PLANT-WIDE OPTIMAL CONTROL WITH DECENTRALIZED MPC

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Abstract: Most standard MPC implementations partition the plant into several units and apply MPC individually to these units. It is known that this strategy can lead to sub-optimal plant-wide control performance, especially if the units interact strongly. This paper tackles the problem of achieving optimal control performance in plants with such an MPC structure. A modeling framework, geared for use in MPC, that incorporates the interactions between the subsystems is employed. One may think that modeling the interactions and communicating the control actions between the controllers is sufficient to improve controller performance. We show that this idea is incorrect and can lead to closed-loop instability. A cooperation based MPC algorithm that converges to the plant-wide optimum is developed. In practical implementations, the cooperation based MPC scheme may have to be terminated before convergence is reached. To permit such flexibility, we propose a feasible cooperation based MPC algorithm. All cooperative iterates in this algorithm are feasible and the resulting MPC controller is closed-loop stable. Two examples comparing the performance of optimal and sub-optimal MPC controllers are presented.

Keywords: MPC, plant-wide control

1. INTRODUCTION

With ever increasing demands on improvements in productivity and efficiency of operation, the chemical industry today places significant importance on plant-wide automation. Improvements in practical control technology can potentially cut costs and raise profits. Over the last decade, Model Predictive Control (MPC) has established itself as one of the popular choices for advanced process control.

A number of articles have focused on improved plant-wide decentralized control. (Sandell-Jr. *et al.*, 1978) provide a survey of decentralized control methods for large scale systems. Some of these decentralized controller design approaches approximate or ignore the interactions between the various subsystems (Lunze, 1992; Siljak, 1991). Other approaches lead to a sub-optimal plant-wide control strategy (Acar and Ozguner, 1988; Samyudia and Kadiman, 2002). (Cui and Jacobsen, 2002) describe performance limitations arising due to the decentralized control framework. Most attention in the area of MPC has focussed on the centralized control framework. However, centralized control may not be practical, especially for large systems. In recent times, there has been some interest in studying plant-wide control within the MPC framework. (Lu, 2000; Kulhavy *et al.*, 2001) outlined the idea of cross-functional integration within the MPC framework and discussed requirements and potential benefits/impact of such technology. (Katebi and Johnson, 1997) proposed a two level de-

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composition coordination strategy for Generalized Predictive Control(GPC) based on the master-slave paradigm. (Zhu *et al.*, 2000; Zhu and Henson, 2002) described a plant-wide control strategy based on the integration of linear and nonlinear MPC coupled with a plant decomposition procedure. A sub-optimal strategy for MPC of interconnected systems was proposed by (Antwerp and Braatz, 2000). While these methods have been demonstrated to work well for the cases considered, no safeguards against failure or closed-loop properties have been established. (Camponogara *et al.*, 2002) proposed a distributed model predictive control scheme to coordinate multiple agents. In their scheme, the MPC controllers exchange state and input trajectory information amongst themselves during a sampling interval. We shall show through examples in section 6 that a strategy based purely on the exchange of trajectory information (i.e. communication) is unreliable and may cause closed-loop instability.

The purpose of this work is to understand the idea of optimal plant-wide control from the MPC framework and provide algorithms that attain the best achievable plant-wide control performance through the suitable integration of the various MPC controllers. The inherent communication capability of the prediction horizons and an enhanced modeling framework are utilized in arriving at these algorithms. For the methods described in this work, we prove nominal properties like optimality, feasibility and closed-loop stability. These properties are central to the practical applicability of plant-wide control methods.

