CONTROL SYSTEM SELECTION: A MEASURE OF CONTROL QUALITY LOSS IN ANALYTICAL CONTROL

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Abstract: A question that often process control engineers face is for what class of processes one should use model predictive control that requires solving numerically a constrained optimization problem repeatedly on-line. The alternative is to use analytical control, such as P, PI, PID and differential geometric control, which do not require the on-line optimization. In other words, for what class of processes can analytical control provide control quality close to the optimal control quality that model predictive control (MPC) can provide? This work presents a measure that allows one to quantify the degradation in closed-loop performance when one implements analytical control instead of MPC. A special case of the measure is used to derive a simple test that can be used to check if a given input-constrained process can be controlled satisfactorily by analytical control. It is shown that processes that exhibit directionality greatly benefit from MPC. In other words, for input-constrained processes whose nonsingular characteristic (decoupling) matrix is independent of manipulated inputs and can be made diagonal by row or column rearrangements, control quality provided by analytical control can be adequate. The measure is used to see if four input-constrained process examples can be controlled satisfactorily by analytical control. Closed-loop responses are shown to confirm the usefulness of the measure.

Keywords: control system selection, model predictive control, analytical control, actuator saturation, directionality, decoupling matrix

1. INTRODUCTION

The degrading effect of constraints on control quality was recognized very early in practice. To minimize the degrading effect, a great number of strategies such as anti-windup schemes and directionality compensators and optimization-based control methodologies such as model predictive control have been developed.

For a given process design, after identifying disturbances and selecting controlled outputs and manipulated inputs, an effective control system is chosen. The control system can be conventional, model-based, analytical (such as P, PI and PID), and/or numerical (model predictive). A question that often process control engineers face is for what class of processes one should use model predictive control that requires solving numerically a constrained optimization problem repeatedly on-line. The alternative is to use analytical control, such as P, PI, PID and differential geometric

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control, which do not require the on-line optimization. In other words, control of what class of processes can significantly benefit from model predictive control?

In current industrial practice, this question is addressed in several ways, which generally depend on:

- The sector of the industry and the type of manufacturing operations (e.g. petrochemical, chemical, or food);
- The acceptance of the technology in operations (typically rooted in the history of introduction of the technology, experience with previous implementations, employee process control skills, and success in maintaining performance over time); and
- How strongly the financial benefits from a migration to MPC manifest themselves.

It is usually an obvious value preposition or a leap of faith in the benefits of the technology that would lead to the development and commissioning of an MPC application and not a detailed analysis.

Few companies have an in-house engineering capability to carry out an operability analysis, which allows for performance benchmarking and assessment of the financial benefits of the MPC migration. This analysis involves the development of a dynamic model of the process in question and the comparative assessment of an MPC versus an analytical implementation. In the best of the cases, a high-fidelity nonlinear simulator of the entire process is interfaced to the commercial MPC controller of preference to conduct the evaluation. The lack of a simple analysis tool with solid theoretical foundations to check whether an input-constrained process can be controlled adequately well by using analytical control is the best motivation for this study.

This work presents a measure that allows one to quantify the degradation in closed-loop performance when one implements analytical control instead of MPC. The measure quantifies the deviation of (a) the response of the process under analytical control from (b) the response of the same process under model predictive control. A special case of the measure is used to derive a simple test that can be used to check if a given input-constrained process can be controlled satisfactorily by analytical control. It is shown that input-constrained processes that exhibit directionality greatly benefit from MPC. The measure is used to check if four process examples can be controlled satisfactorily by analytical control. Closed-loop responses are shown to confirm the usefulness of the measure.

Section 2 describes the scope of the study and some preliminaries. An index of the control quality loss is presented in Section 3. The application of the index is illustrated by four examples in Section 4.

