# CLOSED-LOOP SUBSPACE IDENTIFICATION: AN ORTHOGONAL PROJECTION APPROACH

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Abstract: In this paper, a closed-loop subspace identification approach through an orthogonal projection and subsequent singular value decomposition is proposed. As a by-product of this development, it explains why some existing subspace methods may deliver a bias in the presence of the feedback control and suggests a remedy to eliminate the bias. Furthermore, as the proposed method is a projection based method, it can simultaneously provide extended observability matrix, lower triangular block-Toeplitz matrix, and Kalman filtered state sequences. Therefore, using this method, the system state space matrices can be recovered either from the extended observability matrix/the block-Toeplitz matrix or from the Kalman filter state sequences.

Keywords: Subspace identification, closed-loop identification, projection, instrument variable method, PCA, subspace PCA, singular value decomposition.

## 1. INTRODUCTION

Identification of the subspace matrices from closed loop data has received an increasing attention by a number of researchers. It is found that the regular open-loop subspace identification algorithm yields a biased estimate when applied to closed-loop data(Ljung and McKelvey, 1996). Several modified algorithms have been proposed (Overschee and Moor, 1996; Ljung and McKelvey, 1996; Ljung and McKelvey, 1996). Another class of subspace system identification is called the instrument variable methods (Chou and Verhaegen, 1997; Wang and Qin, 2002). In the class of the instrument variable methods, the effect of disturbances is eliminated by appropriate selection of the instrument variables that are independent of disturbances. Under the framework of MOESP, (Chou and Verhaegen, 1997) developed an instrument variable subspace identification algorithm. They claimed that the algorithm worked for closed-loop systems provided there is at least one sample time delay in the *controller*, which may be restrictive in practice. Aiming at solving the open-loop error in variable (EIV) identification problem, (Wang and Qin, 2002) developed an instrument variable subspace identification method via principal components analysis, which may be applicable to closed-loop identification but only under certain conditions.

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In this paper we develop a closed-loop subspace identification algorithm through an orthogonal subspace projection, and then use either the extended observability matrix/lower block-Toeplitz matrix or the Kalman filter states resulting from the projection to extract system models (Due to the space limit, we will not discuss the procedure to recover the model from the Kalman filter state sequence). Since the model is obtained using projections of subspace, as is the case in most other subspace identification algorithms, it has certain additional properties compared to the instrument methods. It is further shown that the existing instrument subspace identification via PCA, although works well for open-loop EIV systems, yields a biased solution in closed-loop under certain conditions. A remedy to eliminate this bias is also proposed in this paper.

The remainder of this paper is organized as follows. Several subspace notations adopted throughout this paper are revisited in Section 2. Our main results, the proposed subspace closed-loop identification approach and a solution to an existing instrument subspace identification method for closed-loop identification, are discussed in Section 3. Simulation results are presented in Section 4, followed by concluding remarks in Section 5.

## 2. SUBSPACE IDENTIFICATION

Consider a state space model in the innovation form

$$x_{t+1} = Ax_t + Bu_t + Ke_t \tag{1}$$
$$y_t = Cx_t + Du_t + e_t \tag{2}$$

where $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^p$, $y_t \in \mathbb{R}^q$, and $e_t \in \mathbb{R}^q$ is white noise innovation sequence with covariance $\Sigma_e$.

Following the standard subspace notation, one can derive, through the iterative substitution of Eqs(1) and (2), the subspace matrix equations as

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f \tag{3}$$
$$Y_p = \Gamma_i X_p + H_i^d U_p + H_i^s E_p \tag{4}$$
$$X_f = A^i X_p + \Delta_i^d U_p + \Delta_i^s E_p \tag{5}$$

where subscript $p$ stands for the "past" and $f$ for the "future". All notations used in Eqs(3)-(5) are standard and can be found in most of subspace identification literatures. $\Gamma_i$ is extended observability matrix, $\Delta_i^d$ and $\Delta_i^s$ are reversed extended controllability matrices for process input and disturbance input respectively, $H_i^d$ and $H_i^s$ are the lower triangular block-Toeplitz matrices for process input and disturbance input respectively. The past and future input block-Hankel matrices are defined as

$$U_p \triangleq U_{0|i-1} = \begin{pmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{pmatrix} \tag{6}$$

$$U_f \triangleq U_{i|2i-1} = \begin{pmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{pmatrix} \tag{7}$$

where $U_p, U_f \in \mathbb{R}^{pi\times j}$. Note that, in Eqs(6) and (7), the row dimension of $U_f$ is allowed to be different from that of $U_p$, which would provide an extra freedom to tune the identification algorithm.