2. MODELING INTERACTIONS FOR PERFORMANCE IMPROVEMENT

Decentralized models Consider a plant comprising of M subsystems. Let the decentralized (local) model for each subsystem be represented by a discrete linear time invariant (LTI) model of the form

$$x_{ii}(k+1) = A_{ii}x_{ii}(k) + B_{ii}u_i(k), \quad i = 1, 2, \ldots, M \tag{1}$$

in which k is discrete time, and we assume (A_{ii}, B_{ii}, C_{ii}) is a minimal realization for each (u_i, y_i) input-output pair. Owing to material, energy and/or information flows there exists a level of interaction between the subsystems. In the decentralized modeling framework, it is assumed that the interactions have a negligible effect on local variables. This assumption is not reliable in many situations and can lead to deterioration in control performance. The centralized modeling framework, on the other hand, results in models that are larger than necessary. In many cases, centralized control is not feasible for implementation due to its sheer size, multiple time scales of operation, and limited operational flexibility.

Interaction models (IM): We employ IM to quantitatively assess the interactions between the subsystems. Modeling the interactions between subsystems provides a framework for improving plant-wide control performance while retaining most of the advantages of the decentralized control approach.

Consider a subsystem i $(i = 1, 2, \ldots, M)$. We represent the effect of an interacting subsystem j, $j \neq i$ on subsystem i through a discrete LTI model of the form

$$x_{ij}(k+1) = A_{ij}x_{ij}(k) + B_{ij}u_j(k) \quad j = 1, \ldots, M \tag{2}$$

in which B_{ij} represents the effect of the inputs of subsystem j on the states of subsystem i.

For each subsystem i, the decentralized state vector x_{ii} is augmented with states arising due to interactions with other subsystems. Let x_i denote the augmented set of states for subsystem i. The interaction model (IM) for the entire plant can therefore be expressed as a union of decentralized and interaction models. After identification of the significant interactions from closed-loop operating data, we expect that many of the interaction terms will be zero. In the decentralized model, all of the interaction terms are zero.

3. PROBLEM FORMULATION AND ASSUMPTIONS

In this work, we consider five formulations ($\mathcal{P}^1$ to $\mathcal{P}^5$) for unconstrained model predictive control (MPC), or LQR. In each case, a controller is defined by using the first input from the solution to the corresponding optimization problem.

$\mathcal{P}^1$: **Centralized MPC** (3)

$$\begin{aligned} \underset{\boldsymbol{x},\boldsymbol{u}}{\text{Min}} \quad & \Phi(\boldsymbol{x}, \boldsymbol{u}) = \sum_i \Phi_i(\boldsymbol{x}, \boldsymbol{u}) \\ \text{s.t.} \quad & x(t+1) = Ax(t) + Bu(t) \\ & x(k) = \widehat{x}(k) \\ & t = k, k+1, \ldots, k+N-1 \end{aligned}$$

in which

$$\begin{aligned} \boldsymbol{x} &= \{x(k), x(k+1), \ldots, x(k+N)\} \\ \boldsymbol{u} &= \{u(k), u(k+1), \ldots, u(k+N-1)\} \\ & x \in \mathbb{R}^n, u \in \mathbb{R}^m \end{aligned}$$

represent the centralized state and input trajectories through the control horizon. The horizon length is N. The cost function for subsystem i is Φ_i. The vector $\widehat{x}(k)$ represents the current estimate of the centralized model states at discrete time k.

$\mathcal{P}_i^2$: **Decentralized MPC** (4)

$$\begin{aligned} \underset{\boldsymbol{x}_{ii},\boldsymbol{u}_i}{\text{Min}} \quad & \Phi_i(\boldsymbol{x}_{ii}, \boldsymbol{u}_i) \\ \text{s.t.} \quad & x_{ii}(t+1) = A_{ii}x_{ii}(t) + B_{ii}u_i(t) \\ & x_{ii}(k) = \widehat{x}_{ii}(k) \\ & t = k, k+1, \ldots, k+N-1 \end{aligned}$$

in which $\boldsymbol{x}_{ii}$ represents the decentralized state trajectory for subsystem i.

