PREDICTIVE CONTROL OF SWITCHED NONLINEAR PROCESSES WITH SCHEDULED MODE TRANSITIONS [1]

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Abstract: A predictive control framework is proposed for the constrained stabilization of switched nonlinear processes. The problem of stabilization of a switched process, transiting between its constituent modes at prescribed transition times, is considered. The main idea is to design Lyapunov–based predictive controllers for each of the constituent modes in which the switched process operates, and incorporate constraints in the predictive controller design of the individual modes which ensure that transitions between the modes at pre-determined switching times occur in a way so as to guarantee stabilization of the switched closed–loop process. This is achieved as follows: For each constituent mode, a Lyapunov-based predictive controller (MPC) is designed. Then, constraints are incorporated in the MPC design which ensure that: (1) the states of the closed–loop process, at the time of the transition, reside in the stability region of the mode that the process switches into, and (2) the value of the Lyapunov function of the process for each of the modes is nonincreasing whenever the mode is active, thereby guaranteeing stabilization. The proposed control method is demonstrated through application to a chemical process example.

Keywords: Switched Systems, Process Schedule, Nonlinear Processes, Hybrid control, Input constraints, Predictive Control, Bounded Control.

1. INTRODUCTION

Process operation often involves controlled, discrete transitions between multiple, continuous dynamical modes of operation, in order to handle, for example, changes in raw materials, energy sources, product specifications and market demands, giving rise to an overall process behavior that is more appropriately viewed as a hybrid system, i.e., intervals of piecewise continuous behavior interspersed by discrete transitions. Compared to purely continuous processes, the hybrid nature of these systems and their changing dynamics makes them more difficult to describe, analyze, or control.

A class of hybrid systems that has attracted significant attention recently, because it can model several practical control problems that involve integration of supervisory logic-based control schemes and feedback control algorithms, is the class of switched (or multi-modal) processes. For this class, results have been developed for stability analysis using the tools of multiple Lyapunov functions (MLFs) for linear (Peleties and DeCarlo, 1991) and nonlinear (Branicky, 1998) systems, and the concept of dwell time (Hespanha and Morse, 1999); the reader may refer to (Liberzon and Morse, 1999) for a survey of results in this area. These results have motivated the development of methods for control of various classes of switched processes (see, e.g., (Hu *et al.*, 1999)). Despite this progress, however, significant research remains to be done in the direction of control of switched nonlinear processes, especially

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since the majority of practical switched processes exhibit inherently nonlinear dynamics.

Motivated by this, in (El-Farra and Christofides, 2003*a*), a framework for coordinating feedback and switching for control of hybrid nonlinear processes was developed. The key feature of the proposed control methodology is its hierarchical structure involving the integrated synthesis of: (1) a family of lower-level bounded nonlinear controllers that stabilize the continuous dynamical modes, and provide an explicit characterization of the stability region associated with each mode, and (2) upper-level switching laws that ensure stability of the overall switched closed-loop process. While this approach serves to *determine* whether or not a switch can be made at a time without loss of stability guarantees, the controller design does not address the problem of *ensuring* that such a switch can be made safely at a pre-decided time.

Guiding the process through a prescribed trajectory requires a control algorithm that can achieve optimal closed–loop trajectory behavior in the presence of constraints. A practical controller design, for handling of state and input constrains, in an optimal control setting is model predictive control (MPC). One of the important issues that arise in the practical implementation of predictive control policies for the purpose of stabilization, however, is the difficulty they typically encounter in identifying, a priori (i.e., before controller implementation), the set of initial conditions starting from where feasibility and closed-loop stability are guaranteed. This typically results in the need for extensive closed-loop simulations to search over the whole set of possible initial conditions, thus adding to the overall computational load. The fallout of this problem is more pronounced when considering MPC of hybrid processes that involve switching between multiple constrained modes. Re-tuning the parameters of each predictive controller (e.g., horizon length) on-line, or running extensive closed-loop simulations in the midst of mode transitions, to determine the feasibility of switching, becomes computationally intractable, especially if the hybrid process involves a large number of modes with frequent switches.