2. SCOPE AND PRELIMINARIES

Consider nonlinear multivariable processes with a model in the form

$$\begin{aligned} x(k+1) &= \Phi[x(k), u(k)], \qquad x(0) = x_0 \\ y(k) &= h[x(k)] \end{aligned} \tag{1}$$

with the input constraints

$$u_{l_j} \le u_j(k) \le u_{h_j}, \quad j = 1, \cdots, m \tag{2}$$

where $x = [x_1 \cdots x_n]^T$, $u = [u_1 \cdots u_m]^T$, and $y = [y_1 \cdots y_m]^T$ denote the vectors of state variables, manipulated inputs, and controlled outputs, respectively; $\Phi(x, u)$ and $h(x)$ are smooth vector functions; and u_{l_j}, u_{h_j}, Δu_{l_j}, and Δu_{h_j}, $j = 1, \cdots, m$, are constant scalar quantities.

For a process in the form of (1),

- Relative order (degree) of an output y_i with respect to the vector of manipulated inputs is denoted by r_i, where r_i is the smallest integer for which $y_i(k+r_i)$ depends explicitly on the present value of a manipulated input.
- Characteristic (decoupling) matrix

$$\mathcal{C}(x, u) \triangleq \begin{bmatrix} \frac{\partial}{\partial u} h_1^{r_1}(x, u) \\ \vdots \\ \frac{\partial}{\partial u} h_m^{r_m}(x, u) \end{bmatrix}$$

where

$$\begin{aligned} h_i^0(x) &\triangleq h_i(x), \quad i = 1, \cdots, m \\ h_i^\ell(x) &\triangleq h_i^{\ell-1}[\Phi(x, u)], \quad \ell = 1, \cdots, r_i - 1, \\ &\qquad\qquad i = 1, \cdots, m \\ h_i^{r_i}(x, u) &\triangleq h_i^{r_i-1}[\Phi(x, u)], \quad i = 1, \cdots, m \end{aligned}$$

The decoupling (characteristic) matrix has been used to check the degree of dynamic decoupling in processes and specify the class of processes that do not exhibit directionality (Soroush and Valluri, 1999). It has also been called instantaneous process gain, because the strength of a process response over a very short horizon strongly depends on the decoupling matrix of the process. For processes with a diagonal and independent-of-manipulated-inputs decoupling matrix, completely decentralized control is more suitable. The matrix has also been used as basis for input-output pairing.

Let the time-varying static state feedback

$$u^*(k) = \Psi^*[x(k), k, y_{sp}(k)] \tag{3}$$

represent a general model predictive control law for the process of (1) with the input constraints of (2) [that is, the numerical solution to a constrained, moving horizon, optimization problem], and the time-invariant static state feedback

$$w(k) = \Psi[x(k), y_{sp}(k)] \tag{4}$$

denote the same general model predictive control law for the process of (1) without the input constraints of (2) [that is, the numerical solution to an unconstrained, moving horizon, optimization problem].

2.1 Directionality

Directionality is a controller performance degradation that is associated with actuator saturation. As defined in (Soroush and Valluri, 1999; Soroush and Mehranbod, 2002), directionality occurs when for a given command signal $\hat{w}$ the output response of process to sat$(\hat{w})$ is not "closest" (in the controlled output space) to the output response of the process to $\hat{w}$, where

$$\text{sat}_\ell[w] \triangleq \begin{cases} u_{l_\ell}, \; w_\ell < u_{l_\ell} \\ w_\ell, \; u_{l_\ell} \le w_\ell \le u_{h_\ell}, \quad \ell = 1, \cdots, m \\ u_{h_\ell}, \; w_\ell > u_{h_\ell} \end{cases}$$