$$\boldsymbol{x}_{ii} = \{x_{ii}(k), x_{ii}(k+1), \dots, x_{ii}(k+N)\}$$

with $x_{ii}(k) \in \mathbb{R}^{n_{ii}}$. $\widehat{x}_{ii}(k)$ denotes the estimate of the decentralized model states at discrete time k. For communication and cooperation based MPC, an iteration and exchange of variables between subsystems is performed during a sample time. We denote this iteration number as p.

$$\mathcal{P}_i^3: \quad \textbf{Communication based MPC} \qquad (5)$$

$$\begin{aligned} \underset{\boldsymbol{x}_i^p, \boldsymbol{u}_i^p}{\text{Min}} \quad & \Phi_i\left(\boldsymbol{x}_i^p, \boldsymbol{u}_i^p, \boldsymbol{u}_{j\neq i}^{p-1}\right) \\ \text{s.t.} \quad & x_i^p(t+1) = A_i x_i^p(t) + B_{ii} u_i^p(t) \\ & \qquad + \sum_{j\neq i} B_{ij} u_j^{p-1}(t) \\ & x_i(k) = \widehat{x}_i(k) \\ & t = k, k+1, \dots, k+N-1 \end{aligned}$$

in which

$$\begin{aligned} \boldsymbol{x}_i &= \{x_i(k), x_i(k+1), \dots, x_i(k+N)\} \\ \boldsymbol{u}_i^p &= \{u_i^p(k), u_i^p(k+1), \dots, u_i^p(k+N-1)\} \end{aligned}$$

and $\widehat{x}_i(k)$ represents the current estimate of the communication model states. Notice that the input sequence for subsystem i, $\boldsymbol{u}_i^p$, is optimized to produce its value at iteration p, but the other subsystem's inputs are not updated during this optimization; they remain at iterate $p-1$. The objective function is the one for subsystem i only.

We next modify the objective functions of the subsystem's controllers in order to provide a means for cooperative behavior among the controllers. We replace the objective Φ_i with an objective that measures the entire system performance. Many suitable objectives are possible. Here we choose the simplest case, the overall plant objective, which is the weighted sum of all the subsystems' objectives, $\Phi = \sum_i \mathrm{w}_i \Phi_i$. When all the cost functions are quadratic, this formulation exactly matches the centralized cost.

$$\mathcal{P}_i^4: \quad \textbf{Cooperation based MPC} \qquad (6)$$

$$\begin{aligned} \underset{\boldsymbol{x}_i^p, \boldsymbol{u}_i^p}{\text{Min}} \quad & \Phi\left(\boldsymbol{x}_i^p, \boldsymbol{u}_i^p, \boldsymbol{x}_{j\neq i}^{p-1}, \boldsymbol{u}_{j\neq i}^{p-1}\right) \\ \text{s.t.} \quad & x_i^p(t+1) = A_i x_i^p(t) + B_{ii} u_i^p(t) \\ & \qquad + \sum_{j\neq i} B_{ij} u_j^{p-1}(t) \\ & x_i(k) = \widehat{x}_i(k) \\ & t = k, k+1, \dots, k+N-1 \end{aligned}$$

4. COOPERATION BASED CONTROL

Sub-optimality (in the plant-wide sense) of the communication based MPC scheme provides the necessary motivation to seek an alternate approach–one that is plant-wide optimal in the nominal case. We note that while the communication based scheme accounts for the effect of the interacting subsystems, it fails to consider the effect of changes in local variables on other subsystems. Such an exchange scheme can give rise to conflicts between the various communicating controllers and lead to deterioration in control performance (see section 6.1). The cooperation based MPC scheme eliminates the possibility of such controller conflicts by utilizing a cost function that reflects the plant-wide effect of local variable changes. Each local regulator now charts a control path that not only minimizes the local objectives but also a projected cost at the plant-wide level. The algorithm and some useful properties of this method are outlined below.