In this work, a predictive control framework is proposed for the constrained stabilization of switched nonlinear processes. The problem of stabilization of a switched process, transiting between its constituent modes at prescribed transition times, is considered. The main idea is to design Lyapunov–based predictive controllers for each of the constituent modes in which the switched process operates, and incorporate constraints in the predictive controller design of the individual modes which ensure that transitions between the modes at pre-determined switching times occur in a way so as to guarantee stabilization of the switched closed–loop process. This is achieved as follows: For each constituent mode, a Lyapunov-based predictive controller (MPC) is designed. Then, constraints are incorporated in the MPC design which ensure that: (1) the states of the closed–loop process, at the time of the transition, reside in the stability region of the mode that the process switches into, and (2) the value of the Lyapunov function of the process for each of the modes decreases whenever the mode is active, thereby guaranteeing stabilization. The proposed control method is demonstrated through application to a chemical process example.

2. PRELIMINARIES

We consider the class of switched nonlinear processes represented by the following state-space description:

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x(t)) + G_{\sigma(t)}(x(t))u_{\sigma(t)} \\ u_{\sigma(t)} &\in \mathcal{U}_{\sigma} \\ \sigma(t) &\in \mathcal{K} := \{1, \cdots, p\} \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of continuous-time state variables, $u_\sigma(t) = [u^1_\sigma(t) \cdots u^m_\sigma(t)]^T \in \mathcal{U}_\sigma \subset \mathbb{R}^m$ denotes the vector of constrained manipulated inputs taking values in a nonempty compact convex set $\mathcal{U}_\sigma := \{u_\sigma \in \mathbb{R}^m : \|u_\sigma\| \leq u^{max}_\sigma\}$, where the notation $\|\cdot\|$ denotes the euclidian norm, $u^{max}_\sigma > 0$ is the magnitude of the constraints, $\sigma : [0, \infty) \rightarrow \mathcal{K}$ is the switching signal which is assumed to be a piecewise continuous (from the right) function of time, i.e., $\sigma(t_k) = \lim_{t \rightarrow t_k^+} \sigma(t)$ for all k, implying that only a finite number of switches is allowed on any finite interval of time. p is the number of modes of the switched process, $\sigma(t)$, which takes different values in the finite index set $\mathcal{K}$, represents a discrete state that indexes the vector field $f(\cdot)$, the matrix $G(\cdot)$, and the control input $u(\cdot)$, which altogether determine $\dot{x}$. Throughout the paper, we use the notations $t_{k_r^{in}}$ and $t_{k_r^{out}}$ to denote the time at which, for the r-th time, the k-th subsystem is switched in and out, respectively, i.e., $\sigma(t^+_{k_r^{in}}) = \sigma(t^-_{k_r^{out}}) = k$. With this notation, it is understood that the continuous state evolves according to $\dot{x} = f_k(x) + G_k(x)u_k$ for $t_{k_r^{in}} \leq t < t_{k_r^{out}}$. It is assumed that all entries of the vector functions $f_k(x)$, and the $n \times m$ matrices $G_k(x)$, are sufficiently smooth on $\mathbb{R}^m$ and that $f_k(0) = 0$ for all $k \in \mathcal{K}$. Throughout the paper, the notation $L_f\bar{h}$ denotes the standard Lie derivative of a scalar function $\bar{h}(x)$ with respect to the vector function $f(x)$, $L_f\bar{h}(x) = (\partial\bar{h}/\partial x)f(x)$.