The output and innovation block-Hankel matrices $Y_p, Y_f \in \mathbb{R}^{qi\times j}$, $E_p, E_f \in \mathbb{R}^{qi\times j}$, respectively, are defined conformably with $U_p, U_f$.

The states are defined as

$$X_p \triangleq X_0 = \begin{pmatrix} x_0 & x_1 & \cdots & x_{j-1} \end{pmatrix} \tag{8}$$
$$X_f \triangleq X_i = \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \end{pmatrix} \tag{9}$$

where $X_p, X_f \in \mathbb{R}^{n\times j}$.

In subspace identification literature, the following short-hand notation is often used:

$$W_p = \begin{pmatrix} Y_p \\ U_p \end{pmatrix}$$

The following two projections are frequently used throughout this paper.

**Orthogonal projection:** The orthogonal projection of the row space of $A$ onto the row space of $B$ is denoted by $A/B$ and can be calculated through

$$A/B = AB^+B$$

where $B^+$ is the pseudo inverse of $B$.

**Oblique projection:** The oblique projection of the row space of $A \in \mathbb{R}^{p\times j}$ along the row space of $B \in \mathbb{R}^{q\times j}$ on the row space of $C \in \mathbb{R}^{r\times j}$ are defined as $A/_B C$ and can be calculated via (using MATLAB matrix index notation)

$$A/_B C = A \begin{pmatrix} C \\ B \end{pmatrix}^+ (:, 1:r)C$$

Two important properties of the oblique projection are often used and they are

$$A/_A C = 0 \tag{10}$$
$$A/_B A = A \tag{11}$$

These two properties are straightforward results by the definition of the oblique projection.

## 3. CLOSED-LOOP IDENTIFICATION

### *3.1 A preliminary solution*

Let's now revisit Eq(3)

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f \qquad (12)$$

The essential system information is contained in the extended observability matrix $\Gamma_i$ or in the state $X_f$. That is, the first term on the right hand side of Eq(12) deserves our main attention. To calculate, for example, $\Gamma_i$ from Eq(12), one has to get rid of the terms containing $U_f$ and $E_f$. If $E_f$ is independent of past input $U_p$, past output $Y_p$ (or equivalently their combination $W_p$), and future input $U_f$, then one can easily achieve the above objective by performing an oblique projection of Eq(12) along the row space $U_f$ onto the row space of $W_p$, i.e.

$$\begin{aligned} Y_f/_{U_f}W_p = \Gamma_i X_f/_{U_f}W_p + H_i^d U_f/_{U_f}W_p \\ + H_i^s E_f/_{U_f}W_p \end{aligned} \qquad (13)$$

It is easy to see that the last two terms of Eq(13) are zero, $U_f/_{U_f}W_p = 0$ by the property of the oblique projection, Eq(10); $E_f/_{U_f}W_p = 0$ and this is based on the assumption that future disturbance is independent of past input/output and future input. This assumption holds under the open-loop condition. Thus Eq(13) can be simplified to

$$Y_f/_{U_f}W_p = \Gamma_i X_f/_{U_f}W_p \qquad (14)$$

This result indicates that the column space of $\Gamma_i$ is the same as the column space of $Y_f/_{U_f}W_p$, which can be calculated by the SVD decomposition of $Y_f/_{U_f}W_p$. Similarly, the row space of $Y_f/_{U_f}W_p$ is the same as the row space of $X_f/_{U_f}W_p$, which is Kalman filter state solution with $X_p/_{U_f}W_p$ as its initial condition (Favoreel, 1999). Therefore, the Kalman state sequence can also be calculated from the SVD decomposition of $Y_f/_{U_f}W_p$. Subsequently, the system state space matrices can be recovered either from the extended observability matrix or from the Kalman state sequence. This is the solution to open-loop subspace system identification.