Algorithm 1. Given $(\boldsymbol{x}_i^0, \boldsymbol{u}_i^0)$, $\quad i = 1, 2, \dots, M$
and $\quad \epsilon > 0$
$p \leftarrow 1, \quad e_i \leftarrow \Gamma\epsilon \quad \Gamma \gg 1$
while $\quad e_i > \epsilon$
$(\boldsymbol{x}_i^p, \boldsymbol{u}_i^p) = \arg\{\mathcal{P}_i^4\}, \quad i = 1, 2, \dots, M$
$e_i = \|(\boldsymbol{x}_i^p, \boldsymbol{u}_i^p) - (\boldsymbol{x}_i^{p-1}, \boldsymbol{u}_i^{p-1})\|$
$p \leftarrow p + 1$
end (while)

At discrete time k, define

$$\overline{\boldsymbol{x}}_i(k) = [x_i(k+1|k), \dots, x_i(k+N|k)]^T, \qquad (7)$$

$$\overline{\boldsymbol{u}}_i(k) = [u_i(k|k), \dots, u_i(k+N-1|k)]^T \qquad (8)$$

to be the vector of predicted states and inputs respectively, through the control horizon. For the particular case for which all the cost functions are quadratic, we can represent the cost function for the i^{th} subsystem as

$$\Phi_i\left(\overline{\boldsymbol{x}}_i(k), \overline{\boldsymbol{u}}_i(k)\right) = \frac{1}{2}\overline{\boldsymbol{x}}_i^T(k)\mathbb{Q}_i\overline{\boldsymbol{x}}_i(k) + \frac{1}{2}\overline{\boldsymbol{u}}_i^T(k)\mathbb{R}_i\overline{\boldsymbol{u}}_i(k) \qquad (9)$$

in which

$$\begin{aligned} \mathbb{Q}_i &= \text{diag}\left(Q_i(1), Q_i(2), \dots Q_i(N)\right) \quad \text{and} \\ \mathbb{R}_i &= \text{diag}\left(R_i(0), R_i(1), \dots R_i(N-1)\right) \end{aligned}$$

denote the state and input penalties in the regulator through the control horizon. Eliminating the states from the model equations ((1), (2)) and propagating the inputs through the control horizon, we can rewrite the interaction model for subsystem i as

$$\overline{\boldsymbol{x}}_i(k) = E_{ii}\overline{\boldsymbol{u}}_i(k) + \sum_{j=1, j\neq i}^{M} E_{ij}\overline{\boldsymbol{u}}_j(k) + f_i x_i(k) \qquad (10)$$

in which

$$x_i(k) = \widehat{x}_i(k) \qquad (11)$$

$$E_{ij} = \begin{bmatrix} B_{ij} & 0 & \dots & \dots & 0 \\ A_i B_{ij} & B_{ij} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_i^{N-1} B_{ij} & \dots & \dots & \dots & B_{ij} \end{bmatrix} \quad f_i = \begin{bmatrix} A_i \\ A_i^2 \\ \vdots \\ \vdots \\ A_i^N \end{bmatrix} \qquad (12)$$

Theorem 1. Given $Q_i(j) > 0$, $R_i(j) > 0$ and $Q_i(N) > 0 \quad j = 0, 1, 2, \dots N-1$, $\quad i = 1, 2, \dots, M$. Algorithm 1 converges to an optimal limit point.

Theorem 2. If the initial trajectory $\boldsymbol{z}^0 = (\boldsymbol{x}^0, \boldsymbol{u}^0)$ satisfies $\|\boldsymbol{z}^0 - \boldsymbol{z}^*\| \leq \frac{1}{\epsilon^\delta}$ for some $\delta \geq 0$ and $\epsilon > 0$ then we have $\|\boldsymbol{z}^p - \boldsymbol{z}^*\| < \epsilon$ for all $p > \dfrac{(\delta+1)\log\epsilon}{\log\Omega}$, $0 < \lambda < \Omega < 1$ in which $0 < \lambda < 1$ is a system dependent property.

