FAULT DIAGNOSIS AND FAULT TOLERANT CONTROL USING REDUCED ORDER MODELS

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Abstract: The present work considers a model reduction approach for fault diagnosis with generalised likelihood ratio (GLR) method to improve upon its diagnostic performance and computational efficiency in large dimensional applications. Model reduction techniques are widely used in controller design. A similar concept with balanced truncation technique is employed to obtain a reduced order diagnostic model of the process for GLR implementation. The proposed method is incorporated in the fault tolerant control strategy developed by Prakash *et al.* (2002). Simulation results for a binary distillation column demonstrate the efficacy of the proposed fault detection and identification (FDI) method and the fault tolerant control strategy in comparison to the full-scale implementation for a variety of faults.

Keywords: fault detection and identification, GLR method, model reduction, balanced truncation, soft faults, fault tolerant control

1. INTRODUCTION

During the past two decades there has been an increasing trend towards the design and development of fault tolerant control system (FTCS), which can maintain an acceptable control performance despite the occurrence of faults. In an active FTCS design, the controller is designed for normal operation, and as and when a fault is deemed to have occurred, the control law is reformulated or restructured on-line.

An important component of an active FTCS design is the FDI method. Among the FDI approaches, model based methods employing analytical redundancy are found to be most suitable for control applications, since the information it provides facilitates explicit corrective measures. Recently, Prakash *et al.* (2002) developed an active FTCS scheme, which integrates a FDI methodology based on *generalised likelihood ratio* (GLR) method (Willsky and Jones, 1976); (Narasimhan and Mah, 1988) with a controller, through an on-line compensation mechanism. They demonstrated through simulation and experimental studies, the superior performance of the FTCS over the conventional control scheme in the presence of various types of soft faults such as sensor and actuator biases, abrupt changes in unmeasured disturbances and process parameters.

The key component in their FTCS design is the use of a linear model (and a Kalman filter) derived

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from first principles. The *first principles* modeling approach used in their work enables identification of various types of faults. However, the models of process units such as a distillation column or a subsystem consisting of several process units are of high order, involving hundreds of state variables. This can lead to excessive computational demand in real-time application of the proposed FTCS. In addition to this, system observability is a prime requirement in the Kalman filter design for obtaining unique and unbiased estimates of states. Typically, not all the states are observable from the relatively few measurements that are available in such systems. Thus, there is a need to develop an alternate formulation based on the first principles model, which will alleviate computational difficulties and improve the state estimation particularly when the system dimension is large.

In the proposed approach, we still make use of a linearised model of the process developed from first principles in the FDI method, in order to retain the ability to distinguish between different types of faults. However, instead of using the full high order model so developed, we propose to obtain a reduced order normal and fault models of the process (essentially a *diagnostic* model with fault dynamics incorporated) for FDI implementation. An ideal binary distillation column simulation is considered as a prototypical example of a large dimensional process to illustrate the proposed scheme. The objective of this study is to investigate the effectiveness of the proposed approach in comparison to the full-scale implementation.

2. REDUCED MODEL BASED FDI STRATEGY

Fault detection and identification (FDI) is concerned with detecting whether the fault has occurred, and if so, identify the cause and provide an estimate of the fault magnitude. We briefly describe the FDI strategy as developed by Narasimhan and Mah (1988) for identification of various types of faults.

2.1 FDI Strategy

The FDI strategy makes use of process models under normal and faulty operating conditions. A linear discrete stochastic state space equation is used to model the process as follows

$$x(k+1) = \Phi x(k) + \Gamma_u u(k) + w(k) \tag{1}$$

$$y(k) = Cx(k) + v(k) \tag{2}$$

where $x \in R^n$ represents the states, $u \in R^m$ represents the manipulated inputs, $y \in R^r$ represents the measured outputs, $w \in R^q$ and $v \in R^r$ are assumed to be independent zero mean Gaussian white noise sequences with covariance matrix R_1 and R_2 respectively. Under normal operation, the above model can be used to obtain the optimal estimates of the state variables using a Kalman filter (Astrom and Wittenmark, 1994). The innovations (or measurement residuals) generated by the Kalman filter are given as

$$\gamma(k) = y(k) - C\hat{x}(k/k-1) \tag{3}$$

where $\hat{x}(k/k-1)$ denote the state estimates predicted at time k using all measurements made up to time $(k-1)$. Simple *chi-square statistical tests* based on these innovations are employed for fault detection and subsequent fault confirmation over a time window. Fault identification is carried out with the GLR method, where for each hypothesised fault, the characteristic innovations trend termed as *fault signature* is determined from the corresponding fault model and the normal estimator model. For soft faults caused by biases in sensors and actuators which are assumed to occur as step changes, the corresponding fault models can be easily obtained as follows. In the presence of a sensor bias, the measurement model Eq. (2) is given by

$$y(k) = Cx(k) + v(k) + be_{y,i} \tag{4}$$

for $k \geq t_f$, where t_f is the time of occurrence of a fault. Likewise, in the presence of an actuator bias the state transition Eq. (1) is modeled as

$$x(k+1) = \Phi x(k) + \Gamma_u u(k) + w(k) + b\Gamma_u e_{u,i} \tag{5}$$

In the above equations, b represents the magnitude of the fault, $e_{y,i}$ and $e_{u,i}$ are unit vectors of appropriate dimensions, subscripts y, u signifies the fault type with i as the index of the measurement or actuator where the bias occurs.

