PRACTICAL SOLUTIONS TO MULTIVARIATE FEEDBACK CONTROL PERFORMANCE ASSESSMENT

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Abstract: Performance assessment of multivariate feedback control with minimum variance control as the benchmark requires an interactor matrix to filter the closed-loop output. This is to transfer the coordinate of the original variables into a new one in order to separate the control invariant disturbance dynamics from the first few terms of the closed-loop output Markov parameters. There has been a great deal of interest to simplify this approach, in particular, to find methods that do not need the general interactor matrix. With this motivation, this paper discusses practical solutions to multivariate control performance assessment without relying on the general interactor matrices. In particular, we will consider two practical scenarios, 1) no *a priori* knowledge about the process model at all and 2) known time delays between each pairs of inputs and outputs. Solutions to these two scenarios are proposed. Several examples illustrate the feasibility of the proposed approaches.

Keywords: Performance monitoring, performance assessment, control monitoring, multivariate systems.

1. INTRODUCTION

In recent years, there are growing research interests in reducing the complexity of the *a priori* knowledge requirement for multivariate control performance assessment, such as (Ko and Edgar, 2001; Kadali and Huang, 2004; McNabb and Qin, 2001). Although these attempts have reduced the complexity of the *a priori* knowledge requirement to some extent, they all require certain information that is computationally simpler but fundamentally equivalent to the interactor matrices, for example, the open-loop process Markov parameter matrices, the lower triangular Toeplitz matrix, or the multivariate time delay (MTD) matrix. That is, they all require *a priori* knowledge that is beyond the pure time delays between each pairs of the inputs and outputs.

In the univariate case, one interprets output variance under minimum variance control as the variance of the optimal prediction error for the given time delay of a process. One can imagine that if a closed-loop output is highly predictable, then one should be able to do better, i.e. to compensate the predictable content by a well designed controller. Should a better controller be implemented, then the closed-loop output would have been less predictable. Therefore, the high predictability of

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a closed-loop output implies the potential to improve its performance by control re-tuning and/or re-design, or in other word, the existing controller may not have been satisfactory.

The actual process often has time delays, which prevent the complete compensation of the predictable content of the output. For example, if a univariate process has two sample time delays, then the compensation control action will not take effect on the output until two steps later and the one step ahead prediction will not be useful for its compensation. In this case, the best a controller can do is to compensate the predicted content according to the two step optimal prediction (multi-step optimal prediction) and the minimum control error will coincide with the two-step prediction error. Therefore, the two-step optimal prediction error is the lower bound of the output error that can be achieved by a feedback controller. This lower bound is also known as the minimum variance that is often used for control loop performance assessment (Harris, 1989).

Although the same rationale can not be exactly carried over to multivariate processes due to the relatively complex delay structure for multivariate processes, multi step optimal predictions provide useful information about the control performance. We will show that the multi-step optimal prediction error is analogous to closed-loop step response of the univariate process from the white noise to the output. The analogy provides an interesting interpretation of the multi-step optimal prediction error for multivariate processes and results in a new method for performance assessment.

While the prediction error based approach is general, if certain process model knowledge such as time delays between each pairs of inputs and outputs are available, one may be able to do more about control performance assessment than the prediction error based approach. It is known that the diagonal form of the interactor matrix only depends on the pair-wise time delays (delays between each pairs of the inputs and outputs). If one can determine that the process has the diagonal form of the interactor, the computation for the performance assessment can be greatly simplified (Huang and Shah, 1999). Thus there is a need to determine whether a process has a diagonal interactor matrix from the given pair-wise time delays.

Motivated by the above discussions, this paper dedicates to (1) development and analysis of the practical performance assessment approach based on optimal predictions, (2) development of methods to determine whether the process has a simple or diagonal form of interactor matrices. The remainder of this paper is organized as follows. In section 2, results on multivariate feedback control performance assessment without relying on any *a priori* process model knowledge are presented. The performance assessment problem, when the time delays between each pairs of inputs and outputs are known, is discussed in Section 3, followed by concluding remarks in section 4.

