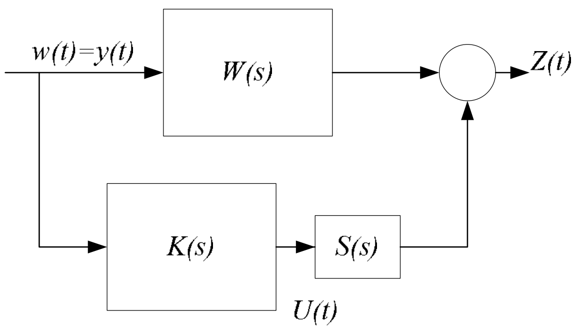
the solution of μ synthesis problem is *D-K* iteration [11]. Due the fact that the structured singular value is computed via its bounds, the *D-K* iteration is the following minimization process.

$$\inf_{\substack{stablizing \\ K(s)}} \inf_{D(s)\in D} \left\| D(s) F_l[G,K] D(s)^{-1} \right\|_\infty \quad (10)$$

The iteration is not guaranteed to reach the global minimum for the reason that the combined optimization isn't convex [8]

4. ANC controller design

A block diagram of ANC system using robust controller is shown in Fig (4) that is same as Fig (1) while the adaptive filter is replaced by robust controller.



Fig(4) : ANC system according robust control diagram

In above figure *W(s)* is primary path transfer function between noise source and residual microphone, and *S(s)* is secondary path transfer function between canceling speaker and residual microphone. The objective of the robust controller is to minimize residual noise in error microphone (*z(t)*) relevant to the *w(t)* considering $\|.\|_\infty$ norm. There are several models for uncertainty in robust theory [6] and robust design is dependent to the uncertainty model. The multiplicative uncertainty model is considered in this paper which represents the high frequency system identification.

The above model can be written for *W(s),S(s)* as

$$W_{real}(s) = W_{no\min al}(s)(1 + w_1 \Delta_1(s)) \quad (11)$$

$$S_{real}(s) = S_{no\min al}(s)(1 + w_2 \Delta_2(s)) \quad (12)$$

where w_1, w_2 are known bound to ensure that $\Delta(s) \le 1$

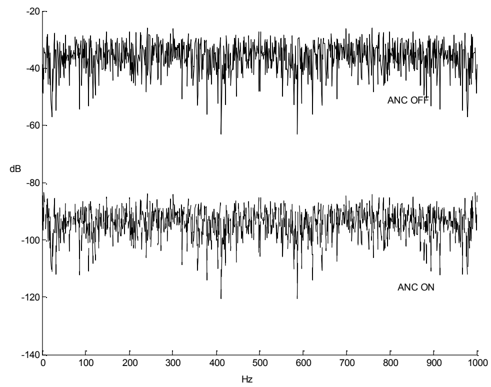
In above equations $W_{no\min al}, S_{no\min al}$ are plant frequency responses in control bandwidths and W_{real}, S_{real} are the physical plants.

The robust controller is designed using MATLAB μ toolbox via *D-K* iteration method [8].

5. Simulation Results

For the simulation the all pass filter is considered as a transfer function between primary source and error microphone (*W(s))* and the secondary path transfer function is considered all pass filter too, because in real systems secondary transfer function is modeled as a pure delay[12],[13]. Fig (5) represents the residual noise in error microphone before and after control respectively.

It is clear that the attenuation is achieved in a relatively wide frequency range while this is not the case in adaptive noise controller.



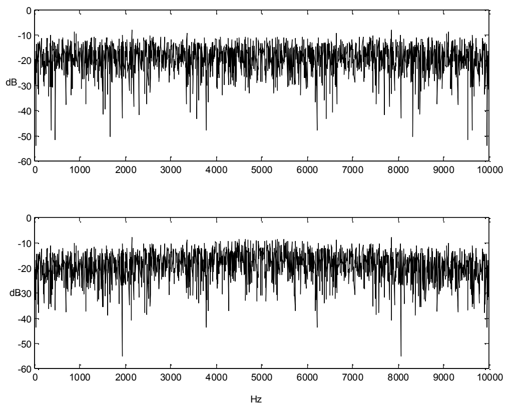
Fig(5): FFT of error signal using robust controller

Fig(6) represents the FFT of error signal using adaptive filters to reduce residual noise.

