WHICH IS THE BEST CRITERION FOR IDENTIFICATION OF DYNAMIC MODELS ?

J. O. Trierweiler* and V. C. Machado

Group of Integration, Modelling, Simulation, Control and Optimization of Processes (GIMSCOP)
Department of Chemical Engineering, Federal University of Rio Grande do Sul (UFRGS)
Rua Duque de Caxias, 1303 apt. 603 CEP: 90010-283 - Porto Alegre - RS - BRAZIL,
Fax: +55 51 3316 3277, Phone: +55 51 3316 4072
E-MAIL: {vinicius, jorge}@enq.ufrgs.br

Abstract: A novel criterion for system identification is proposed. It is based on calculations of the total quadratic error between the derivative of the plant data and the derivative of the model output. This criterion produces much better models when the experimental data are corrupted by unmeasured external disturbance.

Keywords: - system identification, dynamic models, optimization criteria, dynamic models for process control

1 INTRODUCTION

In almost all identification procedure, the optimization problem is formulated as the minimization of a square error criterion, such as: J = *min* $\sum [e(t)]^2$, where the error, *e*, is the difference between the measured (y) and predicted (y_P) output, i.e., $e(t) = y(t)-y_P(t)$. But, this kind of criterion does not give good results, if it is used directly without a filter to identify dynamic models from the plant data corrupted by unmeasured external disturbances. Since unmeasured external disturbances are always present in the industrial scenarios, it would be of the great practical impact if a simple solution for this kind of problem could be found. To go one step ahead in this direction, in this paper, it is analyzed how a criterion based on the square error of derivative signal (i.e., *min* $\sum [de(t)/dt]^2$) can improve the quality of dynamic model.

The paper is structured as follows. In section 2, a motivation example to compare both criteria is presented. In section 3, it is shown how to calculate the derivatives from plant data. In section 4, both criteria are applied to the motivation example. Finally, section 5 compares the quality of the dynamic models identified using the novel optimization criterion.

2 MOTIVATION EXAMPLE

2.1 *Experimental Plant*

To compare and analyze both criteria for identification, it is used plant data from a heating experimental unit consisting of two tanks with constant volumes in series. Figure 1 shows the system schematically. The first tank is fed with a cold water stream (F) at temperature T_0 and has an adjustable electrical heating power, which is used as manipulated variable. The controlled variable is the outlet temperature of second tank, T_2. In this work, the inlet temperature T_0 is considered as unmeasured external disturbance and the inlet flowrate is kept constant during the experiments. The real plant data used in this work are from a unit available at Process Control Group of Dortmund University (Saecker, 1995).

---

* The corresponding author.

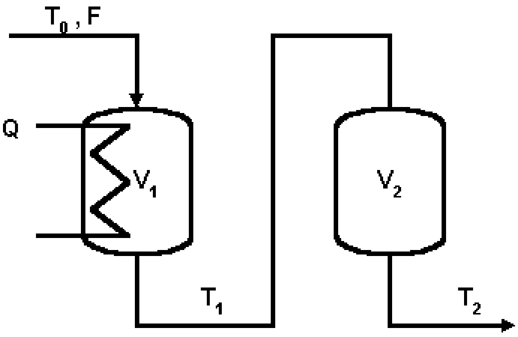

Figure 1. Schematic representation of the heating plant of two tanks.

2.2 Experimental data

An experimental data set with decreasing inlet temperature T_0 was selected and is shown in Figure 2. Although the temperature T_0 is measured, we will assume that this disturbance is not measured. To design a controller for this unit, it is necessary to have a good dynamic model between the manipulated variable (heating power, Q) and the controlled variable T_2. But the measured output temperature is influenced by both Q and T_0. Therefore, the output temperature T_2 is corrupted by the inlet temperature. It is possible to isolate the real effect of the heating duty into the outlet temperature making the temperature difference between inlet and outlet temperatures (i.e., ΔT=T_2-T_0), since the velocity as T_0 varies it is much slower than the dominant system's dynamic. Figure 3 compares the two cases: with and without inlet temperature compensation.

