

IDENTIFICATION FOR DECENTRALIZED MPC

Gudi, R.D.¹, Rawlings, J.B.^{*2}, Venkat, A.² and Jabbar, N.³

¹*Department of Chemical Engineering, Indian Institute of Technology, Bombay, Mumbai 400 076, India.*

^{*}*Department of Chemical and Biological Engineering, University of Wisconsin, Madison 53705, USA.*

³*Jordan Institute of Science and Technology, Irbid 22110, Jordan.*

Abstract: This paper addresses the problem of identifying interaction dynamics that exist between units operating in a decentralized control scheme. Identification of such interaction relationships is crucial to the deployment of coordinated decentralized control schemes. The proposed methodology is based on (i) the use of partial correlation analysis to identify the interacting channels in closed loop, and (ii) closed loop identification of the concerned dynamics using data obtained from suitable dithering. Alternative identification schemes relevant for this scenario are briefly analyzed in this paper and validation studies on a representative system is presented.. *Copyright © 2004 IFAC*

Keywords: Closed loop identification, interaction estimation, partial correlation analysis.

1. INTRODUCTION

Centralized control schemes that are based on a complete description of the cause and effect relationships are known to yield optimized control performance for multivariable systems. However, for optimization and control of large-scale systems, partitioning and decentralized control schemes have been eminently recommended over centralized approaches (Siljak, 1996). This choice is due to a number of factors. Firstly, the computational intensity for control is known to grow quite rapidly than size. Therefore, it is practical to design and implement control schemes for smaller sub-systems by looking at their local cause and effect relationships and the effect of interactions from other subsystems. The second and perhaps more important limitation for centralized control of large-scale systems is from a modeling and identification perspective. It is widely recognized that identification of cause-effect relationships in large-scale systems is a relatively difficult task. While such relationships are identified easily at a local level, for example at a unit level in a chemical or power generation plant, the interaction between such levels is usually associated with a lot of uncertainty. Often times, such interactions are also not perceivable during direct modeling, but manifest when the local control loops are closed. Identification of such interaction dynamics is a critical requirement for implementing coordinated decentralized schemes, which are known to yield

closed loop performance that approaches that of a centralized control scheme.

As indicated earlier, the interacting dynamics are often not easily characterized, unless all available control inputs for the large scale system are perturbed for identification. Therefore, such dynamics are best identified under closed loop conditions, wherein the large-scale system is partitioned into smaller sub-systems with local controllers. Having identified these interactions, co-ordinated and decentralized control can then be achieved with these local controllers by implementing a higher-level coordinator or a peer-level communication (Mesarovic *et al.* 1970, Katebi and Johnson (1997)). These coordination mechanisms thus require knowledge of the interaction to achieve the desired level of coordination. This interaction thus needs to be identified under controlled conditions using closed-loop identification methodologies.

Closed-loop identification strategies have been extensively proposed in the literature and an excellent review of the state of the art can be found in Forsell and Ljung (1999). The primary motivation in these strategies has been towards identifying direct models between the inputs and outputs using closed loop data, when for example, the plant is open loop unstable or there are inherent feedback mechanisms implemented for safety reasons. Another important reason for identifying such direct models has been towards obtaining reduced order models with a view

to achieve better control(Gevers and Ljung, 1986). Broadly, three different approaches to closed loop identification of the direct dynamics, viz (i) the direct, (ii) the indirect and (iii) the joint input-output methods, have been proposed. These methods differ in terms of apriori knowledge assumed about the nature of the controller and the assumptions made regarding the noise models. The relative merits of these strategies, in terms of consistency of estimates and the applicability of these methods (depending on the accuracy of the noise models) are discussed in Ljung (1999), Forsell and Ljung (1999).