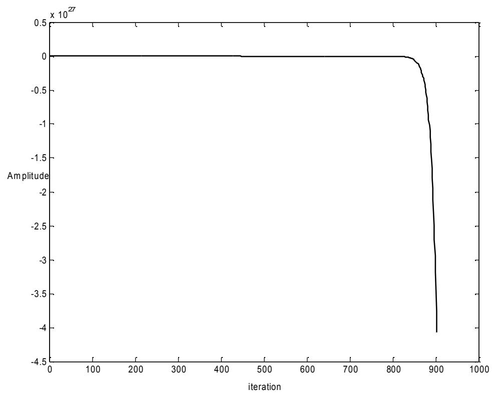
In this case it is clear that the maximum attenuation depends to the noise frequency and because of model error (Fig (1)) the system couldn't achieves the proper reduction

It was shown [14], [15] that the stability and system behavior are dependent to secondary transfer function modeling and the large modeling error can lead to instability [16]. In contrast with adaptive filters, robust controller consider the modeling error in uncertainty blocks and the design procedure will be based on stable controller [17]

Fig (6) is the error signal when adaptive ANC system is derived by large modeling error.



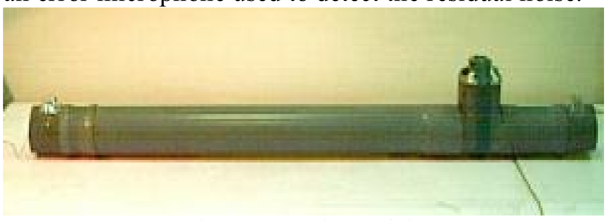
Fig(6) FFT of error signal using adaptive filters before ANC(above) after ANC(below)



Fig(6): error signal of adaptive ANC system with large model error

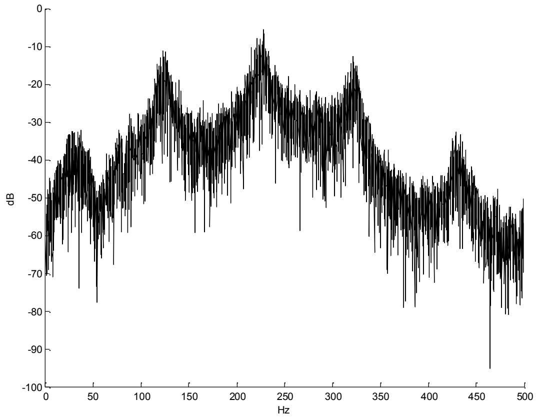
6. Experimental Results

The laboratory set up used to implement the ANC system pictured in Fig. (7), consists of an open ended polyvinyl chloride (PVC) duct with the following major elements: actuating device named the primary speaker, a compensating device named the secondary speaker and an error microphone used to detect the residual noise.

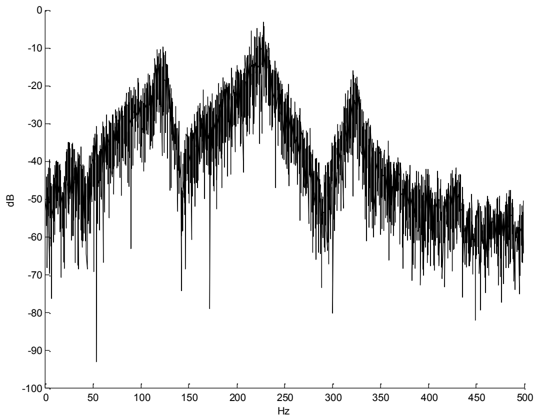


Fig(7): Experimental duct

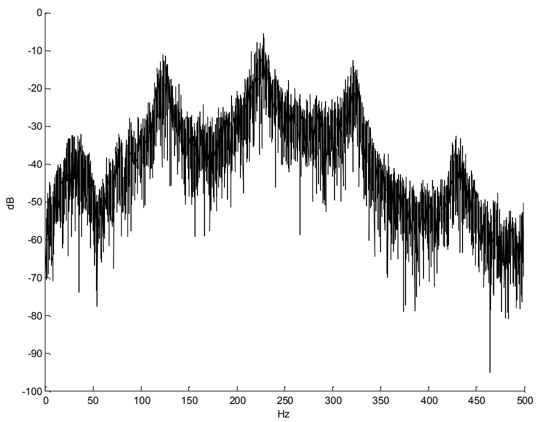
at first to design controller the channel estimation should be done. Our estimation was based on white noise input signal and using an adaptive filter of order 100 the estimation was obtained. Fig (8) and Fig (9) show the primary and secondary channel frequency response respectively. The FFT of error signal is shown in Fig(10),(11) when ANC system is off and on.