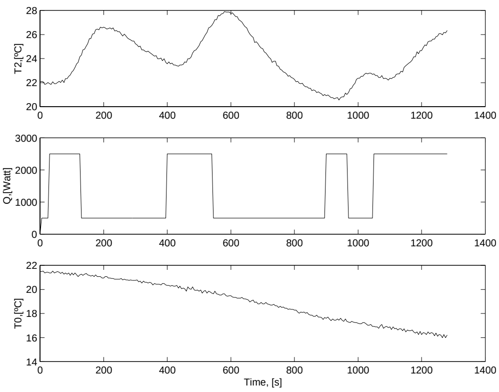

Figure 2. Experimental data set with decreasing inlet temperature T_0 (disturbance), heating duty Q (manipulated variable), and outlet temperature T_2.

Next section shows the model identification for these two data sets, applying Box-Jenkins and Output Error structures.

2.3 Black-box Identification

Now the two data sets shown in Figure 3 are identified using the Box-Jenkins and Output Error model structures available in the MATLAB System Identification Toolbox (Ljung, 1997). Box-Jenkins models (BJ) give good results when unmeasured disturbance like T_0 are present, while the Output Error (OE) models can only succeed when the discrepancy between the model and experimental data is produced by normal distributed measurement noise.

Figure 4 shows the comparison between BJ, OE, and plant data using identified models from the data set $\Delta\Delta T$ (cf. Figure 3). Since this data set the inlet temperature effect T_0 is compensate, both model structures produce very good results.

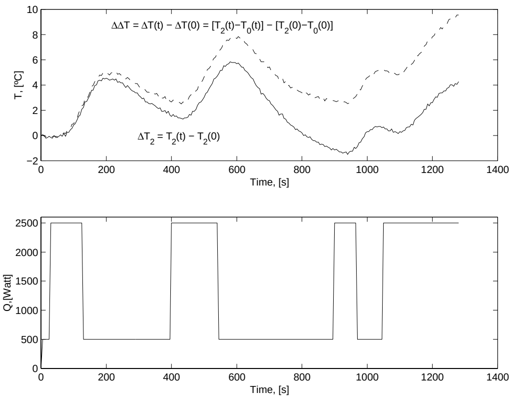

Figure 3. Data sets that are used in the identification procedure: (1) ΔT_2=$T_2(t)$-$T_2(0)$ (solid line) data without compensation of the inlet temperature effect and (2) $\Delta\Delta T = \Delta T(t)$-$\Delta T(0) = [T_2(t)$-$T_0(t)]$-$[T_2(0)$-$T_0(0)]$ (dashed line) data with compensation of the inlet temperature effect.

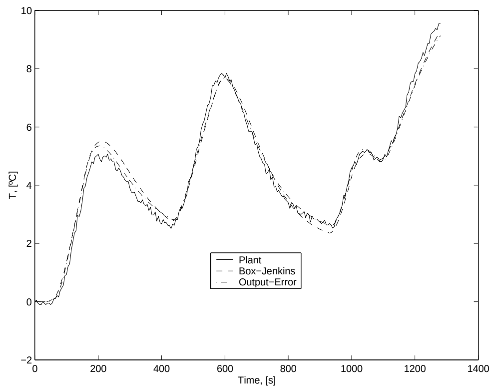

Figure 4. Comparison between plant data without effect of the variations in T_0 (i.e., $\Delta\Delta T$ of Figure 3) and the identified models OE and BJ.

The situation is completely different when the data set ΔT_2 is used. Figure 5 clearly shows that the model BJ has captured the tendency of the model, although has the larger square error if compared to the OE model. On the other hand, the OE model cannot capture the tendency as well as the BJ model. At this point an important question arises: "which is the best model?" If we compare with the results shown in

Figure 4, we can conclude that the BJ model is the correct one. Therefore, it is much better to follow the tendency (the derivative) than the output signal, in other words, the discrepancy between the model and experimental data is better described as a function of $[\mathrm{d}e(t)/\mathrm{d}t]^2$ instead of $[e(t)]^2$. In the next section, we will see how we can calculate $\mathrm{d}e(t)/\mathrm{d}t$ from experimental data.