In this work, we consider the problem of stabilization of continuous-time nonlinear processes where mode transitions are decided and executed at pre-determined times. In order to provide the necessary background for our main results in section 3, we will briefly review in the remainder of this section the design procedure for, and the stability properties of a bounded controller design, stability properties of which are then exploited in the design of a Lyapunov–based model predictive controller that guarantees stability for an explicitly characterized set of initial conditions. For simplicity, we focus on the state feedback problem assuming measurements of $x(t)$ to be available for all t.

2.1 *Bounded Lyapunov-based control*

Consider the system of Eq.1, for a fixed $\sigma(t) = k$, where $\mathcal{U}_k = \{u \in \mathbb{R}^m : \|u\| \leq u_k^{max}\}$, and a control Lyapunov function V_k for which, we construct, using the results in (Lin and Sontag, 1991) (see also (El-Farra and Christofides, 2001; El-Farra and Christofides, 2003*b*)), the following continuous bounded control law

$$u_k(x) = -k_k(x)(L_{G_k}V_k)^T(x) \equiv b_k(x) \quad (2)$$

where $k_k(\cdot) =$

$$\frac{L_fV_k + \sqrt{L_fV_k^2 + (u_k^{max}\|(L_{G_k}V_k)^T\|)^4}}{\|(L_{G_k}V_k)^T\|^2\left[1 + \sqrt{1 + (u_k^{max}\|(L_{G_k}V_k)^T\|)^2}\right]} \quad (3)$$

and $L_{G_k}V_k(x) = [L_{g_k^1}V_k \cdots L_{g_k^m}V_k]$ is a row vector where g_k^i is the ith column of G_k. For the above controller, one can show, using standard Lyapunov arguments, that whenever the closed–loop state trajectory, x, evolves within the region described by the set $\Phi_k(u_k^{max}) =$

$$\{x \in \mathbb{R}^n : L_fV_k(x) < u_k^{max}\|(L_{G_k}V_k)^T(x)\|\} \quad (4)$$

then the controller satisfies the constraints, and the time-derivative of the Lyapunov function is negative-definite. An estimate of the stability region can be obtained by using the level sets of V_k, i.e.,

$$\Omega_k(u_k^{max}) = \{x \in \mathbb{R}^n : V_k(x) \leq c_k^{max}\} \quad (5)$$

where $c_k^{max} > 0$ is the largest number for which $\Phi_k(u_k^{max}) \supset \Omega_k(u_k^{max})\backslash\{0\}$. $\Omega_k(u_k^{max})$ then provides an estimate of the stability region, starting from where the origin of the constrained closed–loop process for the mode k, under the control law of Eqs.2-3, is guaranteed to be asymptotically stable.

The bounded controller of Eqs.2-3 possesses a robustness property with respect to measurement errors, that preserves closed–loop stability when the control action is implemented in a discrete (sample and hold) fashion with a sufficiently small hold time, Δ. Specifically, given the size of a neighborhood of the origin (characterized by δ'), one can compute a positive real number Δ^*, such that if Δ is less than Δ^*, then for all initial conditions in Ω_k, the closed–loop state remains in Ω_k and eventually converges to the specified neighborhood of the origin (for proof, see (Mhaskar *et al.*, 2004*b*)). This robustness property will be exploited in the Lyapunov-based predictive controller design of section 2.2.

2.2 *Model predictive control*

In this section, we consider model predictive control of the system of Eq.1, for a fixed $\sigma(t) = k$ for some $k \in \mathcal{K}$. We present here a Lyapunov–based design of MPC that guarantees feasibility of the optimization problem and hence constrained stabilization of the closed–loop process from an explicitly characterized set of initial conditions. For this MPC design, the control action at state x and time t is obtained by solving, on-line, a finite horizon optimal control problem of the form:

$$P(x,t) : \min\{J(x,t,u_k(\cdot))|u_k(\cdot) \in S_k\} \quad (6)$$

$$s.t. \ \dot{x} = f_k(x) + G_k(x)u_k \quad (7)$$

$$V_k(x(t+\Delta)) < V_k(x(t)) \ \ if \ \ V_k(x(t)) > \delta_k' \quad (8)$$

$$V_k(x(t+\Delta)) \leq V_k(x(t)) \ \ if \ \ V_k(x(t)) \leq \delta_k' \quad (9)$$