2. ASSESSMENT OF MULTIVARIATE CONTROL PERFORMANCE WITHOUT ANY A PRIORI KNOWLEDGE OF PROCESS MODEL

2.1 Performance assessment using optimal multi step prediction errors

The following multivariate process will be considered in this paper:

$$Y_t = TU_t + Na_t \tag{1}$$

where $T$and N are proper (causal), rational transfer function matrices in the backshift operator q^{-1}; Y_t, U_t and a_t are output, input and noise vectors of appropriate dimensions. a_t is further assumed to be white noise with zero mean and $Var(a_t) = \Sigma_a = I$. However, if $\Sigma_a \neq I$, one can always normalize N such that $N \leftarrow N\Sigma_a^{1/2}$ and $a_t \leftarrow \Sigma_a^{-1/2} a_t$, and then the new a_t will satisfy $\Sigma_a = I$. Furthermore, we assume that T does not have nonmimimum phase zeros in multivariable sense (Huang and Shah, 1999). The interactor matrix of T is denoted as D with order d and the interactor matrix has three possible forms: simple, diagonal and general. For details on the three forms of the interactor matrix, readers are referred to (Huang and Shah, 1999).

If the process (1) is controlled by a linear feedback controller, the closed-loop model from a_t to Y_t can be represented by a moving average or a Markov parameter form:

$$Y_t = F_0 a_t + \cdots + F_{i-1} a_{t-(i-1)} + F_i a_{t-i} + \cdots \tag{2}$$

This moving average model can be estimated from routine operating data without any *a priori* knowledge about the process.

Since a_t is white noise, the optimal *ith* step prediction is given by

$$Y_{t|t-i} = F_i a_{t-i} + F_{i+1} a_{t-i-1} + \cdots \tag{3}$$

and the prediction error $e_{t|t-i} = Y_t - Y_{t|t-i}$ is given by

$$e_{t|t-i} = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} \tag{4}$$

The covariance of the prediction error can be calculated as

$$Cov(e_{t|t-i}) = F_0 F_0^T + F_1 F_1^T + \cdots + F_{i-1} F_{i-1}^T$$

and its scalar measure

$$s_i \triangleq tr[Cov(e_{t|t-i})] = tr(F_0 F_0^T + \cdots + F_{i-1}F_{i-1}^T)$$

The incremental of the prediction error can be calculated as

$$r_i \triangleq tr[Cov(e_{t|t-i}) - Cov(e_{t|t-(i-1)})] = tr(F_{i-1}F_{i-1}^T)$$

If we plot s_i versus i, then the plot reflects how the prediction error increases with the prediction horizon. Note that as $i \to \infty$, $Cov(e_{t|t-i}) \to Cov(Y_t)$. This fact can be seen by comparing eqn(2) and (4).

A plot of r_i versus i indicates how the incremental of the prediction error changes with the prediction horizon i. These two plots (r_i and s_i) will be shown to be useful for the interpretation of the control performance.

To understand s_i and r_i, a deterministic interpretation of eqn(2) is helpful. The moving average model of eqn(2) is applicable to both stochastic and deterministic input. Considering that a_t is a unit impulse, then the coefficient matrices $F_0, F_1, \cdots, F_i, \cdots$ are the impulse response coefficients of the closed-loop system to the impulse disturbance. Since $r_i = tr(F_i F_i^T)$, r_i is a 2-norm measure of the impulse response coefficients and is analogous to the squared impulse response coefficients of a univariate process. Therefore a plot of r_i versus i reflects how the disturbance is regulated by the controller, i.e. how the control error is reduced versus time. Each point of the plot, for example r_i, represents the squared error of the closed-loop response at time i due to an impulse disturbance. Therefore, this plot of r_i versus i, is a good indication of closed-loop performance of a multivariate controller, which has been suggested in the literature for multivariate control performance monitoring (Shah *et al.*, 2002). However, we will show that a s_i plot is more useful.

Since $s_i = tr(F_0 F_0^T + F_1 F_1^T + \cdots + F_i F_i^T)$, s_i is nothing but the sum of squared error of the closed-loop response to an impulse disturbance up to time i. If we plot s_i versus i, each point of the plot, for example s_i, represents the sum of squared error (SSE) of the closed-loop response up to time i. If, after time i, the disturbance can be completely controlled (by a deadbeat control for example), then the total error of the response will remain to be s_i, which is the i step optimal prediction error from stochastic view point. There are two characters in a s_i plot worth noting. 1) It is monotonically increasing or non-decreasing curve. 2) Its steady state is $tr[Cov(Y_t)]$, the actual variance of the output. Since s_i is the integration of the squared closed-loop impulse response, it is analogous to the step response and can be used to determine dynamic information such as the settling time of the closed-loop response to the disturbance.