Definition 1 (Soroush and Valluri, 1999; Soroush and Mehranbod, 2002): A process in the form of (1) does not exhibit directionality over time horizons $N_1, \cdots, N_m \ge 0$, if and only if for every sequence $\hat{w}(k) \in \Re^m$, $k = 0, 1, \cdots, N$, and for every initial condition $\hat{x}_0 \in \Re^n$, $u(k) = \text{sat}\{\hat{w}(k)\}$ minimizes

$$\sum_{i=1}^{m} \sum_{k=0}^{N_i} [\hat{y}_i(k) - \hat{y}_i^*(k)]^2$$

subject to $u_{l_\ell} \le u_\ell(k) \le u_{h_\ell}$, $\ell = 1, \cdots, m$, where $N = \max(N_1, \cdots, N_m)$,

$$\left.\begin{aligned} \hat{x}^*(k+1) &= \Phi[\hat{x}^*(k), \hat{w}(k)], \quad \hat{x}^*(0) = \hat{x}_0 \\ \hat{y}^*(k) &= h[\hat{x}^*(k)] \end{aligned}\right\}$$

$$\left.\begin{aligned} \hat{x}(k+1) &= \Phi[\hat{x}(k), u(k)], \quad \hat{x}(0) = \hat{x}_0 \\ \hat{y}(k) &= h[\hat{x}(k)] \end{aligned}\right\}$$

According to this definition, directionality can also occur in a SISO process that is not affine in its manipulated input.

Remark 1: The class of processes whose nonsingular characteristic (decoupling) matrix is independent of u and can be made diagonal by row or column rearrangements does not exhibit the directionality over the very small time horizons $r_1, \cdots, r_m$.

Remark 2: For linear processes in the form:

$$\left.\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k), \quad \hat{x}(0) = \hat{x}_0 \\ \hat{y}(k) &= C\hat{x}(k) \end{aligned}\right\} \tag{5}$$

where A, B and C are $n \times n$, $n \times m$ and $m \times n$ matrices respectively, the characteristic (decoupling) matrix

$$\mathcal{C} = \begin{bmatrix} c_1 A^{r_1 - 1} B \\ \vdots \\ c_m A^{r_m - 1} B \end{bmatrix}$$

where c_i is the ith row of the matrix C. Linear processes whose characteristic matrix can be made diagonal by row or column rearrangements do not exhibit the directionality over the horizons $r_1, \cdots, r_m$.

Remark 3: The characteristic matrix of a linear process can be identified easily by subjecting the process at steady state $x = 0$ to step changes in its inputs. For example, the characteristic matrix entry $\mathcal{C}_{ij}$ can be calculated by subjecting the process at steady state $x = 0$ to a step change in u_j only and measuring the first nonzero value of y_i. In this case, $\mathcal{C}_{ij}$= [first nonzero value of y_i]/[size of the step change in u_j], $i = 1, \cdots, m$, $j = 1, \cdots, m$. In practice, the characteristic-matrix identification step may need to be repeated, if the process moves significantly away from the nominal steady state.

3. CONTROL QUALITY LOSS IN ANALYTICAL CONTROL

For linear processes, the general unconstrained MPC law of (4) is a linear state feedback that can be written in an analytical form. In the case of nonlinear processes, one may be able to find an analytical state feedback equivalent to the general unconstrained MPC law of (4). Thus, the general unconstrained MPC law of (4), in general, can be viewed as an analytical time-invariant state feedback. A question that one might ask is when the analytical state feedback of (4) together with clipping (the controller output) is adequate for constrained processes of the form (1). In other words, for what class of processes can this analytical control scheme provide a satisfactory control quality [which is not much less than the quality provided by the numerical state feedback of (3)]? To address this question, let us define the following index of control quality loss.

Definition 2: The index of control quality loss in analytical control over time horizons $N_1, \cdots, N_m$, is defined as:

$$I_N = \max_k \sum_{i=1}^{m} \sum_{\ell=0}^{N_i} \left[\frac{y_i(k+\ell)}{y_i^*(k+\ell)} - 1 \right]^2, \quad k = 0, \cdots, \infty$$

where $N_1 \ge r_1, \cdots, N_m \ge r_m$,

$$\left.\begin{aligned} x^*(k+1) &= \Phi[x^*(k), u^*(k)], \quad x^*(0) = x_0 \\ y^*(k) &= h[x^*(k)] \end{aligned}\right\}$$

$$\left.\begin{aligned} x(k+1) &= \Phi[x(k), sat\{w(k)\}], \quad x(0) = x_0 \\ y(k) &= h[x(k)] \end{aligned}\right\}$$

Special Case: The index of control quality loss in analytical control over time horizons $r_1, \cdots, r_m$, takes the form:

$$I_r = \max_k \sum_{i=1}^{m} \sum_{\ell=0}^{r_i} \left[\frac{y_i(k+\ell)}{y_i^*(k+\ell)} - 1\right]^2, \quad k = 0, \cdots, \infty$$

The definition of the directionality over short horizons of $r_1, \cdots, r_m$, can be used to identify the class of processes that have a control-quality loss index of 0.