The situation becomes more complex for closed-loop identification where the future disturbance $E_f$ is no longer independent of the future input $U_f$ due to the feedback. The implication of this dependency is that the oblique projection of $E_f$ along $U_f$ onto $W_p$ is no longer zero although the orthogonal projection of $E_f$ onto $W_p$ is zero.

To solve this problem, by adopting the EIV structure, we move the term related to $U_f$ into the left hand side of Eq(12) as it would be a troublesome term if left in the right hand side of the equation. This yields a new equation with both input and output variables in the same side of the equation.

$$\left[\, I \;\; -H_i^d \,\right] \begin{pmatrix} Y_f \\ U_f \end{pmatrix} = \Gamma_i X_f + H_i^s E_f \qquad (15)$$

Using the short-hand notation

$$W_f = \begin{pmatrix} Y_f \\ U_f \end{pmatrix}$$

Eq(15) can be simplified to

$$\left[\, I \;\; -H_i^d \,\right] W_f = \Gamma_i X_f + H_i^s E_f \qquad (16)$$

Performing an *orthogonal projection* of Eq(16) onto the row space of $W_p$ yields

$$\left[\, I \;\; -H_i^d \,\right] W_f/W_p = \Gamma_i X_f/W_p + H_i^s E_f/W_p \qquad (17)$$

The last term of Eq(17) is an orthogonal projection of the future disturbance (white noise) onto the row space of past input and output matrix $W_p$, which is zero. Therefore, Eq(17) can be simplified to

$$\left[\, I \;\; -H_i^d \,\right] W_f/W_p = \Gamma_i X_f/W_p = \Gamma_i \hat{X}_f \qquad (18)$$

*Remark 1.* Eq(18) is a natural result through the projection as is often done in subspace system identification literature. The orthogonal projection of Eq(16) onto the row space of $W_p$ results in Eq(18), which includes a multiplication term between the extended observability matrix $\Gamma_i$ and non-steady state Kalman state $\hat{X}_f$. On the other hand, (Wang and Qin, 2002) used an instrument variable method to arrive an equation as

$$\left[\, I \;\; -H_i^d \,\right] W_f W_p^T = \Gamma_i X_f W_p^T \qquad (19)$$

where the instrument variable is the past input and output $W_p$. This equation is derived by multiplying Eq(16) by $W_p^T$ and noticing the independency between $W_p$ and $E_f$.

The projection method may also be mathematically regarded as an instrument variable method where the instrument is $W_p^T(W_p W_p^T)^{-1}W_p$. However, by using the projection, it results in a multiplication term between the extended observability matrix and the Kalman filtered state, which provides some additional feature and performance. We shall call the proposed projection method as the subspace orthogonal projection identification method, abbreviated SOPIM, while the subspace identification method via PCA of (Wang and Qin, 2002) is abbreviated as SIMPCA.

Now multiplying both sides of Eq(18) by the orthogonal column space of $\Gamma_i$, denoted by $\Gamma_i^\perp$, yields

$$(\Gamma_i^\perp)^T \left[\, I \;\; -H_i^d \,\right] W_f/W_p = 0 \qquad (20)$$

Denoting $Z = W_f/W_p$, the problem is transferred to finding the orthogonal column space of $Z$, which should equal to the column space of $\left( (\Gamma_i^\perp)^T \left[\, I \;\; -H_i^d \,\right] \right)^T$.

Perform SVD decomposition of $Z$ as

$$Z = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} \Sigma_1 & \\ & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \tag{21}$$

where, in practice, $Z$ is not singular and one has to determine its rank by checking its singular values. In theory, its rank should be $pi + n$(Wang and Qin, 2002) assuming that the external excitation is persistent excitation. The rank determination is equivalent to the determination of system orders.

With Eq(21), one can easily find the orthogonal column space of $Z$, which is $U_2$. Therefore

$$\left( (\Gamma_i^{\perp})^T \left[ I \; -H_i^d \right] \right)^T = U_2 M \tag{22}$$

where $M$ is any constant matrix. Partition

$$U_2 M = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

Then Eq(22) can be written as

$$\begin{pmatrix} (\Gamma_i^{\perp}) \\ -(H_i^d)^T \Gamma_i^{\perp} \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} \tag{23}$$

Therefore,

$$\Gamma_i = P_1^{\perp} \tag{24}$$

$$-(P_1)^T H_i^d = P_2^T \tag{25}$$

The remaining problem is to solve for $\Gamma_i$ and $H_i^d$, and then to extract the system matrices $A, B, C, D$ from $\Gamma_i$ and $H_i^d$. Many methods are available for this purpose, for example, (Wang and Qin, 2002).