Motivated by the interpretation of s_i, we define the closed-loop potential p_i as

$$p_i \triangleq \frac{s_\infty - s_i}{s_\infty} \tag{5}$$

Since s_i is monotonically increasing with i, p_i is monotonically decreasing. Since $s_0 = tr[Cov(Y_t - Y_{t|t})] = 0$, $p_0 = 1$. Therefore, p_i starts from 1 at $i = 0$ and monotonically decreases to 0 and $0 \le p_i \le 1$. p_i can be interpreted as follows: If a deadbeat control action can be applied from time i, then the process output SSE can be reduced by $100 \times p_i$ percent. From stochastic view point, if i is greater than the interactor order d (Huang and Shah, 1999), there exists a control such that the variance of the multivariate output can be reduced by $100 \times p_i$ percent of the current variance (see Lemma 1 next for this control law). Since the order of the actual interactor matrix is usually unknown, one would look for the trajectory of the closed-loop potential versus a range of possible d. Potential plots such as those illustrated in Fig. 2 are useful. Faster the potential decays to zero, less the possibility to improve control performance. Due to the monotonically decreasing nature of the potentials and fixed starting and ending values of the potentials, the area below the potential plot well reflects the rate of its decaying. Therefore, it is possible to define a scalar index to monitor the change of the closed-loop potential. This index is called relative potential index and can be calculated as

$$\eta_p = \frac{\int p_i^{(2)}}{\int p_i^{(1)}} - 1 \tag{6}$$

where $p_i^{(1)}$ is a reference potential calculated, for example, from the data sampled before control tuning, and $p_i^{(2)}$ is calculated from data sampled after the tuning. The value of η_p gives the percent change of the closed-loop potential with the positive sign indicating an increased potential and the negative sign indicating a decreased potential. Note that an increase of the potential implies a deteriorated tuning while a decrease of the potential implies an improved tuning.

Lemma 1. If there is a controller with $Q^* = Q_0^*$ expressed in the IMC framework such that its closed-loop output can be written as

$$Y_t = \underbrace{F_0 + \cdots + F_{i-1}q^{-(i-1)}}_{F} + q^{-i}R \tag{7}$$

where $i \ge d$ and R is a proper rational transfer function matrix, then there exists another physically realizable controller with $Q^* = Q_i^*$ such that its closed-loop output can be written as

$$Y_t = \underbrace{F_0 + \cdots + F_{i-1}q^{-(i-1)}}_{F} \quad (8)$$

and the controller can be written as

$$Q_i^* = Q_0^* + \tilde{T}^{-1}(q^{-i}D)RN^{-1} \quad (9)$$

where, the relation between the IMC control, Q^*, and the actual feedback control, Q, is given by

$$Q = Q^*(I - TQ^*)^{-1} \quad (10)$$

PROOF. Omitted due to space limit.

This lemma tells us that s_i, the i-step prediction error or the sum squared error, is achievable by a physically implementable controller if $i \geq d$.

2.2 Example

Example 2. Consider a 2×2 multivariable process with the open-loop transfer function matrix T and disturbance transfer function matrix N given by

$$T = \begin{bmatrix} \frac{q^{-1}}{1-0.4q^{-1}} & \frac{0.5q^{-2}}{1-0.1q^{-1}} \\ \frac{0.3q^{-1}}{1-0.4q^{-1}} & \frac{q^{-2}}{1-0.8q^{-1}} \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{1-0.5q^{-1}} & \frac{-q^{-1}}{1-0.6q^{-1}} \\ \frac{q^{-1}}{1-0.7q^{-1}} & \frac{1.0}{1-0.8q^{-1}} \end{bmatrix}$$

The white noise excitation, a_t, is a two-dimensional normally-distributed white noise sequence with $\Sigma_a = I$.