where $S_k = S_k(t,T)$ is the family of piecewise continuous functions (functions continuous from the right), with period Δ less than Δ^*, mapping $[t, t+T]$ into $\mathcal{U}_k$ and T is the specified horizon. A control $u_k(\cdot)$ in S_k is characterized by the sequence $\{u_k[j]\}$ where $u_k[j] := u_k(j\Delta)$ and satisfies $u_k(t) = u_k[j]$ for all $t \in [j\Delta, (j+1)\Delta)$. The performance index is given by

$$J(x,t,u_k(\cdot)) = \int_t^{t+T} \left[\|x^u(s;x,t)\|_Q^2 + \|u_k(s)\|_R^2\right] ds$$

where R and Q are strictly positive definite, symmetric matrices and $x^u(s;x,t)$ denotes the solution of Eq.1, due to control u, with initial state x at time t. The minimizing control $u_k^0(\cdot) \in S_k$ is then applied to the plant over the interval $[j\Delta, (j+1)\Delta)$ and the procedure is repeated indefinitely. This defines an implicit model predictive control law

$$M_k(x) := argmin(J(x,t,u_k(\cdot))) = u_k^0(t;x,t) \quad (10)$$

The predictive controller formulation of Eqs.6–10 requires that the value of the Lyapunov function decrease after the first step only. Since the optimization problem is guaranteed to be initially and successively feasible for all initial conditions in Ω_k, every control move that is implemented, enforces a decay in the value of the Lyapunov function, leading to stability (for a proof and further details, see (Mhaskar *et al.*, 2004*b*)).

3. PREDICTIVE CONTROL OF SWITCHED NONLINEAR PROCESSES

Consider now the nonlinear switched process of Eq.1. The control problem is formulated as the one of designing a Lyapunov-based predictive controller that guides the closed–loop process trajectory in a way that the schedule, defined by the sets $\mathcal{T}_{k,in}$ and $\mathcal{T}_{k,out}$, for all $k \in \mathcal{K}$, is followed while also, stability of the closed–loop process is achieved. The implementation of the predictive controller that addresses this problem is described algorithmically below (for a proof and further details, see (Mhaskar *et al.*, 2004*b*)). Without loss of generality, it is assumed that the switching schedule dictates initializing the process in mode 1:

(1) Given the process model of Eq.1 and the constraints on the input, design the bounded controller of Eqs.2–3 for each mode and compute

the stability region estimate, $\Omega_k(u_k^{max})$, for the bounded controller using Eq.4-5.

(2) Given the size of the neighborhood, that the state is required to converge to, compute Δ^* such that the predictive controller for each of the individual modes can drive the process to the given neighborhood of the origin for discretization times less than Δ^*, and use a $\Delta \in (0, \Delta^*]$ for the purpose of MPC implementation.

(3) Consider the time $t_{k_r^{in}}$, which designates the time that the closed–loop process is in the k-th mode for the r-th time, and the state belongs to the stability region of the k-th mode (for the purpose of initialization, i.e., at $t = 0$, this would correspond to $t_{1_1^{in}}$, and the state belonging to the stability region $\Omega_1(u_1^{max})$).

(4) If at any time, the switching sequence is aborted, go to step 7; else continue to step 5.

(5) From the set of prescribed switching times, pick $t_{m_j^{in}} = t_{k_r^{out}}$ ($t_{m_j^{in}}$, therefore, is the time that the next switch takes place, and that the process, upon exiting from the current mode enters mode m for the j-th time).