The identification problem considered in this paper involves the characterization of the interaction dynamics between decentralized control loops. The challenges encountered are somewhat similar to what one would expect for identification of the direct plant in closed loop, discussed in the earlier paragraph. However, some key differences exist. Firstly, as is the case with closed loop identification, lack of informative data for identification is a key problem. This problem is overcome in regular closed loop identification, via the use of a dither signal applied either at the controller output or at the setpoint. However, for the case of identification of interaction involving several decentralized loops, dithering of each of the controller outputs is not practical. A more systematic method to prune down this set of variables is necessary. Secondly, in regular closed loop identification, the direct method (i.e. ignoring the presence of feedback) has been recommended in view of its optimal statistical properties. For the case of interaction identification, the direct method is not applicable because (as will be shown later) the inter-relationships between any two interacting decentralized loops also involves the individual loop sensitivities. Thus, one has to resort to other closed loop identification methods to first explicitly estimate the individual loop properties and then factor them out. Finally, a recognized drawback of the indirect method of closed loop identification is that it requires knowledge of the feedback controller mechanisms. In the identification problem considered in this paper, each of the individual decentralized controllers is multivariable and perhaps constrained in nature, and therefore the use of the indirect method is precluded. Also, in view of the multivariable nature of the concerned loops and the interacting dynamics, the use of subspace based state-space identification methods (Verdult and Verhaegen (2002)) greatly facilitates the identification procedure.

This paper analyzes the above mentioned problems encountered in identifying the interaction dynamics in large scale systems, with a view to achieving co-ordinated decentralized control. The approach proposed here to identify the interaction is based on using closed loop data. As mentioned before, dithering all the control inputs regardless of whether they contribute to interaction is perhaps not practical. The interaction between loops in large scale systems

is generally sparse. Therefore, towards isolating the channels that contribute to interaction and minimize the number of control inputs that need to be dithered, we propose the use of partial correlation analysis. Having dithered a smaller set of relevant manipulative variables and generated sufficiently rich closed loop data, the problem of identifying the interacting dynamics needs to be addressed. Towards this end, we present and analyze three methods for identification of the interacting dynamics from the viewpoints of apriori knowledge necessary as well as their applicability in the constrained controller case. The proposed identification strategies have been validated on representative examples taken from the literature.

2. PROBLEM DEFINITION

The identification problem that we seek to address in the paper is shown in Figure 1. For purposes of explanation, we consider the case of two decentralized loops and seek to identify the interacting dynamics $G_{dj}(\mathbf{q})$ between them.

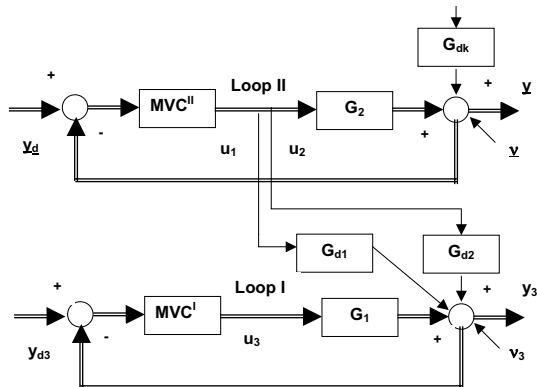


Figure 1: Schematic of interacting multivariable controllers

Each of the individual loop outputs is assumed to be affected by noise and unmeasured disturbances \mathbf{v} . Each of the individual controllers are assumed to be multivariable in nature. For purposes of explanation again, we assume that the controllers involved with loop I and loop II are multivariable and of size $n \times n$. These controllers are designed based on the local dynamics $G_1(\mathbf{q})$ and $G_2(\mathbf{q})$, which are assumed to be known. We further assume that the interaction dynamics G_d is sparse. This is a realistic assumption because if the structure was full, one could deploy a centralized (rather than decentralized) scheme that is based on the complete enumeration of all the cause and effect relationships. In general, no other apriori knowledge is assumed about the interactions and the channels in which they could exist. We seek to estimate the interacting dynamics $G_d(\mathbf{q})$ under closed loop conditions. The intent is to then use knowledge of the interacting dynamics in co-ordinated control schemes so as to achieve centralized performance but using decentralized control structures as shown above.