Consider that the following multiloop controller is implemented in the process:

$$Q = k\begin{bmatrix} \frac{0.5-0.20q^{-1}}{1-0.5q^{-1}} & 0 \\ 0 & \frac{0.25-0.200q^{-1}}{(1-0.5q^{-1})(1+0.5q^{-1})} \end{bmatrix}$$

In this example, three control gains, $k = 2.8, 3, 3.2$ respectively, are considered. s_i and r_i for $i = 1, 2, \cdots, 10$ are calculated and plotted in Fig.1. The s_i plot (top sub-figure) indicates that the closed-loop settling time increases with the increasing of the control gain, so does the SSE. For example, the settling time for $k = 2.8$ is about 5 samples while the settling time for $k = 3.2$ is more than 10 samples. The r_i plot shown in the bottom sub-figure of Fig.1 presents the similar information as s_i such as the information about the

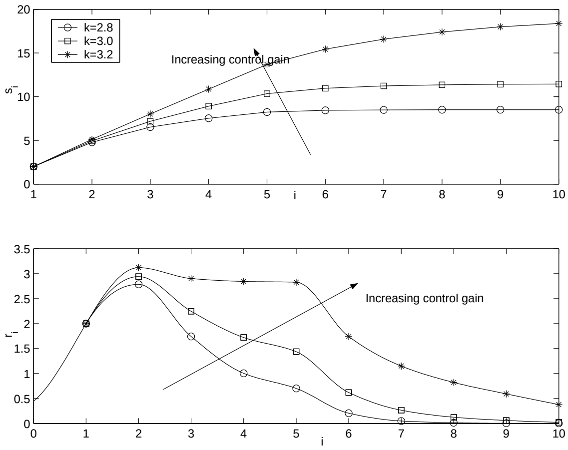

Fig. 1. s_i and r_i plots.

settling time. However, unlike the s_i plot which is monotonically increasing, the r_i plot has a more complicated and hard-to-interpret shape. We will therefore recommend to use the s_i plot and the p_i plot(to be discussed next).

The potential plot of p_i shown in Fig.2 is possibly more useful in the interpretation of control performance. For example, s_i for $k = 3.2$ has a slowest rate to approach its steady state and thus its potential decreases to zero at the slowest rate. For a considerable range of the process delays (expressed by interactor order d for example), its potential is significantly different from zero. For example, for an interactor order up to 5 samples, the potential is larger than 0.3, i.e. 30% reduction of variance is possible for the interactor order up to 5. On the other hand, for the tuning of $k = 2.8$, the potential dies to zero quickly. In this case, there is not much potential left after the interactor order is greater than 3.

For control tuning of multivariate systems or control upgrading from multiloop control to multivariable control such as MPC, one is interested in whether control performance is indeed improved. If an existing control gain is $k = 3$, assume that the gain is tuned to 2.8 or 3.2 and representative closed-loop data are sampled before and after the tuning. Then the scalar measures of the relative closed-loop potentials calculated from the data are -0.21 or 0.39 for tunings $k = 2.8$ or 3.2 with $k = 3.0$ as the reference. These results indicate that 1) if the controller gain increases to 3.2, then the resulting system has increased closed-loop potential by 39%, indicating a deteriorated performance; 2) if the controller gain decreases to 2.8, then the resulting system has reduced closed-loop potential by 21%, indicating an improved performance.

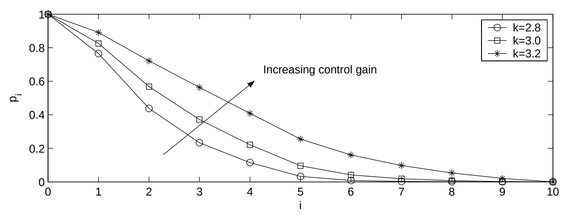

Fig. 2. Potential p_i plot.

3. ASSESSMENT OF MULTIVARIATE CONTROL PERFORMANCE WITH KNOWN PAIR-WISE TIME DELAYS

If the time delays between each pair of inputs and outputs are known *a priori*, we should search for a possible simple or diagonal structure of the interactor matrix, which can greatly simplify the computation of the multivariate minimum variance. Both the simple and the diagonal interactor matrices can be calculated from the time delays between each pairs of inputs and outputs of the process. Once the interactor matrix is known, multivariate control performance assessment problem is readily solved following the procedure of (Harris *et al.*, 1996; Huang and Shah, 1999). One may surprisingly find that the simple and diagonal interactor matrices are not uncommon, particularly in industrial process, where the sparse structure of the transfer function matrix is often observed. The sparse structure also facilitates the determination of the interactor structure.