Can SOPIM and SIMPCA work under closed-loop conditions? Up to now, it appears that there should be no question for both to work under closed-loop conditions as the input $U_f$, the correlation of which with the future disturbance $E_f$ is the key problem in closed-loop identification, is not involved in the projection or in the instrument variable. Both equations (18) and (19) appear to be able to uniquely determine the process model irrespective of open or closed loop. However, simulation results indicate that this is not the case although both work for open-loop systems.

### *3.2 The problem and the solution*

The problem must have resulted from the controller since they both work well under the open-loop condition. Without presenting a full version of analytical derivation due to the space limit, in this section, we will make a heuristic analysis of the problem and then present our solution.

To find the problem, let's consider that the controller is described by the following state space model:

$$x_{t+1}^c = A_c x_t^c + B_c (r_t - y_t) \tag{26}$$

$$u_t = C_c x_t^c + D_c (r_t - y_t) \tag{27}$$

where $r$ is the setpoint excitation. Using subspace notations, we should have the controller expressed as

$$U_f = \Gamma_i^c X_f^c + H_i^c (R_f - Y_f) \tag{28}$$

$$U_p = \Gamma_i^c X_p^c + H_i^c (R_p - Y_p) \tag{29}$$

where $R_p$ and $R_f$ are data Hankel matrices of the setpoint, $X_p^c$ and $X_f^c$ are the state matrices of the controller, $\Gamma_i^c$ is the extended observability matrix, and $H_i^c$ is the lower triangular block-Toeplitz matrix, of the controller.

Eq(28) can be re-arranged to give

$$\left[ H_i^c \; I \right] W_f = \Gamma_i^c X_f^c + H_i^c R_f \tag{30}$$

Projecting Eq(30) to $W_p$ yields

$$\left[ H_i^c \; I \right] W_f / W_p = \Gamma_i^c X_f^c / W_p + H_i^c R_f / W_p \tag{31}$$

Comparing Eq(31) with Eq(18), one can find the overlap of two subspaces represented by the two equations, one for process model subspace and the other for controller model subspace, if $H_i^c R_f / W_p \rightarrow 0$ or $R_f W_p^T \rightarrow 0$ (in the case of SIMPCA). This situation can occur, for example, when $R_f$ is white noise excitation.

Our solution to this problem (omitting details due to space limit) is: for closed-loop identification, one should replace $W_p$ by $W_{pr}$ according to Eq(32)

$$W_{pr} \triangleq \begin{pmatrix} R_f \\ W_p \end{pmatrix} \tag{32}$$

This will guarantee $H_i^c R_f / W_p \neq 0$ or $R_f W_p^T \neq 0$. With the modification, all the computation procedures discussed in the last section are valid for closed-loop identification after replacing $W_p$ by $W_{pr}$, and then both SOPIM and SIMPCA can be truly applied to closed-loop data. We shall call the modified algorithms as closed-loop SOPIM (abbreviated as CSOPIM) and closed-loop SIMPCA (abbreviated as CSIMPCA), respectively.

*Remark 2.* From the above discussion, one can see that the undesired effect of the feedback control on the identifiability of open-loop instrument and/or projection subspace methods may also be alleviated if the setpoint $r_t$ "stays away" from whiteness, i.e. if they are (strongly) autocorrelated or colored. But this "non-witeness" is ambiguous and there is no *a priori* indication on how $R_f / W_p$ or $R_f W_p^T$ will be different from zero even if $r_t$ is not white. In addition, the effect of whiteness or non-whiteness on the identifiability also depends on the controller in the feedback and the disturbances that are affecting the process. For example, if the output can not follow the setpoint closely due to large disturbances, then $W_p$ can have a little correlation with $R_f$ even if the setpoint is not white, resulting $R_f W_p^T \rightarrow 0$ in SIMPCA or $R_f/W_p \rightarrow 0$ in SOPIM. Consequently, $W_p$ may not be suitable to be an instrument in this case even though the external excitation is non-white.