In general, closed loop data is known to be less informative from an identification viewpoint. Hence

a dither signal \mathbf{d} at the controller output is commonly employed. Figure 2 shows the block diagram of a single decentralized loop with the introduction of the dither signal

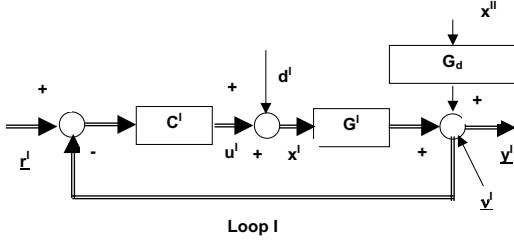


Figure 2: Signals associated with a single loop.

3. PARTIAL CORRELATION ANALYSIS

Consider the block diagram shown in Figure 2. During normal plant operation, the dither signal \mathbf{d}^I is zero and therefore the signals in the vectors \mathbf{x} and \mathbf{u} are the same. Routine operating data is collected on each of the variables in \mathbf{u} and \mathbf{y} . We seek to use this routine operating data to assess the existence of interaction, i.e. we test the correlation between any signal p in \mathbf{x}^I and a set of signals q, s and t (say) in \mathbf{x}^{II} . Since we would like to test this correlation in a dynamic sense, (i.e. the cause and effect relationships are dynamic in nature, rather than static, with at least a single lag due to zero order hold), we consider a dynamic map of these signals. Let Y denote the vector of measured observations of p and X denote the matrix of measured and lagged observations of signals q, s and t , with n being the number of lags in each of the latter signals. In other words, the k^{th} row of Y would then consist of $p(k)$ and that of X would contain $[q(k-1) \dots q(k-n), s(k-1), \dots, s(k-n), t(k-1) \dots t(k-n)]$. A regular correlation analysis based on the evaluation of the Spearman correlation coefficient (Draper and Smith(1981), would essentially look at a matrix $Z=[Y \ X]$ and evaluate the correlation coefficients in terms of the covariance of Z . These coefficients would then be tested for significance using the t-test. However, due to the possibility of the variables in X being correlated, we propose to use the partial correlation analysis. The partial correlation coefficient analysis can be used to suppress the effect of correlation and clearly discriminate the influence of correlated causal variables in X on the effect variable in Y .

Let us assume that the X matrix has dependent columns and therefore a reduced rank. Consider the problem of assessing the correlation between any column variable x_i with the variable Y . Let matrix X_R contain all variable x_j ($j \neq i$). We first find a vector e_{x_i} that contains the negation of the effect of all other variables on x_i . To do this, we first consider the regression problem between x_i and X_R . Assuming a linear relationship, the regression can be written as,

$$x_i = X_R \theta_1 + E_1 \quad (1)$$

This regression problem can and can be solved for θ_1 using the simple Moore-Penrose inverse as,

$$\theta_1 = (X_R^T X_R)^{-1} X_R^T x_i \quad (2)$$

We now evaluate the prediction errors for the above model as,

$$e_{x_i} = x_i - X_R (X_R^T X_R)^{-1} X_R^T x_i \quad (3)$$

In the next step, the influence of X_R on Y is again estimated using a regression model. The relevant equations can be written as,

$$Y = X_R \theta_2 + E_2 \quad (4)$$

from which θ_2 can be estimated as,

$$\theta_2 = (X_R^T X_R)^{-1} X_R^T Y \quad (5)$$

and the prediction errors for y can be estimated as,

$$e_y = y - X_R (X_R^T X_R)^{-1} X_R^T Y \quad (6)$$

In the above equations, e_{x_i} and e_y can be assumed to consist of those components of x_i and Y that are independent of X_R . These quantities can be used to check for the partial correlation of x_i with y . This partial correlation can therefore be written as a regular correlation between e_{x_i} and e_y as,

$$pr_{x_i, y} = \frac{e_{x_i}^T e_y}{(m-1) \sigma_{e_{x_i}} \sigma_{e_y}} \quad (7)$$

It can be seen that, if X is full rank, i.e. if x_i is completely uncorrelated with all columns in X_R ; then the predictions in Equations 3 will be white and so e_{x_i} will be equal to x_i itself. Similarly, e_y will contain only the effect of x_i on y (i.e. the effect of X_R on y will be suppressed). The partial correlation can thus give an assessment of the existence of a linear dependence between the cause and effect variables in the presence of correlated cause variables.