Remark 4 (Soroush and Mehranbod, 2002): For processes that do not exhibit the directionality over the time horizons $r_1, \cdots, r_m$, the index of control quality loss, $I_r = 0$. The class of processes whose nonsingular characteristic matrix is independent of u and can be made diagonal by row or column rearrangements does not exhibit directionality over the time horizons $r_1, \cdots, r_m$.

4. EXAMPLES

4.1 Example 1

Consider the two-input two-output, time-invariant, linear process:

$$G(z) = \begin{bmatrix} \dfrac{1}{z-0.5} & \dfrac{1000(z-1)}{(z-0.5)^2} \\ \dfrac{z-1}{(z-0.5)^2} & \dfrac{2}{z-0.5} \end{bmatrix}$$

The steady state gain matrix (K_p) and the characteristic matrix ($\mathcal{C}$) of the process are:

$$K_p = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 1 & 1000 \\ 1 & 2 \end{bmatrix}$$

The steady state gain matrix is diagonal, implying that the process is "statically", completely decoupled. The process, however, is dynamically, strongly coupled, since its characteristic matrix is not diagonal; this process exhibits the directionality. Thus, or this process $I_r \neq 0$, implying that if MPC is not used, the loss of control quality will be significant.

4.2 Example 2

Consider the two-input two-output, time-invariant, linear process:

$$G(z) = \begin{bmatrix} \dfrac{1}{z-0.5} & \dfrac{1000}{(z-0.5)^2} \\ \dfrac{1}{(z-0.5)^2} & \dfrac{2}{z-0.5} \end{bmatrix}$$

The steady state gain matrix (K_p) and the characteristic matrix ($\mathcal{C}$) of the process are:

$$K_p = \begin{bmatrix} 2 & 4000 \\ 4 & 4 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

implying that the process is "statically", highly coupled but is dynamically, weakly coupled (characteristic matrix is diagonal); the process does not exhibit the directionality. Thus, for this process $I_r = 0$, implying that if analytical control is used, the loss of control quality will not be significant.

Examples 1 and 2 confirmed that control system selection should be on the basis of dynamic rather than steady-state characteristics of a process. In particular, it is the nature of the characteristic matrix not that of the steady state gain matrix that determines when analytical control is adequate.

4.3 Example 3: SISO Nonlinear Chemical Reactor

Consider an isothermal continuous stirred-tank reactor in which a non-elementary chemical reaction $A \rightarrow B$ takes place. The rate of production of B is given by:

$$R_B[C_A] = 0.036C_A^3 - 0.78C_A^2 + 4.4C_A - 1.0$$

where C_A denotes the concentration of the reactant. The reactor dynamics are described by

$$\frac{dC_B}{dt} = 0.036C_A^3 - 0.78C_A^2 + 4.4C_A - 1.0 - 0.625C_B \quad (6)$$

with $C_B(0) = 0$ and $0 \leq C_A \leq 10$, where C_B denotes the concentration of the product. Exact time-discretization (with a sampling period of 0.1 *hour*) of the reactor model leads to

$$C_B(k+1) = 0.9394C_B(k) + 0.0969R_B[C_A(k)] \quad (7)$$

The control objective is to maintain the concentration of the product, C_B, at a desired level ($C_{B_{sp}} = 8.0\ kmol.m^{-3}$) by manipulating the reactant concentration, C_A. Here, the characteristic matrix

$$\mathcal{C} = 0.108u^2 - 1.56u + 4.4$$

Because $\mathcal{C}$ depends on u, the process has the directionality and can, therefore, benefit greatly from model predictive control.