Consider a multivariable transfer function matrix of dimension $n \times m$ given by

$$T = \begin{pmatrix} T_{11}q^{-d_{11}} & T_{12}q^{-d_{12}} & \cdots & T_{1m}q^{-d_{1m}} \\ T_{21}q^{-d_{21}} & T_{22}q^{-d_{22}} & \cdots & T_{2m}q^{-d_{2m}} \\ \cdots & \cdots & \cdots & \cdots \\ T_{n1}q^{-d_{n1}} & T_{n2}q^{-d_{n2}} & \cdots & T_{nm}q^{-d_{nm}} \end{pmatrix} \quad (11)$$

where T_{ij} is a scalar transfer function from the jth input to the ith output. Define a delay matrix

$$\Psi = \begin{pmatrix} t_{11}q^{-d_{11}} & t_{12}q^{-d_{12}} & \cdots & t_{1m}q^{-d_{1m}} \\ t_{21}q^{-d_{21}} & t_{22}q^{-d_{22}} & \cdots & t_{2m}q^{-d_{2m}} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n1}q^{-d_{n1}} & t_{n2}q^{-d_{n2}} & \cdots & t_{nm}q^{-d_{nm}} \end{pmatrix} \quad (12)$$

where d_{ij}'s are time delays that are assumed known; t_{ij} is the first non-zero impulse response coefficient from the jth input to the ith output, which is typically unknown. From Ψ, we can arrive a diagonal matrix

$$\Theta = \begin{pmatrix} q^{d_1} & & & \\ & q^{d_2} & & \\ & & \ddots & \\ & & & q^{d_n} \end{pmatrix} \quad (13)$$

where $d_i = min\{d_{ij} : j = 1, \cdots, m\}$. Then the following lemma is true:

Lemma 3. If T has a diagonal interactor matrix D then $D = \Theta$, where T and Θ are given by eqn(11), and (13), respectively.

PROOF. Omitted due to space limit.

The following lemma provides a useful criterion to determine whether a process T has a diagonal interactor matrix:

Lemma 4. If $K = \lim_{q^{-1}\to 0} \Theta\Psi$ is of full rank for all $t_{ij} \neq 0$, then 1) the interactor matrix is diagonal and 2) $D = \Theta$.

PROOF. The first part can be proved by noticing the fact that $\lim_{q^{-1}\to 0} \Theta T = \lim_{q^{-1}\to 0} \Theta\Psi = K$. By the definition of the interactor matrix, if K is of full rank, Θ must be an interactor matrix of T. Based on the result of the first part, the second part of the proof directly follows from Lemma 3.

Lemma 4 provides a sufficient condition for the determination of the interactor structure. In practice, one can relax this condition by checking the determinant of K (if K is not a square matrix, one has to use the singular values instead), to determine conditions for the singularity. MATLAB symbolic toolbox is useful for such an application. The procedure is as follows: 1) calculate the determinant or singular values, 2) find condition for the determinant to be zero or the singular values to be zero, and 3) check whether these conditions hold. Next, we will demonstrate the method of determining the structure of the interactor matrix using an industrial example.

Example 5. Consider an industrial 6×6 process presented in (Gao *et al.*, 2003), which has the following pair-wise delay matrix:

$$\Psi = \begin{pmatrix} q^{-1} & 0 & 0 & 0 & 0 & 0 \\ q^{-1} & q^{-1} & q^{-1} & 0 & q^{-1} & 0 \\ q^{-1} & q^{-1} & q^{-1} & 0 & q^{-1} & 0 \\ 0 & 0 & 0 & 0 & q^{-1} & 0 \\ q^{-2} & q^{-2} & 0 & q^{-2} & q^{-2} & q^{-2} \\ 0 & 0 & 0 & q^{-1} & q^{-1} & 0 \end{pmatrix} \quad (14)$$