## 4. SIMULATION

In this section, we will use a benchmark problem to evaluate the proposed approach and compare it with other existing subspace identification algorithms. We will apply the following representative subspace algorithms in the literature: closed-loop algorithm by (Overschee and Moor, 1996), closed-loop algorithm by (Verhaegen, 1993), closed-loop algorithm by (Ljung and McKelvey, 1996), and the two classical subspace algorithms, N4SID, MOESP and two versions of CVA, i.e. MATLAB N4SID with CVA weighting and CVA according to (Larimore, 1990). To comply with the standard practice in subspace identification literature (Overschee and Moor, 1996), we will perform Monte-Carlo simulations and the averaged Bode magnitude plot from the Monte-Carlo simulation will be used to represent bias error while the scatter plot of estimated poles will be used to represent the variance error of the estimation.

The system to be considered(Verhaegen, 1993; Overschee and Moor, 1996), expressed in the innovation state space form (Overschee and Moor, 1996), is given by Eqs(1) and (2) with the following numerical values

$$A = \begin{pmatrix} 4.40 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4.00 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.0033 \\ -0.00002 \end{pmatrix},$$

$$C^T = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, K = \begin{pmatrix} 2.3 \\ -6.64 \\ 7.515 \\ -4.0146 \\ 0.86336 \end{pmatrix}$$

The state space model of the feedback control has the following values:

$$A_c = \begin{pmatrix} 2.65 & -3.11 & 1.75 & -0.39 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, B_c = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} -0.4135 & 0.8629 & -0.7625 & 0.2521 \end{pmatrix}$$

with $D_c = 0.61$. The simulation conditions exactly resemble those used by (Overschee and Moor, 1996): $e_t$ is a Gaussian white noise sequence with variance 1/9; the reference signal $r_t$ is a Gaussian white noise sequence with variance 1, injected after the controller and before the plant. Each simulation run generates 1200 data points. We generate 100 data sets, each time with the same reference input $r_t$ but with a different noise sequence $e_t$.

(Overschee and Moor, 1996) used this example to compare several closed-loop identification algorithms. The conclusion from their simulations was that algorithm 1 of Van Overschee and De Moor is superior to other algorithms in terms of performance. However, the algorithms proposed in (Overschee and Moor, 1996) required the precise information of at least first $i$ Markove parameters of the controller model, while others did not.

In this simulation, we will reproduce the results of (Overschee and Moor, 1996) and compare them with the most representative algorithm proposed in this paper, CSOPIM. The simulation results are shown in Fig.1. From this figure, we can see that the proposed algorithm CSOPIM has an almost identical performance as that of algorithm Van Overschee and De Moor in both bias and variance aspects. However, the proposed algorithm has the advantage over algorithm Van Overschee and De Moor in the sense that the proposed algorithm does not need any knowledge about the controller model while algorithm Van Overschee and De Moor does. To see the further advantage of the proposed algorithm, we consider EIV case by adding white noises to measurements of both $u_t$ and $y_t$. We do two Monte-Carlo simulations for the EIV case, one with measurement noise variance 0.2 and the other 0.5. The comparison results are shown in Fig.2. From this figure, one can see that the proposed algorithm performs better than algorithm Van Overschee and De Moor in the presence of measurement noises.

To appreciate closed-loop subspace identification algorithms, we have also applied classical subspace algorithms, N4SID, MOESP, two CVAs (CVA according to (Larimore, 1990) and MATLAB N4SID with CVA weighting) to closed-loop data. Our results indicate that N4SID, MOESP and MATLAB based CVA deliver essentially the same performance and all are biased in the presence of feedback control. The CVA programmed according to (Larimore, 1990) gives some improved performance compared to the MATLAB N4SID with CVA weighting but the bias error remains.