For this process, we use the two model predictive controllers (Grantz et al., 1998):

$$\begin{aligned} &\eta(k+1) = 0.9\eta(k) + 0.0394C_B(k) + 0.0969R_B[u(k)] \\ &\min_{u(k)} \{R_B[u(k)] - 1.0320[e(k) + \eta(k)] - 0.4066C_B(k)\}^2 \\ &\text{subject to} \qquad 0 \leq u(k) \leq 10 \end{aligned} \quad (8)$$

and

$$\eta(k+1) = 0.9\eta(k) + 0.0394C_B(k) + 0.09694I$$

$$\min_{w(k)} \{R_B[w(k)] - 1.0320[e(k) + \eta(k)] - 0.4066$$

$$u(k) = \mathrm{sat}(w(k))$$

with $\eta(0) = C_B(0)$ and $e = 8 - C_B$. Numer simulations are carried out for the following tl cases:

- Case C1: the process of (7) without the in constraints, under the controller of (8) or
- Case C2: the process of (7) with the in constraints, under the controller of (8).
- Case C3: the process of (7) with the in constraints, under the controller of (9).

In the absence of the input constraints, the c trollers of (8) and (9) induce the first-order lir input-output response

$$C_B(k+1) - 0.9C_B(k) = 0.8 \qquad ($$

As shown in Figure 1, the closed-loop respoi are considerably different for the three ca The response in case C1 has, of course, lowest possible integral of squared error [I (ISE=3.47×10^2). The advantage of constrai MPC (case C2 with an ISE = 3.90×10^2) c unconstrained MPC with clipping (case C3 v an ISE = 1.45×10^3) is quite obvious.

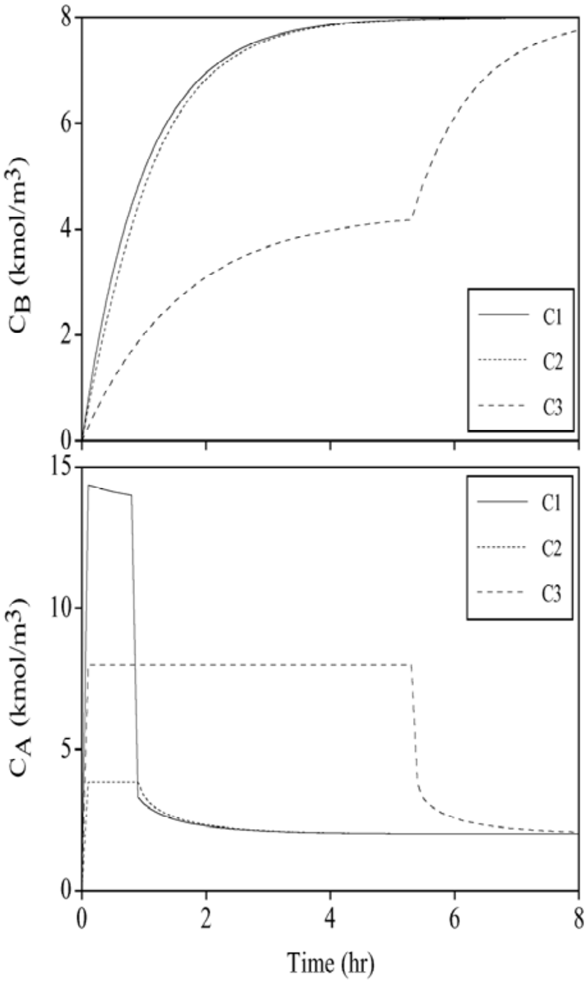

Fig. 1. Controlled output and manipulated input of Example 3.