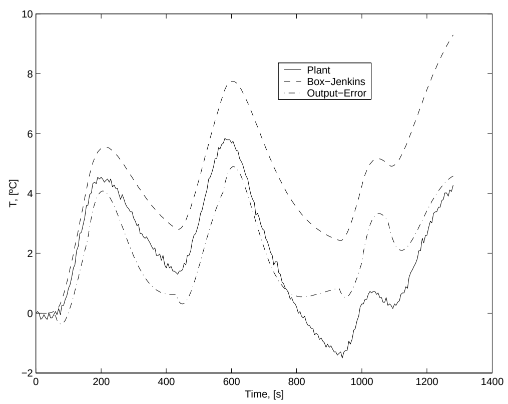

Figure 5. Comparison between plant data without compensation of T_0 (i.e., ΔT_2) and the identified models OE and BJ.

3 METHODS FOR DERIVATIVE CALCULATION

In most of the industrial cases, plant data are corrupted by noise, which increases the difficult to yield the correct derivative signal. An example of this is showed in Figure 6.

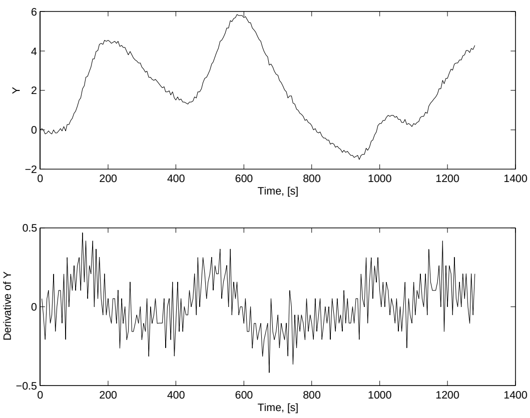

Figure 6. Plant data and its derivative.

To avoid this problem, it is necessary to filter the data before to apply the derivative. Here two different forms to filter the plant data are compared: low pass and Wavelets filters. These methodologies are implemented in MATLAB. After filtering the data, the derivative can be easily estimated, by finite differences method, for instance.

3.1 Low pass filter (Idfilt (Ljung,1997))

Filtering data with classical low pass filters as Butterworth filters usually given good results. These filters are implemented in (Ljung, 1997) and (Signal TB). In this paper the version of Butterworth filter implemented in the matlab function idfilt (Ljung,1997) was used.

3.2 Wavelets (Misra et al., 1999)

Wavelets provide a joint time-scale representation of a signal through its projection onto nested orthogonal subspaces; finer subspaces containing coarser subspaces, in a multi-resolution framework. Signal at each level of resolution is projected on an approximation subspace and a detail subspace. Differently of Fourier analysis, the Wavelets analysis is based on infinite sets of base functions, not only sines and cosines, which allow representing a noisy signal as a sum of other signals with different frequencies. If the objective is to remove noise, high frequencies component of the signal must be removed, in general.

To compare two methods to filter the data, Figure 7 shows the derivative of the filtered data by Idfilt and Wavelets algorithms. The data set is the same as in the Figure 6, but to make clearer the comparison, only the first 200s are shown.

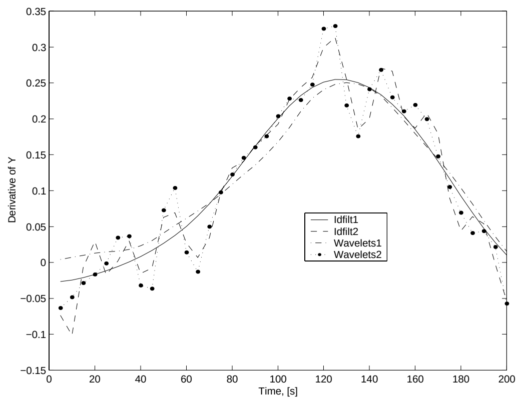

Figure 7. Calculated derivatives from filtered data by Idfilt and Wavelets algorithms. Only the first 200 s was plotted to increase the visual quality. Idfilt1 (filter order = 2, cut-off frequency = 0.1), Wavelets1 (decomposition level = 50), Idfilt2 (filter order = 2, cut-off frequency = 0.5), and Wavelets2 (decomposition level = 1).