(6) Implement the predictive controller of Eqs.6-10, together with the transition constraint $V_m(t_{m_j^{in}}) < V_m(t_{m_{j-1}^{in}})$ that requires that when the closed–loop process enters the mode m, the value of V_m is less than what it was at the time that the process last entered mode m (this is a version of the multiple Lyapunov function stability condition, see (Branicky, 1998)). If the process has never entered mode m before, i.e., for $j = 1$, set $V_m(t_{m_{j-1}^{in}}) = c_m^{max}$ (this requires that the first time the process enters mode m, it should enter in a way that the state belongs to the stability region corresponding to mode m). If the closed–loop state has already entered the desired ball around the origin, implement $V_m(t_{m_j^{in}}) < \delta_m^{'}$, that ensures that the state stays within the ball. Implement the predictive controller up-to time $t_{k_r^{out}}$, i.e., until the time that the process switches into mode m. Go back to step 2 to proceed with the rest of the switching sequence.

(7) Implement the Lyapunov-based predictive controller of Eqs.6-10 for the current mode to stabilize the closed–loop process.

Remark 1: The closed–loop state evolves in the stability region of the current mode (the mode in which the switched process is operating at a given time) in such a way that at the transition times the state resides in the stability region of the target mode. If at any time, the switching sequence is aborted, the predictive control algorithm is able to stabilize the closed-loop process, because by virtue of the constraint of Eqs.8-9, the state evolves such that it remains in the stability region of the current mode also. The requirement that the Lyapunov function value of the mode, upon entry, be less than that when the process last exited this mode, serves to ensure stabilization of the closed-loop process when the switching sequence is infinite (see (Branicky, 1998) for details). Additionally, the transition constraint serves to fulfil the requirement that the state belong to the stability region of the target mode at the time of the switch.

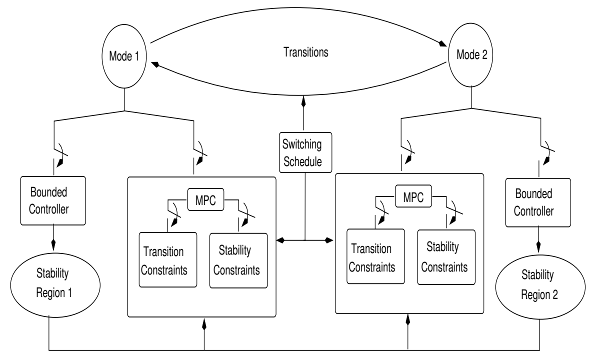

Fig. 1. A schematic representation of the predictive control structure comprising of the predictive and bounded controllers for the individual modes, together with transition constraints.

Remark 2: The constraints of Eqs.8-9 require that the closed–loop state evolves so that the value of the Lyapunov function for the current mode continues to decay, and therefore that the closed–loop state trajectory evolves in a way that it enters the stability region of the target mode (due to the transition constraint), while continuing to evolve in the stability region of the current mode. This condition may be restrictive, and hamper the feasibility of the optimization problem, especially if the stability regions of the constituent modes do not have a significant overlap. When dealing with a finite switching sequence, these constraints can be relaxed for all times before the final switch takes place. For example, a relaxed version of the transition constraint that may be used up to the time of the terminal switch, requires that $V_m(t_{m_j^{in}}) < c_m^{max}$ (i.e., the closed–loop state resides in the stability region of the target mode).

Remark 3: For linear processes, the problem of control of hybrid processes can be handled by formulating it in the form of a mixed logical dynamical system and solving a mixed-integer linear program to come up with an optimal switching sequence and switching times (see (Bemporad and Morari, 1999)). For nonlinear processes, while one can, in principle, still set up the mixed integer nonlinear programming problem, the solution to such a problem is hard to compute and online implementation of such a control algorithm is not possible. The proposed approach provides a systematic framework for control of switched nonlinear processes by decoupling the problem through appropriate constraints on the control implemented on the control action in each of the individual modes. Note also, that if it is possible to solve the mixed integer optimization problem off-line, it can be used to provide a set of optimal switching times, and an associated switching sequence, which can be then implemented online using the proposed approach.