Remark 1: It must be mentioned since we use lagged variables in X , the resulting indices would indicate the existence of a linear, dynamic relationship between the signals in \mathbf{x}^{II} (i.e. q, s and t) and p in \mathbf{x}^I . For the variables amongst q, s and t that are correlated with p , one would obtain symmetric blocks of non-zero correlation indices of size n , which would indicate the evidence of a non-zero interacting dynamics in G_d for the assumed pair of signals.

Remark 2: It is possible that the reduced signals e_{x_i} and e_y might also reflect the effect of unmeasured disturbances that routinely affect the loop. These effects would corrupt the interpretation from the above analysis. However, if there is any evidence of such disturbances affecting the loop, then the regression problems posed in Equation (1) and (4) may be solved using more sophisticated methods such as the PEM or the instrumental variables (IV)

method, rather than the simple Moore-Penrose method as indicated above.

Remark 3: If the matrix X contains perfectly correlated variables (*i.e.* if the signals g , s and t are perfectly correlated), the above method would still not be able to discriminate between signals that have a dependent relationship from those that do not. However, perfect correlation is difficult to exist/obtain in practice. Even an uncorrelatedness in the measurement noise associated with the signals g , s and t could be adequate to discriminate between these signals.

4. CLOSED LOOP IDENTIFICATION

Having isolated the channels in which interaction exists, we now focus on the problem of identifying the interacting dynamics. For purposes of explanation we consider that only one signal p in \mathbf{x}^I affects the loop I (see Figure 2) through some disturbance dynamics, which needs to be estimated using closed loop identification methods. It is now assumed that the signal p can be perturbed as a dither signal at the output of controller II (MVC₂) (see Figure 1), in a way that is uncorrelated with loop I set-point changes. As well, through regular target specifications by the upper LP layer in a typical MPC setting (Shah *et al.* (2002)), it is assumed that sufficient excitation exists at the set-point for Loop I.

For loop I, the relationship between the two independent, deterministic signals p and \mathbf{r}^I and the dependent signals can be written \mathbf{x}^I and \mathbf{y}^I as,

$$\mathbf{x}^I = (I + C^I G^I)^{-1} C^I \mathbf{r}^I + (I + C^I G^I)^{-1} C^I G_d p + (I + C^I G^I)^{-1} d^I \quad (8)$$

$$\mathbf{y}^I = (I + G^I C^I)^{-1} G^I C^I \mathbf{r}^I + (I + G^I C^I)^{-1} G_d p + (I + G^I C^I)^{-1} G^I d^I \quad (9)$$

In the above equations, the effect of stochastic and unknown noise elements can also be added in the above equation but are omitted for brevity. Also, it is important to note that the loop sensitivities associated with the loop are different for the general multivariable case and are related by the equation,

$$(I + C^I G^I)^{-1} C^I = C^I (I + G^I C^I)^{-1} \quad (10)$$

As mentioned before, in regular closed loop identification using the direct method, the effect of feedback is ignored and the signals \mathbf{y}^I and \mathbf{u}^I are used to estimate the transfer function \mathbf{G}^I using Equation 9. In the above problem, we seek to estimate G_d and in this case the direct method of closed loop identification is not really an option. This is because the direct method based on the signals p and \mathbf{y}^I (or \mathbf{x}^I) will only yield the product of G_d and one of the loop sensitivities; it is therefore necessary to explicitly estimate and factor out the sensitivity dynamics to generate an estimate of G_d . In earlier approaches (Van den Hof and Schrama (1993), Huang and Shah (1997)), an alternative two step, indirect method of closed loop identification of \mathbf{G}^I was proposed with some of its theoretical properties being reviewed subsequently by Gevers *et al.* (2001). Here we propose to use both single step as well as two step

methods for the identification of the interacting dynamics G_d assuming that the plant dynamics \mathbf{G}^I is known. This latter assumption is valid considering that cause and effect relationships can more easily be characterized in smaller decomposed sub-systems. The methods for closed loop identification can be presented as follows:

Method 1: Single Step Method: Consider that the closed loop system is simultaneously perturbed both at the set-point \mathbf{r}^I and at the output of C^{II} for signal p in an uncorrelated fashion. The dither input d^I is assumed to be zero in Figure 2. Using Equation (8), it can be seen that this results in a multi-input single output, open-loop identification problem in the variables \mathbf{r}^I and p as inputs and \mathbf{x}^I as the output. This identification problem can be solved in the presence of stochastic disturbances affecting loop I, using the prediction error method (PEM) to generate unbiased estimates of the dynamics $G_{cl} = (I + C^I G^I)^{-1} C^I$ and the $G_{d,cl} = (I + C^I G^I)^{-1} C^I G_d$. Having obtained an estimate of these dynamics, the interacting dynamics can be obtained as,

$$G_d = (G_{cl})^{-1} G_{d,cl} \quad (11)$$

An important aspect that must be recognized towards realization of G_d as above is that Loops I and II are multivariable in nature and could involve a good number of manipulated and controlled variables. Towards ease of parameterization of the dynamics in G_{cl} and $G_{d,cl}$ during identification, it is more suitable to use appropriate order state space models, identify them using suitable sub-space methods and then realize the transfer function representations of G_d , if necessary as indicated by Equation 11 above. However, as will be seen in the next section, inverses are also easily realizable in state space forms.

Method 2: Two step method

Equation 8 is again the starting point for the two step method. Again considering that the dither d^I is zero, routine plant data involving the variables \mathbf{r}^I and \mathbf{x}^I can be collected. This routine plant data would contain the effect of interactions from Loop II through signal p as well as effect of other stochastic disturbances. In the first step of the two step identification method, the dynamics G_{cl} can be estimated by considering the signals \mathbf{r}^I and \mathbf{x}^I and by using the output-error method (OEM) which is a variant of the PEM method. The OEM is a special case of the PEM method with noise parameterized as unity, and it has been shown (Soderstrom and Stoica (1989)) that (i) if the true dynamics belongs to the model set, and (ii) the driving signals are orthogonal to the noise, unbiased estimates of the concerned dynamics can be obtained. Thus an unbiased estimate of the dynamics in state space form of G_{cl} can be obtained in the first step, using a suitable sub-space method. Some theoretical considerations related to analysis of bias in the estimates of G_{cl} in the presence of interacting dynamics G_d is presented in Gudi and Rawlings (2003).

In the second step, the dither at the set-point \mathbf{r}^I is set to zero and the signal p is dithered. The signals \mathbf{x}^I and

p_d can now be used to estimate G_d . Towards this end, a new signal x_f^I is generated as a filtered version of x^I as given by,

$$x_f^I = \{(I + C^I G^I)^{-1} C^I\}^{-1} x^I \quad (12)$$

This filtered signal x_f^I is then used along with the measurements of p to generate an estimate of the interacting dynamics G_d using the OEM or the PEM methodology. A critical step in the above filtering is the realization of the inverse of $G_{cl}^I = (I + C^I G^I)^{-1} C^I$ as required in Equation 12. In this context as well, parameterization of state space models for capturing the dynamics is more desirable, as state space representations are relatively easier to invert than transfer function representations. It can be shown that the relationship between the state space matrices A,B,C,D of any forward model are related to the state space matrices Φ, Γ, C_i and D_i of the inverse model as given by the relationships,

$$\begin{aligned} \Phi &= A - B D^{-1} C; \Gamma = B D^{-1} \\ C_i &= -D^{-1} C; D_i = D^{-1} \end{aligned}$$

It is also possible to show that for the identification of the loop sensitivities, the inverse of D always exists and so these transformations are realizable.