Due to good results and simplicity, the Idfilt with filter order=2 and cut-off frequency=0.1 was selected to be used in the rest of the paper. For data collected at a corrected sampling rate (i.e., sampling time based on the process dynamic), these parameters for the filter can be in general applied and will produce good results. Therefore, we recommend using a Butterworth's second order filter with 0.1 cut-off frequency as a default and starting point selection.

4 APPLICATION IN EXAMPLE

Figure 5 shows the quality of the identified models OE and BJ when the effect of the unmeasured disturbance is not compensated. For BJ mode, as a disturbance model is simultaneously identified with the process model, can be expected better model than for OE model structure. Unfortunately, the better quality of the BJ model is masked if it is directly compared with the original data (cf. Figure 5). On the other hand, if the results are compared in a derivative basis, the BJ becomes much closer to the derivative data, as it is shown in Figure 8. Note that the OE model has a bigger discrepancy here than in the original data. To check this affirmation, the sum of the quadratic errors between the derivative of the model and the derivative of the plant data (i.e.,$\sum$ [d$e(t)$/dt]^2) and the sum of the quadratic errors (i.e.,$\sum$ [$e(t)$]^2) were calculated. The results are summarized in Table 1 and confirm our visual impression, that the OE models fit better the original data and the BJ model the derivative data.

Table 1. Sum of the quadratic errors obtained from the Figures 7 and 8 for the OE and BJ models.

Sum of quadratic error $\sum [e(t)]^2$		Sum of quadratic derivative error ($\sum$ [d$e(t)$/dt]^2)	
OE	BJ	OE	BJ
$0.75\text{x}10^5$	$4.25\text{x}10^5$	$1.73\text{x}10^2$	$0.32\text{x}10^2$

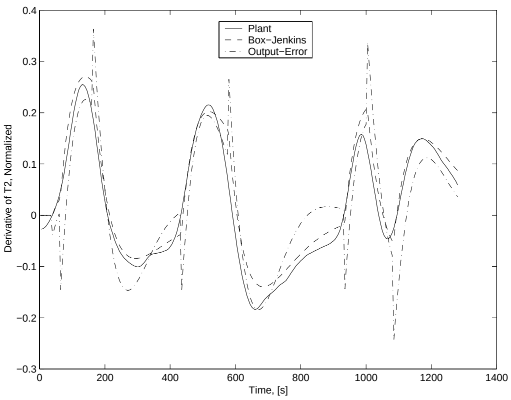

Figure 8: Comparison between the derivative of the plant data filtered by Idfilt and the derivatives of the identified models OE and BJ.

When data are affected by load disturbances, it is possible to affirm that quadratic error criterion brings wrong conclusions about the quality of the identified models. In the studied example, BJ model was better than OE model, and not the opposite, as quadratic error criterion would indicate. On the other hand, the quadratic derivative error criterion has reflected better the quality of the identified model. This makes us to think about to use a different criterion for system identification based on *min* $\sum$ [d$e(t)$/dt]^2. The next section explores this idea.