Remark 4: For purely continuous processes, the problem of implementing MPC with guaranteed stability regions was recently addressed for linear processes under state (El-Farra *et al.*, 2004*b*) and output feedback (Mhaskar *et al.*, 2004*a*) and in (El-Farra *et al.*, 2004*a*) for nonlinear processes, by means of a hybrid control structure that unites bounded control and MPC. The hybrid control structure was used to embed the implementation of MPC within the stability region of a Lyapunov-based bounded controller which serves as a fall-back component that can be switched to in the event of infeasibility or instability of the predictive controller. In this work, the bounded controller design is used for the purpose of providing an estimate of the stability region for the Lyapunov-based predictive controller, and feasible initial guesses for the control moves (the decision variables in the optimization problem). In the event, however, that the MPC solver malfunctions, and fails to yield a solution (due, for example, to numerical problems), the switching schedule needs to be aborted and the bounded controller can be switched in to preserve stability.

4. APPLICATION TO A CHEMICAL PROCESS EXAMPLE

We consider a continuous stirred tank reactor where an irreversible, first-order exothermic reaction of the form $A \xrightarrow{k} B$ takes place. The operation schedule requires switching between two available inlet streams consisting of pure A at flow rates F_1, F_2, concentrations C_{A1}, C_{A2}, and temperatures T_{A1}, T_{A2}, respectively. For each mode of operation, the mathematical model for the process takes the form:

$$\begin{aligned} \dot{C}_A &= \frac{F_\sigma}{V}(C_{A\sigma} - C_A) - k_0 e^{\frac{-E}{RT_R}} C_A \\ \dot{T}_R &= \frac{F_\sigma}{V}(T_{A\sigma} - T_R) + \frac{(-\Delta H)}{\rho c_p} k_0 e^{\frac{-E}{RT_R}} C_A \\ &\quad + \frac{Q_\sigma}{\rho c_p V} \end{aligned} \tag{11}$$

where C_A denotes the concentration of the species A, T_R denotes the temperature of the reactor, Q_σ is the heat removed from the reactor, V is the volume of the reactor, k_0, E, ΔH are the pre-exponential constant, the activation energy, and the enthalpy of the reaction, c_p and ρ, are the heat capacity and fluid density in the reactor and $\sigma(t) \in \{1,\ 2\}$ is the discrete variable. The values of all process parameters can be found in Table 1. The control objective is to stabilize the reactor at the unstable equilibrium point $(C_A^s,\ T_R^s) = (0.57\ mol/l, 395.3\ K)$ using the rate of heat input, Q_σ, and change in inlet concentration of species A, $\Delta C_{A\sigma} = C_{A\sigma} - C_{A\sigma_s}$ as manipulated inputs with constraints: $|Q_\sigma| \leq 1KJ/hr$ and $|\Delta C_{A\sigma}| \leq 1\ mol/l$, $\sigma = 1, 2$. For each mode, we construct a bounded controller and compute its stability region estimate,

Table 1. Process parameters and steady–state values.

V	=	0.1	m^3
R	=	8.314	$kJ/kmol \cdot K$
C_{A1_s}	=	0.79	$kmol/m^3$
C_{A2_s}	=	1.0	$kmol/m^3$
T_{A1}	=	352.6	K
T_{A2}	=	310.0	K
Q_{1_s}	=	0.0	KJ/hr
Q_{2_s}	=	0.0	KJ/hr
ΔH	=	-4.78×10^4	$kJ/kmol$
k_0	=	1.2×10^9	s^{-1}
E	=	8.314×10^4	$kJ/kmol$
c_p	=	0.239	$kJ/kg \cdot K$
ρ	=	1000.0	kg/m^3
F_1	=	3.34×10^{-3}	m^3/s
F_2	=	1.67×10^{-3}	m^3/s
T_{Rs}	=	395.33	K
C_{As}	=	0.57	$kmol/m^3$