5. FEASIBLE COOPERATION BASED CONTROL

Theorem 2 provides a conservative lower bound on the number of cooperative iterates required for convergence. It is possible that the process sampling time is shorter than the time required for convergence of the cooperative iterates. To facilitate the practical use of the cooperative control methodology, we need the iterates generated by the cooperation based MPC scheme to be feasible and closed-loop stable. The above two properties permit us to terminate the cooperative scheme at the end of each sampling interval and inject the final iterate into the plant even if convergence has not been attained. In order to satisfy feasibility and closed-loop stability, we propose modifications to the existing cooperation based MPC framework. For the purpose of this study, we restrict ourselves to MPC controllers with quadratic cost functions.

To guarantee feasibility, we eliminate the model constraints and solve the cooperation based MPC problem as a collection of unconstrained optimization problems of the form

$\mathcal{P}_i^5$: **Feasible cooperation based MPC**

$$\min_{\boldsymbol{u}_i^p} \quad \Phi_i^p(x_i(k), \overline{\boldsymbol{u}}_{j\neq i}^{p-1}) = \frac{1}{2}\overline{\boldsymbol{u}}_i^{p^T}(k)\mathfrak{R}_i\overline{\boldsymbol{u}}_i^p(k) + \left(r_i(k) + \sum_{j=1,j\neq i}^{M} H_{ij}\overline{\boldsymbol{u}}_j^{p-1}(k)\right)^T \overline{\boldsymbol{u}}_i^p(k) \tag{13}$$

in which

$$\mathfrak{R}_i = \mathbb{R}_i + E_{ii}^T\mathbb{Q}_iE_{ii} + \sum_{j\neq i}^{M} E_{ji}^T\mathbb{Q}_jE_{ji} \tag{14}$$

$$r_i(k) = E_{ii}^T\mathbb{Q}_if_ix_i(k) + \sum_{j\neq i}^{M} E_{ji}^T\mathbb{Q}_jf_ix_j(k) \tag{15}$$

$$H_{ij} = E_{ii}^T\mathbb{Q}_iE_{ij} + E_{ji}^T\mathbb{Q}_jE_{jj} \tag{16}$$

$$x_j(k) = \widehat{x}_j(k) \tag{17}$$

We have the following results for feasible cooperation based MPC

Lemma 3. Given the MPC formulation $\mathcal{P}_i^5$, the sequence of cost functions

$$\{\Phi^p\} = \{\sum_{i=1}^{M} \Phi_i^p\left(x_i(k), \overline{u}_{j\neq i}^{p-1}\right)\}$$

is a non-increasing function of p.

At time k, let

$$\overline{\boldsymbol{u}}_i^{p*}\left(x_i(k), \overline{\boldsymbol{u}}_{j\neq i}^{p-1}\right) = \left[u_i^{p*}(k|k), \ldots, u_i^{p*}(k+N-1|k)\right]^T$$

represent the solution to the optimization problem $\mathcal{P}_i^5$ at iteration number p. The corresponding optimum value of the cost function is denoted as $\Phi_i^{p*}(x_i(k), \overline{\boldsymbol{u}}_{j\neq i}^{p-1})$. The control law is obtained through a receding horizon implementation of optimal control whereby the input applied to subsystem i is $u_i^p(x_i(k), \overline{\boldsymbol{u}}_{j\neq i}^{p-1}) = u_i^{p*}(k|k)$. Lemma 3 leads to the following theorem on closed-loop stability.