5 SQUARE DERIVATIVE ERROR AS CRITERION FOR IDENTIFICATION OF DYNAMIC MODELS

The most common cost function for system identification is to minimize the square error, i.e.

$$S1 = \min \sum_{k=1}^{n} \left(y_k - \hat{y}_{k|k-1} \right)^2 \qquad (1)$$

where n is the number of experimental data points, y_k is the k value of the controlled variable, and $\hat{y}_{k|k-1}$ is the predicted value of the output variable (model). Here, we propose to use the square derivative error as a new criterion for system identification, i.e.

$$S2 = \min \sum_{k=1}^{n} \left(\frac{d}{dt} y_k - \frac{d}{dt} \hat{y}_{k|k-1} \right)^2 . \qquad (2)$$

Just to show the benefices of the new criterion, we apply it to identify the parameter $a0$, $a1$, $a2$, $a3$, and $a4$ of the following second order transfer function with pure time delay:

$$\hat{G}(s) = \frac{(a0 \cdot s + a1) \cdot e^{-a4 \cdot s}}{a2 \cdot s^2 + a3 \cdot s + 1} \qquad (3)$$

To illustrate the potentialities of the new criterion, it will be applied to two examples.

5.1 The two tanks heating system

This system was already as introduced as motivation example in section 2. Here, the criterion $S1$ (eq. 1) and $S2$ (eq. 2) are applied to the data set with and without inlet temperature compensation.

The $\Delta\Delta T$ data set (with inlet temperature compensation). Table 2 summarizes the parameters of the identified models. The transfer function $G1$ was identified using a discrete OE model structure. The discrete model was converted to continuos and the corresponding parameters of (3) are listed in the table. The solution of the nonlinear optimization problems $S1$ and $S2$ for the second order transfer function (3) gave the models $G2$ and $G3$ of Table 2, respectively. The optimization problems were solved using standard and generic optimization methods implemented in the *lsqnonlin* function of the Matlab Optimization Toolbox (Coleman et al., 1997).

Figure 9 shows that all three models are very similar what is also seen in Figure 10 where the corresponding step responses are presented. These results confirm that for experimental data without significant unmeasured external disturbance the optimization criteria $S1$ and $S2$ are similar.

Table 2. Transfer function coefficients were obtained from the $\Delta\Delta T$ experimental data

TF	Parameters of equation (3)				
	$a0$	$a1$	$a2$	$a3$	$a4$
$G1$ / OE	-0,01	$4.4.10^{-3}$	4997.5	154.1	30.0
$G2$ / S1	0.0	$4.3.10^{-3}$	5541.0	148.9	28.0
$G3$ / S2	0.0	$4.3.10^{-3}$	5663.0	150.5	28.0

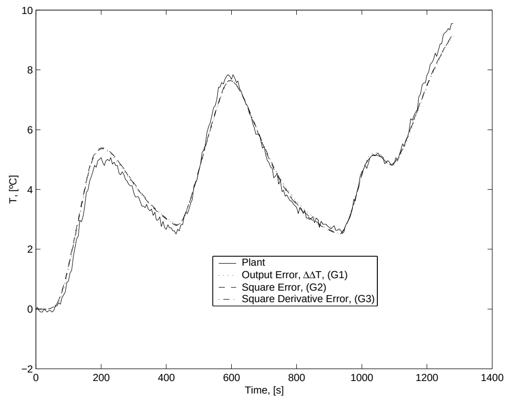

Figure 9. Models were identified using the $\Delta\Delta T$ experimental data set: $G1$ identified with OE (is assumed as true model, see fig. 4), $G2$ identified with criterion $S1$ and $G3$ with $S2$.

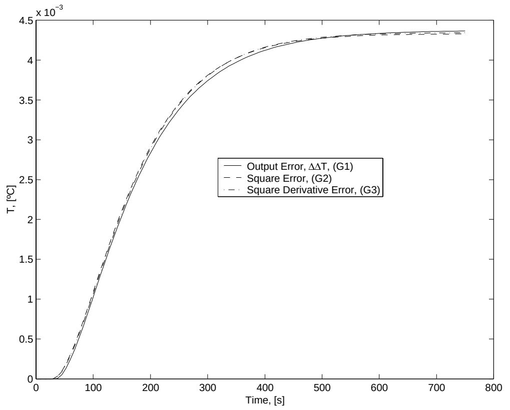

Figure 10. Step responses of $G1$, $G2$, and $G3$.