Ω_1 and Ω_2, shown in Fig.2. The parameters in the objective function of Eq.2.2 are chosen as $Q = qI$, with $q = 1$, and $R = rI$, with $r = 1.0$. The constrained nonlinear optimization problem is solved using the MATLAB subroutine fmincon, and the set of ODEs is integrated using the MATLAB solver ODE45. We first demonstrate the implementation of the Lyapunov-based predictive controller to a single mode operation of the chemical reactor, i.e., one in which the process is operated for all times in mode 1. To this end, we consider an initial condition that belongs to the stability region of the predictive controller for mode 1. As shown by the solid line in Fig.2, starting from the initial condition $(C_A,\ T_R) = (0.14\ mol/l,\ 404.9\ K)$, which belongs to the stability region of the predictive controller for mode 1, successful stabilization of the closed–loop process is achieved. The corresponding state and input profiles are shown in Fig.3.

To demonstrate the need to implement the algorithm proposed in section 3 for stabilization when switching is involved, we choose a schedule involving a switch from inlet stream 1 (mode 1) to inlet stream 2 (mode 2) at time $t = 0.1\ hr$. Once again the process is initialized within the stability region of mode 1, and the predictive controller for mode 1 is implemented. Up until $t = 0.1\ hr$, the state of the closed–loop process moves towards the desired steady state (as seen from the dashed–lines in Fig.2); however, when the process switches to mode 2, the MPC controller, designed for the stabilization of the process in mode 2, does not yield a feasible solution. If the bounded controller for mode 2 is implemented, the resulting control action is not able to stabilize the closed–loop process (dashed lines in Figs.2–3). This happens because at the time of the transition, the state of the closed–loop process (marked by the o in Fig.2) does not belong to the stability region of mode 2. Note also that while the predictive formulation of Eqs.6-10 guarantees stabilization for all initial conditions belonging to the stability region of mode 1, it does not incorporate constraints which enable or ensure a safe transition to mode 2.

Finally, the predictive control algorithm described in section 3 (which incorporates constraints that account for switching) is implemented (dash-dotted lines in Figs.2–3). The MPC controller of mode 1 is designed to drive the state of the closed–loop process such that the state belongs to the stability region of mode 2 at the switching time. Consequently, when the process switches to mode 2 at $t = 0.1\ hr$, the closed–loop process state at the switching time (marked by the $\diamondsuit$ in Fig.2) belongs to the stability region of the MPC designed for mode 2. At this time, when the process switches to mode 2 and the corresponding predictive controller is implemented, closed–loop stability is achieved.

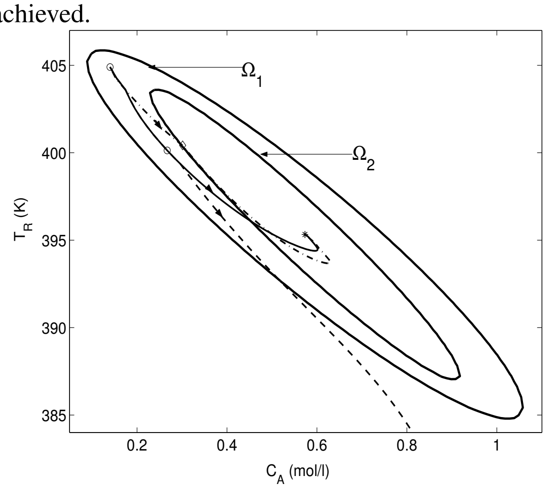

Fig. 2. Closed–loop state trajectory when the reactor is operated in mode 1 for all times under the stabilizing MPC formulation of Eqs.6-10 (solid line), when the reactor operation involves switching from mode 1 to mode 2 at $t = 0.1\ hr$, under the predictive controller design of Eqs.6-10 (dashed line), and when the reactor operation involves switching from mode 1 to mode 2 at $t = 0.1\ hr$, under the predictive controller described in section 3 (dashed–dotted line).