Theorem 4. Given the MPC formulation $\mathcal{P}_i^5 \quad i = 1, 2, \ldots, M$. Suppose the following assumptions are satisfied

- (A_i, B_i) stabilizable
- Perfect knowledge of the states at each sampling instant k (state feedback).
- $Q_i(0) = Q_i(1) = \cdots = Q_i(N-1) = Q_i$
- $R_i(0) = R_i(1) = \cdots = R_i(N-1) = R_i$
- Stage cost $L_i(k+j|k) = \frac{1}{2}x_i(k+j|k)^TQ_i(j)x_i(k+j|k) + \frac{1}{2}u_i(k+j|k)^TR_i(j)u_i(k+j|k) > 0, \quad j = 0, 1, \ldots, N-1$.
- Terminal state constraint $x_i(k+N|k) = 0$
- $u_i(k+j|k) = 0, j \geq N$

then the origin is an asymptotically stable equilibrium point for the closed-loop system $x(k+1) = Ax(k) + Bu^p(x(k), \overline{\boldsymbol{u}}^{p-1})$, in which $u^p(x(k), \overline{\boldsymbol{u}}^{p-1}) = \left[u_1^p(x_1(k), \overline{\boldsymbol{u}}_{j\neq 1}^{p-1}), \ldots, u_M^p(x_M(k), \overline{\boldsymbol{u}}_{j\neq M}^{p-1})\right]^T$, for all $x(k)$ and all $p = 1, 2, \ldots$.

6. EXAMPLES

We present two examples to demonstrate and compare the control performance of the various MPC schemes described in this paper ($\mathcal{P}^1 - \mathcal{P}^5$). To provide a consistent platform for comparison, we ensure the following

- Perfect state feedback.
- Each regulator utilizes the optimal targets i.e. the centralized model targets, in evaluating its optimal control trajectory.
- No disturbances affect the plant.

To assess controller performance , we use the cumulative stage cost (CSC) as an index to quantify controller performance. The CSC index is given by $\sum_k \sum_{i=1}^{M} L_i(k|k)$. In addition, the iterative schemes for methods $\mathcal{P}_i^3, \mathcal{P}_i^4$ and $\mathcal{P}_i^5$ are initialized with the corresponding decentralized control solution.

6.1 Example 1

We consider a plant constituted by two subsystems. The first subsystem is represented by a LTI state space model comprising of 3 inputs, 3 outputs and 5 states. The LTI model for the second

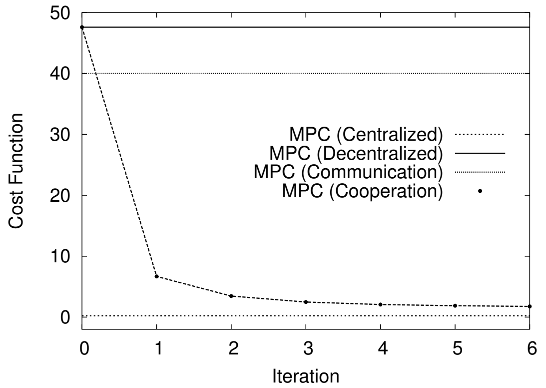

Fig. 1. Performance of the various schemes within a sampling interval

Table 1. Controller performance

MPC Configuration	Cumulative Stage Cost
Decentralized MPC	344.45
Communication based MPC	∞ (Unstable)
Cooperation based MPC (1 iterate)	5.581
Cooperation based MPC (10 iterates)	3.307
Cooperation based MPC (convergence)	2.654
Centralized MPC	2.654

subsystem consists of 4 inputs, 4 outputs and 8 states. The two units are such that the behavior of one subsystem affects the performance of the other.

We investigate the performance of the decentralized, centralized, communication and cooperation based MPC frameworks when a set point change is made to the outputs of subsystems 1 and 2. For scheme $\mathcal{P}_i^5$, the terminal state constraint (Theorem 4) is employed to guarantee closed-loop stability. Table 1 summarizes the controller performance for the various schemes considered ($\mathcal{P}^1 - \mathcal{P}_i^5$).