Method 3: Direct Method: Constrained controller case

In the presence of constraints, it is well-known (Zafriou (1990)) that the controller dynamics and hence the loop sensitivities are no longer linear. In such a case, it is inappropriate to use the methods proposed in the earlier sub-sections, which inherently are based on the assumption of linearity. In such a case, it is important to exploit the fact that the plant dynamics G^I is well characterized. Defining the prediction error, $\varepsilon = y^I - G_m^I x^I$, where G_m^I is the model of the plant G^I , one can write the closed loop relationships between ε and the independent deterministic loop inputs as,

$$\varepsilon = (S_0 G^I C^I - G_m^I R_0^I) r + (S_0 + G_m^I R_0^I) G_d p \quad (13)$$

In the above equation, $S_0 = (I + G^I C^I)^{-1}$, $S_0^I = (I + C^I G^I)^{-1}$ and $R_0^I = (I + C^I G^I)^{-1} C^I$ and it can be shown that for $G_m^I = G^I$ the deterministic relationship between ε and p can be written as,

$$\varepsilon = G_d p \quad (14)$$

Thus, in the presence of stochastic disturbances affecting at the output, a prediction error method based on the measured signals ε and p can be used to estimate the dynamics G_d .

A relative assessment of the above three methods of identifying the interacting dynamics can be presented as follows: In the presence of a perfect model G_m^I , one would always prefer Method 3 as it is a single step, direct method that accommodates the most general constrained controller case. It does not require any other knowledge of the controller mechanisms or the loop sensitivities and as such is a parallel counterpart to the Direct Identification scheme proposed for identifying direct models in closed loop. However, it is based on the key assumption of a perfect plant model. If this

assumption cannot be justified, one would then choose amongst the first two methods. The merits of the first method is that it is a one step method that just uses closed loop data to simultaneously identify the concerned dynamics in a MISO framework. However, the drawback is that sufficiently strong and uncorrelated dither has to be applied simultaneously at both the set-point and at the manipulated variables, to generate unbiased models. This may pose problems in terms of the acceptance of this strategy at the plant level. The second method on the other hand is a two step method which involves the identification of the loop sensitivities in the first step. This identification may be done without strong dithering at the set-point. Regular set-point updating by the upper LP layers could be sufficient to provide the necessary excitation. The identification could be performed using data collected over extended periods of time; for the closed loop case, availability of such data is not a constraint. After this first step, regular dithering of the manipulated variables of the other controllers over a short period of time is adequate to generate the excitation necessary of identification of the interacting dynamics. Thus, the second method would be preferable when dithering at the setpoint is not permissible.

5. CASE STUDY

We consider the following 3 x 3 transfer function and choose to partition the system for decentralized control as shown below:

$$G(s) = \begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{-18.9}{21s+1} & \frac{-1}{3.3s+1} \\ \frac{6.6}{6.6} & \frac{-19.4}{-19.4} & \frac{2}{2} \\ \frac{10.9s+1}{-1.333 * K_1} & \frac{14.4s+1}{-0.875 * K_2} & \frac{5s+1}{21 * K_3} \\ \frac{3.33s+1}{3.33s+1} & \frac{1.25s+1}{1.25s+1} & \frac{10s+1}{10s+1} \end{bmatrix}$$

The input-output pair $\{y_1, y_2 - u_1, u_2\}$ forms a decentralized sub-system which is controlled by two PI controllers ($K_c=0.47$ and $\tau_i = 3.26$ for y_1-u_1 pairing and $K_c=-0.09$ and $\tau_i = 9.35$ for the y_2-u_2 pairing) with decouplers. The pair y_3-u_3 forms another decentralized sub-system that is also regulated by a PI controller with $K_c=0.09$ and $\tau_i = 10$. The two decentralized loops were assumed to be corrupted at the output with random noise having variance of 0.1. It was proposed to first evaluate the partial correlation analysis for identifying the interactions for zero and non-zero values of the parameter K_i (see the y_i-u_i dynamics). For closed loop identification, the set-points and the controller outputs were dithered using random signals having variance 0.3. These were designed to be uncorrelated whenever necessary, as for example in Method 1. For method 2, simple step like set-point changes were introduced in the first step to estimate the sensitivities. For method 3, the controller outputs were clipped using an appropriate saturation block, to simulate

constrained controller action. The canonical variate analysis (CVA) based sub-space method was used for the identification of the state space models using closed loop data. The model order for the state space models were selected based on the Akaike Information Criteria and the models identified were cross-validated using closed loop data.