## 5. CONCLUSION

In this paper, by adopting the EIV model structure, a subspace orthogonal projection identification method (SOPIM) is proposed. It, however, yields a bias for closed-loop identification

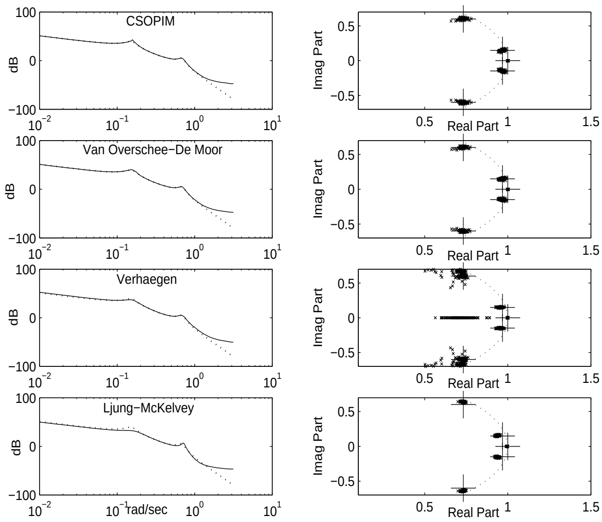


Fig. 1. Closed-loop Monte-Carlo simulations. The left column is Bode magnitude plots; the dotted lines are the true values and the solid lines are the estimated values averaged from 100 runs. The right column is the scatter plots of the eigenvalues of the estimated $A$ matrix.

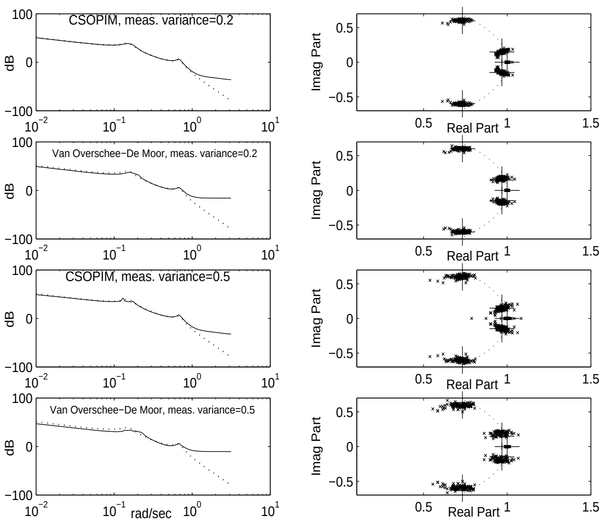


Fig. 2. Closed-loop Monte-Carlo simulations for the EIV system. The left column is Bode magnitude plots; the dotted lines are the true values and the solid lines are the estimated values averaged from 100 runs. The right column is the scatter plots of the eigenvalues of the estimated $A$ matrix.

at least under the condition that the external excitation is white. Through analysis of the bias error of SOPIM under closed-loop conditions, it is discovered that other existing instrument subspace methods in the literature may also suffer from the same bias error and therefore, these methods only yield a partial solution to closed-loop identification. Motivated by this discovery, a remedy is derived to eliminate bias error for SOPIM as well as for one of the existing instrument subspace identification algorithms, SIMPCA, for the sake of closed-loop identification. As a result two new subspace closed-loop identification algorithms CSOPIM and CSIMPCA, named after SOPIM and SIMPCA, respectively, are developed. In addition, the orthogonal projection method proposed in this paper provides both extended observability matrix and Kalman filter state sequence. Therefore, system models may also be recovered from the estimated Kalman state sequence. Simulations based on a benchmark problem compare the performance of the proposed algorithms with a number of well-known subspace identification algorithms and verify the feasibility and closed-loop applicability of the proposed algorithms.

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## REFERENCES

Chou, C.T. and M. Verhaegen (1997). Subspace algorithms for the identification of multivariable dynamic errors-in-variables models. *Automatica* **33(10)**, 1857–1869.

Favoreel, W. (1999). Subspace methods for identification and control of linear and bilinear systems. PhD thesis. Katholieke Universiteit Leuven.

Larimore, W.E. (1990). Canonical variate analysis in identification, filtering, and adaptive control. In: *Proceedings of the 29th Conference on Decision and Control*. pp. 596–604.

Ljung, L. and T. McKelvey (1996). Subspace identification from closed loop data. *Signal Processing* **52**, 209–215.

Overschee, P. Van and B. De Moor (1996). Closed-loop subspace system identification. Technical Report ESAT-SISTA/TR 1996-52I. Katholieke Universiteit Leuven.

Verhaegen, Michel (1993). Application of a subspace model identification technique to identify LTI systems operating on closed-loop. *Automatica* **29(4)**, 1027–1040.

Wang, J. and J. Qin (2002). A new subspace identification approach based on principal component analysis. *Journal of Process Control* **12**, 841–855.