The decentralized MPC regulators attempt to track their respective set points with no knowledge of the effect of the interacting subsystem. Control performance consequently suffers and the deterioration in control performance is reflected as a relatively large CSC index. The centralized MPC regulator on the other hand is equipped with the complete model and all interactions are accounted for exactly. This controller therefore achieves the optimal nominal plant-wide control performance. The sequence of iterates generated by the communication based MPC scheme converge to a non-optimal limit point. In this case, the resulting communication based MPC controller leads to unstable closed-loop behavior. This example illustrates the unreliability of the communication based MPC scheme ($\mathcal{P}_i^3$). In accordance with Theorem 1, the cooperation based MPC scheme $\mathcal{P}_i^4$ converges and the performance is identical to the centralized MPC controller. For scheme $\mathcal{P}_i^5$, we examine closed loop performance when the cooperative scheme is terminated after 1 iterate and when the scheme is terminated after 10 iterates. As expected from lemma 3, the CSC index decreases with the iteration number until we reach the centralized CSC index. Figure 1 shows the performance of the various schemes within a sampling interval. It is verified that the controller resulting from scheme $\mathcal{P}_i^5$ stabilizes the plant in closed-loop for all values of p.

6.2 Example 2

Table 2. Controller performance

MPC Configuration	Cumulative Stage Cost
Decentralized MPC	76.953
Communication based MPC	52.773
Cooperation based MPC (1 iterate)	51.142
Cooperation based MPC (10 iterates)	50.529
Cooperation based MPC (convergence)	50.495
Centralized MPC	50.495

The second example is a reactor separator with recycle. The control challenges posed by the dynamics of the recycle stream has been studied by (Luyben, 1993*a*), (Luyben, 1993*b*) and (Luyben, 1994). (Wu *et al.*, 2002) and (Monroy-Loperena *et al.*, 2004) have addressed the problem of plant-wide control structure selection for reactor separator processes with recycle. For the process considered, fresh feed of reactant A enters a CSTR where an exothermic reaction A $\rightarrow$ B takes place. The outlet stream from the reactor is a mixture of product B and unreacted A. This stream is fed to a distillation column where the separation of the two occurs. The distillate, rich in reactant A, is recycled back to the reactor. The bottoms stream is almost pure B and is drawn out. It is desired to maintain the reactor temperature at 331.5^oC. The flow rate to the reactor is manipulated to achieve this objective. The control objective for the distillation column is to maintain the top and bottom compositions of A at their desired specs by manipulating the reflux to the column and the vapor boil up flow rate.

The flow of the reactor outlet stream to the distillation column and the recycle of the distillate back to the reactor are sources of complex interaction. A change in the reactor operating conditions affects the performance of the distillation column and vice-versa. For the purpose of this study, a linearized model around the desired steady state is used to represent the plant.

Due to changes in operating conditions, it is desired to decrease the reactor temperature by 5^oC and increase the composition of A in the distillate by 0.05. Decentralized MPC gives unsatisfactory performance as this mode of operation does not consider the two-way interaction between the reactor and the distillation column. In comparison, communication based MPC leads to an improvement in control performance. However, as observed in section 6.1, communication based MPC is an unreliable control strategy as it lacks well defined convergence and closed-loop properties.

With cooperation based MPC, we require a maximum of 28 cooperative iterates to achieve convergence. To permit real-time implementation, we employ the feasible cooperation based MPC controller in conjunction with Theorem 4. This controller is guaranteed to be feasible and closed-loop stable. In accordance with Lemma 3, the cost function is observed to be a non-increasing function of the iteration number p at each time step. The performance of the various MPC controllers are summarized in Table 2. For this example, we note that a single cooperative exchange leads to a 33.5 percent improvement in control performance in comparison with decentralized MPC.

7. CONCLUSIONS

In this work, the problem of integrating several MPC controllers to achieve optimal plant-wide control was addressed. The proposed cooperative MPC methodology is plant-wide optimal at convergence. To facilitate real time implementation, we propose a feasible cooperative MPC strategy. All iterates generated by this scheme are feasible and the resulting MPC controller is closed-loop stable. The theoretical aspects and performance of the proposed methods are illustrated through two examples.

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