An unitary interactor matrix was calculated in (Gao *et al.*, 2003) using *complete knowledge* of the process transfer function matrix. The result was

$$D = \begin{pmatrix} 0.059q & 0.72q & 0.4q & -0.56 & 0 & 0 \\ 0.006q & 0.42q & -0.90q & -0.09q & 0 & 0 \\ -0.023q & -0.31q & -0.098q & -0.47q & 0 & 0.82q \\ -0.033q & -0.44q & -0.14q & -0.67q & 0 & -0.57q \\ -0.997q & 0.068q & 0.025q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q^2 & 0 \end{pmatrix} \quad (15)$$

Using the method discussed in this section, we would get

$$\Theta = \begin{pmatrix} q & & & & & \\ & q & & & & \\ & & q & & & \\ & & & q & & \\ & & & & q^2 & \\ & & & & & q \end{pmatrix} \quad (16)$$

and K matrix

$$K = \begin{pmatrix} t_{11} & 0 & 0 & 0 & 0 & 0 \\ t_{21} & t_{22} & t_{23} & 0 & t_{25} & 0 \\ t_{31} & t_{32} & t_{33} & 0 & t_{35} & 0 \\ 0 & 0 & 0 & 0 & t_{45} & 0 \\ t_{51} & t_{52} & 0 & t_{54} & t_{55} & t_{56} \\ 0 & 0 & 0 & t_{64} & t_{65} & 0 \end{pmatrix} \quad (17)$$

The determinant of K can be calculated as $det(K) = t_{25}t_{64}t_{56}(t_{22}t_{33} - t_{32}t_{23})$. Due to the sparse structure of industrial processes, the determinants often have such a simple structure. Since $t_{ij} \neq 0$ by the definition, the condition for the determinant to be zero is $\frac{t_{22}}{t_{32}} = \frac{t_{23}}{t_{33}}$. Whether this condition holds can be easily determined by checking the variables of the process. As discussed in (Gao *et al.*, 2003), CV_2 is temperature, CV_3 internal reflux ratio, MV_2 reboiler stem flow, and MV_3 internal reflux flow. The chance for the condition to be true has the probability $\rightarrow 0$. Therefore, the interactor matrix should have a diagonal structure and the complete knowledge of the multivariate process, which was assumed in (Gao *et al.*, 2003), is not necessary. In the worst case, if one is not able to determine whether the condition is true, one would at most need to find out the first non-zero impulse response coefficients of four sub-transfer functions, a significant reduction of the *a priori* knowledge than the complete transfer function matrices.

Is this result in a contradiction to that of (Gao *et al.*, 2003) shown in eqn(15)? It has been shown in (Huang and Shah, 1999) that the two unitary interactor matrices are equivalent if

$$\bar{D} = \Gamma D \quad (18)$$

where Γ is a unitary constant matrix. It can be shown by using QR decomposition that if

$$\Gamma = \begin{pmatrix} -0.0592 & -0.0065 & 0.0236 & 0.0330 & 0.9974 & 0 \\ -0.7234 & -0.4251 & 0.3103 & 0.4417 & -0.0679 & 0 \\ 0.4000 & -0.9003 & -0.0976 & -0.1389 & 0.0249 & 0 \\ -0.5596 & -0.0934 & -0.4734 & -0.6738 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.8183 & -0.5748 & 0 & 0 \end{pmatrix} (19)$$

pre-multiplying eqn(15) by Γ results in eqn(16), a diagonal interactor matrix! Therefore, difficult to see or not, eqn(15) is in fact a diagonal interactor matrix. The thumb of rule is that higher the dimension, higher the chance to have a diagonal interactor structure.

Once the diagonal structure of the ineractor matrix is determined, the multivariate minimum variance can be determined by following the approaches discussed in (Harris *et al.*, 1996; Huang and Shah, 1999) in addition to the multi-step optimal prediction based approach for performance assessment.

4. CONCLUSION

In this paper, we have discussed two practical approaches for multivariate feedback control performance assessment. The first approach assumes that there is no any *a priori* knowledge about the process model. The solution is based on the multi-step optimal prediction error. This approach is general and applicable to processes with or without *a priori* model information. The second approach assumes that time delays between each pair of inputs and outputs are available. Then the structure of the interactor matrices may be determined from the given time delays. The simulation examples have shown the feasibility of the proposed algorithms.

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