The ΔT_2 data set (without inlet temperature compensation). For this experimental data, the situation is completely different to the case before. Here, due to the external unmeasured/uncompensated external disturbance, the optimization criteria $S1$ and $S2$ converge to different parameters as shown in Table 3. In this table, $G4$ was identified using a discrete BJ model structure. The discrete model was converted to continuos and the corresponding parameters of (3) are listed. The solution of the nonlinear optimization problems $S1$ and $S2$ are $G5$ and $G6$, respectively.

Figure 11 shows clearly that the square derivative error criterion, i.e., $S2$, is the best criterion for identification, since it is closed to the Box-Jenkins model (assumed as the true model, for the case with load disturbances). The step responses shown in Figure 12 make clearer the difference between the models. Note that the model based on the square derivative error ($G6$/$S2$) has almost the same parameters and response as BJ and is almost unaffected by the presence of disturbance as we can see with the comparison between the models $G3$ and $G6$.

Table 3: Transfer function coefficients were obtained from the ΔT_2 experimental data

TF	Parameters of equation (3)				
	$a0$	$a1$	$a2$	$a3$	$a4$
$G4$ / BJ	-0,006	$4.4.10^{-3}$	6013.2	142.6	30.0
$G5$ / S1	0.0	$1.9.10^{-3}$	3158.0	112.4	30.0
$G6$ / S2	0.0	$4.0.10^{-3}$	5156.0	143.6	30.0

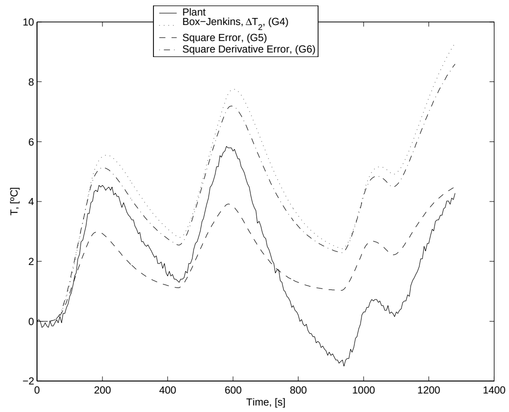

Figure 11: Models were identified using the ΔT_2 experimental data set: $G4$ identified with BJ (is assumed as true model, see fig. 5), $G5$ (dashed line) identified with criterion $S1$ and $G6$ (dashdot line) with $S2$.

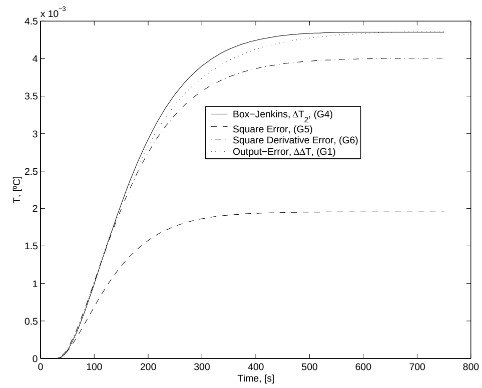

Figure 12. Step responses of $G4$, $G5$, and $G6$.

5.2 Additional example

The following LTI model is used as the true plant in this example:

$$G(s) = \frac{1 \cdot e^{-5 \cdot s}}{s^2 + 2s + 1} \quad (4)$$

Figure 13 shows the block diagram used to generate the data to be used in the identification procedure. The plant output *y*, the PRBS input *u*, and the output disturbance *d* used as experimental data are shown in Figure 14. It is assumed that the output disturbance is not measured, i.e., in the identification procedure only the signal *u* and *y* are used.