Fig(8):Frequency response of primary path

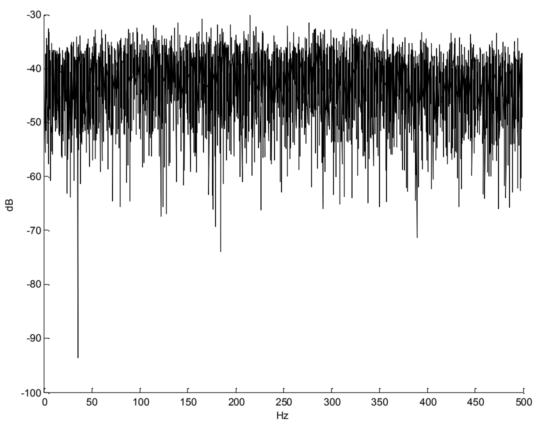


Fig(9):Frequency response of secondary path

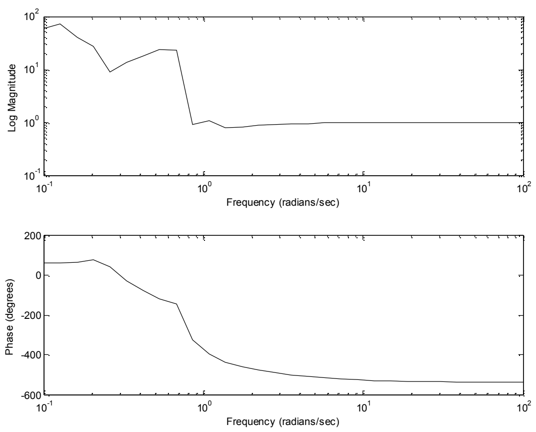


Fig(10):FFT of error signal ANC off

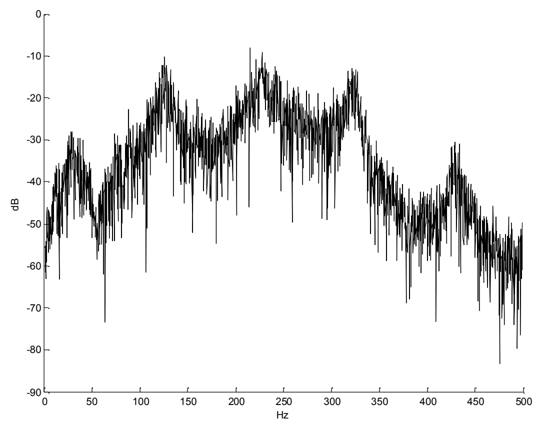
In Fig (12) the frequency response of controller is shown. It is clear that the controller has output signal in low frequency more than high frequency. Fig (13) depicts the FFT of error signal when adaptive controller is used instead of robust controller. Comparing Fig (11) and Fig (13) it is clear that robust controller could achieve better performance in wide frequency range than the ordinary adaptive controllers. Fig (14) shows the convergence behavior of adaptive controller of order 100. Considering Fig (14) it is clear that the system shows weak convergence behavior and with large secondary path estimation error this behavior might lead to divergence.



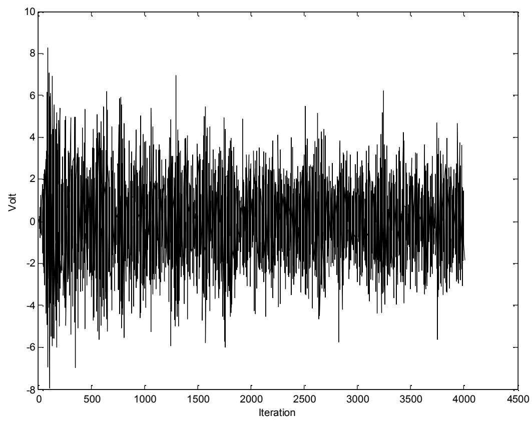
Fig(11):FFT of error signal ANC on (robust controller)



Fig(12):Frequency response of controller



Fig(13):FFT of error signal ANC on (Adaptive controller)



Fig(14): error signal **,** ANC on (Adaptive controller)

7. Conclusion

In this paper a new method based on robust control theory is proposed to control the unwanted noise in ANC systems. It was shown that in contrast with the adaptive filters, this type of controller provide the acceptable attenuation in the large frequency range. In addition the robust controller is chosen as to the whole system remains stable while in adaptive ANC system stability highly depends to modeling error. Simulation and experimental results confirm our claims.

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