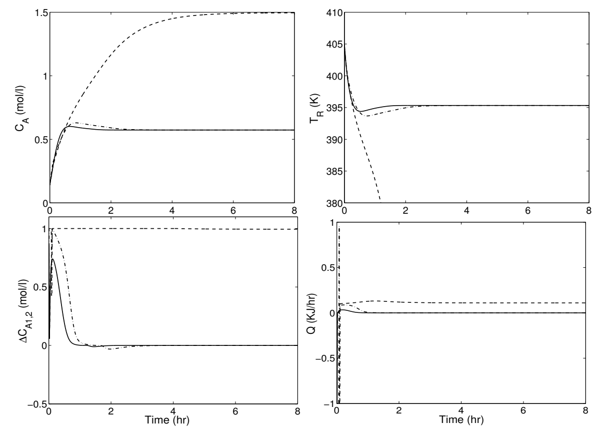

Fig. 3. Closed-loop state (top plots) and manipulated input (bottom plots) profiles: when the reactor is operated in mode 1 for all times under the stabilizing MPC formulation of Eqs.6-10 (solid line), when the reactor operation involves switching from mode 1 to mode 2 at $t = 0.1\ hr$, under the predictive controller design of Eqs.6-10 (dashed line), and when the reactor operation involves switching from mode 1 to mode 2 at $t = 0.1\ hr$, under the predictive controller described in section 3 (dashed–dotted line).

5. REFERENCES

Bemporad, A. and M. Morari (1999). Control of systems integrating logic, dynamics and constraints. *Automatica* **35**, 407–427.

Branicky, M. S. (1998). Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Trans. Automat. Contr.* **43**, 475–482.

El-Farra, N. and P. D. Christofides (2003*a*). Coordinated feedback and switching for control of hybrid nonlinear processes. *AIChE J.* **49**, 2079–2098.

El-Farra, N. H. and P. D. Christofides (2001). Integrating robustness, optimality and constraints in control of nonlinear processes. *Chem. Eng. Sci.* **56**, 1841–1868.

El-Farra, N. H. and P. D. Christofides (2003*b*). Bounded robust control of constrained multivariabe nonlinear processes. *Chem. Eng. Sci.* **58**, 3025–3047.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2004*a*). Hybrid predictive control of nonlinear systems: Method and applications to chemical processes. *Int. J. Rob. & Non. Contr.* **4**, 199–225.

El-Farra, N. H., P. Mhaskar and P. D. Christofides (2004*b*). Uniting bounded control and MPC for stabilization of constrained linear systems. *Automatica* **40**, 101–110.

Hespanha, J. P. and A. S. Morse (1999). Stability of switched systems with average dwell time. In: *Proceedings of 38th IEEE Conference on Decision and Control*. Phoenix, AZ. pp. 2655–2660.

Hu, B., X. Xu, P. J. Antsaklis and A. N. Michel (1999). Robust stabilizing control law for a class of second-order switched systems. *Syst. Contr. Lett.* **38**, 197–207.

Liberzon, D. and A. S. Morse (1999). Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine* **19**, 59–70.

Lin, Y. and E. D. Sontag (1991). A universal formula for stabilization with bounded controls. *Syst. & Contr. Lett.* **16**, 393–397.

Mhaskar, P., N. H. El-Farra and P. D. Christofides (2004*a*). Hybrid predictive control of process systems. *AIChE J., in Press*.

Mhaskar, P., N. H. El-Farra and P. D. Christofides (2004*b*). Predictive control of switched nonlinear systems with scheduled mode transitions. *IEEE Trans. Automat. Contr., submitted*.

Peleties, P. and R. DeCarlo (1991). Asymptotic stability of m-switched systems using Lyapunov-like functions. In: *Proceedings of American Control Conference*. Boston, MA. pp. 1679–1684.