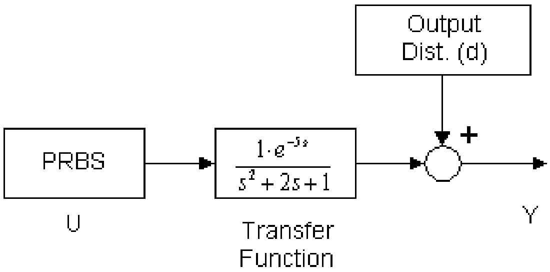

Figure 13. Block diagram used to generate the identification data

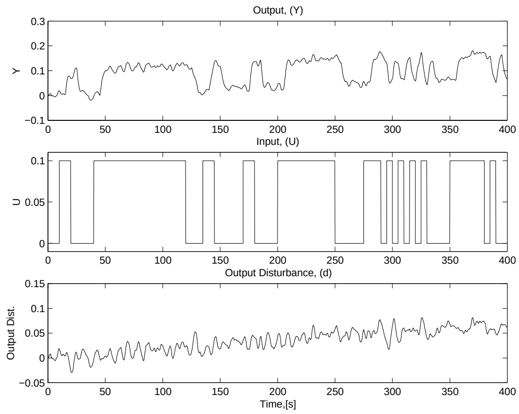

Figure 14. Open-loop data used in the identification test

Table 4 summarizes the parameters of the identified models. The transfer function *G*7 was identified using BJ model. The solution of the nonlinear optimization problems *S*1 and *S*2 gave the models *G*8 and *G*9, respectively.

Table 4: Transfer function coefficients for the additional example

TF	Parameters of equation (3)				
	a0	a1	a2	a3	a4
Plant	0	1	1	2	5
*G*7 / BJ	0,063	1.265	1.304	2.152	5
*G*8 / S1	0.0	1.436	4.871	4.414	3,5
*G*9 / S2	0.0	0.991	1.059	2.058	5

Figure 15 shows the step responses of the models. Note that the model *G*9 agrees very well with the true plant. Even the BJ model does not produce good results. This result clearly shows that the proposed square derivative error criterion (S2) is much better than the traditional square error criterion (S1).

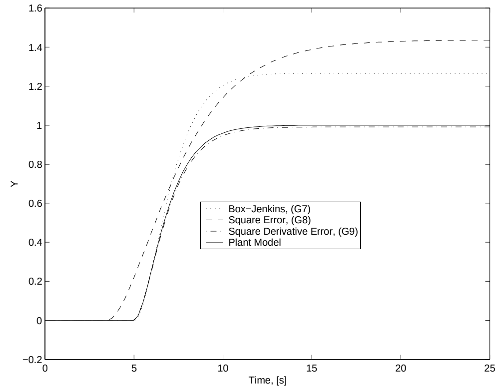

Figure 15. Step responses of the identified models: true plant (solid line), *G*7/BJ (dotted line), *G*8/S1 (dashdot line), and *G*9/S2 (dashed line).

6 CONCLUSIONS

The paper proposed a new criterion for system identification based on the square derivative error (i.e., $min \sum [de(t)/dt]^2$). It was shown that, special when the plant data is corrupted by unmeasured external disturbances, the $min \sum [de(t)/dt]^2$ criterion produces much better results than the traditional square error criterion (i.e., $J = min \sum [e(t)]^2$). The reason why the new criterion produces better results is its capacity to capture the process tendency instead of trying to pass through all experimental points. Since unmeasured external disturbances are always present in the industrial scenarios, the new criterion can have significant impact in real process identification problems.

ACKNOWLEDGMENT

The authors thank to Prof. Dr. S. Engell and to the Process Control Group of Dortmund University for the plant data used in this work. The authors also thank the financial support given by PETROBRAS/FINEP.

REFERENCES

Coleman, T.; Branch, M. A.; Grace A. Optimization Toolbox for Matlab, Version 2.0, 1997.

Ljung, L., MATLAB System Identification Toolbox, User's Guide, Mathworks, Version 4.0.5. , (1997)

MATLAB Signal Processing Toolbox, User's Guide, Mathworks, Version 4.3., (1997)

Misra, M.; Kumar, S.; Joe Qin, S; Seemann, D.; "Error based criterion for on-line wavelet data compression.", *Journal of Process Control,* ***11***, pp.717-731, (1999).

Saecker, D., Studienarbeit, Universität Dortmund, (1995)