Isolating interacting channels

The partial correlation analysis was used to assess the existence of interaction between the two sub-systems. Towards this end, K_2 was deliberately set to zero in the plant and K_1 and K_3 were set to 1. Closed loop data for u_3 and u_1, u_2 was used along with partial correlation analysis to infer the presence of interaction. A total of 10 sets of data consisting of 1000 points (sampling time of one time unit) was used to perform both the regular and partial correlation analysis.

It was found that the Spearman correlation coefficient based on the regular correlation analysis was not able to differentiate the two channels u_1 - u_3 (interaction exists) and u_2 - u_3 (absence of interaction) in the closed loop for a majority of the sets. This was obviously because both u_1 and u_2 were correlated amongst themselves. In fact, the Spearman correlation coefficient for the pair u_1 and u_2 was consistently above 0.8 for most of the sets indicating a strong correlation between these variables. However, partial correlation analysis was able to remove the correlation between these variables and consistently assess the underlying closed loop correlation between u_3 and u_1, u_2 .

Estimating interacting dynamics

Both the single step and two step methods were able to accurately identify the interacting dynamics between u_1 and y_3 . For method 2, the slower set-point excitation in the first step did require a larger data set to provide consistent estimates of the sensitivities. As mentioned before, this requirement of a larger data set is not a drawback in closed loop. However, in both the methods, the identified loop sensitivities as well as the interaction dynamics matched quite closely with the true values. The direct method (Method 3) was a simple open loop identification problem because of the assumption of a perfect model. The interaction model between u_1 and y_3 was also accurately estimated as expected. However, in terms of applicability this requirement of a perfect model is a constraint.

6. CONCLUSIONS

The focus of this paper was on the identification of interacting dynamics in large decentralized multivariable controllers. Partial correlation analysis was proposed to isolate the interacting channels and three methods of closed loop identification was proposed. A representative case study was presented to highlight the practicality of the proposed methodology.

REFERENCES

- Siljak, D.D, "Decentralized Control and Computations: Status and Prospects", Annual Reviews in Control, **20**, 131-142, (1996)
- Mesarovic, M.D., Macko D. and Takahara, "Two coordination principles and their application in large scale systems control", Automatica, **6**, 2647-2473, (1999).
- Katebi, M.R and Johnson, M.A., "Predictive Control Design for Large Scale Systems", Automatica, **33** (3), pp. 421-425, (1997).
- Forsell, U. and Ljung, L., "Closed Loop Identification revisited", Automatica, **35**(7), pp. 1215-1241, (1999).
- Gevers, M. and Ljung, L. "Optimal Experiment Designs with respect to intended model application", Automatica, **5**, 543-554, (1986).
- Ljung, L., "System Identification: Theory for the user", Prentice Hall, (1999).
- Verdult, V. and Verhaegen, M., Subspace identification of multivariable linear parameter-varying systems, Automatica, **38**(5), pp. 805-814, (2002).
- Draper, N. R., and Smith, H. "Applied Regression Analysis," 2nd ed., New York, John Wiley and Sons, 1981.
- Shah, S.L., Patwardhan, R. and Huang, B. "Multivariable Controller Performance Analysis", CPC – VI, Proceedings of sixth international conference on Chemical Process Control, Rawlings, J.B., Ogunnaike B.A., Eaton, J.W.(Eds.), AIChE symposium series, Vol. 98, 2002.
- Van den Hof, P.M.J, and Schrama, R.J.P. "An indirect method for transfer function estimation using closed loop data", Automatica, **29**(6), 1523-1527 (1993).
- Huang, B. and Shah, S.L., "Closed Loop Identification – A two step approach", Journal of Process Control, **7**, pp. 425-438, (1997).
- Gevers M., Ljung, L and Van den Hof, P., "Asymptotic variance expressions for closed loop identification", Automatica, **37**, 781-786, (2001).
- Soderstrom, T. and Stoica, P., "System Identification", Prentice-Hall, (1989).
- Gudi, R.D. and Rawlings, J.B, "Issues in identification of interacting dynamics in large decentralized multivariable systems", (In preparation) (2003).