4.4 Example 4: Multivariable Linear Example

Consider the two-input two-output, time-invar linear process:

$$\left.\begin{aligned} x_1(k+1) &= 0.99x_1(k) + 0.40u_1(k) - 3.00u_2(k) \\ x_2(k+1) &= 0.99x_2(k) - 0.01u_1(k) + 0.40u_2(k) \\ y_1(k) &= x_1(k) \\ y_2(k) &= x_2(k) \end{aligned}\right\} \quad (11)$$

with $x_1(0) = 0$, $x_2(0) = 0$, $|u_1(k)| \leq 1$, and $|u_2(k)| \leq 1$.

For this linear example,

$$\mathcal{C} = \begin{bmatrix} 0.40 & -3.00 \\ -0.01 & 0.40 \end{bmatrix}$$

which is not diagonal, and thus the process cannot be regulated effectively by analytical control.

The performance of a model predictive controller is compared with that of two completely-decentralized proportional-integral (PI) controllers with conditional integration (to prevent integral windup) with the set point values $y_{sp_1} = 8$ and $y_{sp_2} = 3$. Figure 2 depicts the closed-loop process input and output responses under the two PI controllers (dashed line) and the model predictive controller (dotted line). In the absence of the input constraints, the closed-loop responses (solid line) under the decentralized PI controllers and the model predictive controller are the same. In the presence of the input constraints, however, while the PI controllers show poor responses in y_1, the model predictive controller provides a significantly better closed-loop performance. In this example, it happens that in the presence of the constraints the y_2 response is better when the PI controllers are used. With the model predictive controller, a y_2 response closer to the unconstrained one can be obtained by placing a higher weight on y_2.

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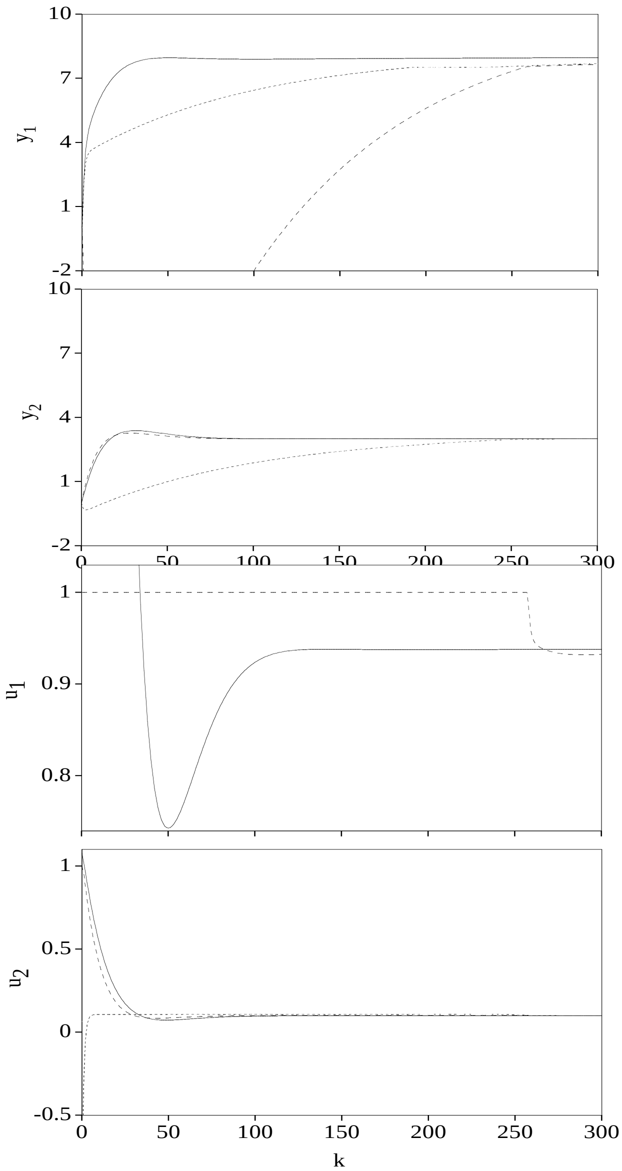

Fig. 2. Controlled outputs and manipulated inputs of Example 4.

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