The above fault models for sensor and actuator biases can be used whether the normal process model (defined by Eqs. 1 and 2) is derived from first principles, or obtained from input-output data using system identification methods. *However, if a step change in a process parameter or disturbance variable occurs, then the appropriate fault model can be derived, provided a first principles modeling approach is used (Prakash et al., 2002).* The fault models for these type of faults can be obtained as

$$x(k+1) = \Phi x(k) + \Gamma_u u(k) + w(k) + b\Gamma_{f'} e_{f',i} \tag{6}$$

where the matrix $\Gamma_{f'}$ depends on the type of fault that has occurred (disturbance or parameter) with $f' \in \{d, p\}$.

From the linearity of the system and filter, the effect of each fault on the expected values of the innovations at any time can be obtained as

$$E\left(\gamma(k)\right) = \left\{ \begin{array}{ll} 0 & k < t_f \\ bG_f(k;t_f)g_{f,i} & k \geq t_f \end{array} \right\} \quad (7)$$

where the subscript $f \in \{y, u, d, p\}$ denotes the fault type. Here, the matrix $G_f(k;t_f)$ is referred to as signature matrix and depends upon the time t_f at which the fault has occurred and time k at which the innovations are computed. The vector $g_{f,i}$ which we refer to as the fault signature vector, depends upon the fault type and location. The computational details of these signature matrices and signature vectors for different faults are given in Prakash et al. (2002). *The GLR method essentially identifies the fault whose signature best fits the observed innovations pattern.*

2.2 Reduced order FDI development

Model reduction techniques are widely used in controller design. Among the various approaches, the *balanced truncation* technique (Moore, 1981), removes after suitable transformations, the states that are difficult to control or observe. Since such states contribute little to the understanding of the process input-output behavior, their removal does not significantly alter the quality of the model predictions. The distinct advantage of this reduction approach is that one can obtain an *observable subsystem* for the Kalman filter design to be used in FDI development.

As explained in the preceding section, the GLR based FDI strategy makes use of process models under normal as well as faulty operating conditions. Therefore, the requirement in fault diagnosis is to adequately capture *fault-output dynamics* when compared to a controller design where the effect of a manipulated input on an output is important. The standard approach for obtaining reduced order models to design controllers needs to be modified.

A generic state space process model which describes the effect of manipulated inputs, parametric and disturbance faults, can be obtained by combining the normal and fault model (Eqs. 1, 5 and 6) as follows

$$x(k+1) = \Phi x(k) + \Gamma_u u_c(k) \quad (8)$$

$$y(k) = Cx(k) \quad (9)$$

where

$$u_c = \left[\, u^T \;\; d^T \;\; p^T \,\right]^T \quad (10)$$

where $x \in R^n$, $y \in R^r$ are variables as defined with Eqs 1 and 2. The input vector $u_c \in R^{m_c}$ denotes a complete input set with $\Gamma = \left[\, \Gamma_u \;\; \Gamma_d \;\; \Gamma_p \,\right]$ as the corresponding input coupling matrix with individual components as explained in Eqs 5 and 6. Let ζ be a vector of variables related to x by a balancing transformation $x = T\zeta$. The state space model Eqs. (8) and (9) in transformed domain is given as

$$\zeta(k+1) = \hat{\Phi}\zeta(k) + \hat{\Gamma}u_c(k) \quad (11)$$

$$y(k) = \hat{C}\zeta(k) \quad (12)$$

where $\hat{\Phi} = T^{-1}\Phi T$; $\hat{\Gamma} = T^{-1}\Gamma$ and $\hat{C} = CT$. In a balanced realisation, each state in the transformed domain ζ_i is just as controllable as it is observable. A measure of the state joint controllability and observability is given by the size of its associated Hankel singular value σ_i. The size of σ_i is a relative measure of the contribution that state ζ_i makes to the input-output behavior. The balanced truncation approach removes the states corresponding to the small Hankel singular values. Partition the state vector ζ of dimension n into ζ_1 and ζ_2 where ζ_2 is a vector of $(n-l)$ states whose effect is negligible compared to ζ_1 on the input-output response. With appropriate partitioning of $\hat{\Phi}$, $\hat{\Gamma}$ and $\hat{C}$ the state space equations becomes

$$\left[\begin{array}{c} \zeta_1(k+1) \\ \zeta_2(k+1) \end{array} \right] = \left[\begin{array}{cc} \hat{\Phi}_{11} & \hat{\Phi}_{12} \\ \hat{\Phi}_{21} & \hat{\Phi}_{22} \end{array} \right] \left[\begin{array}{c} \zeta_1(k) \\ \zeta_2(k) \end{array} \right] + \left[\begin{array}{c} \hat{\Gamma}_1 \\ \hat{\Gamma}_2 \end{array} \right] u_c(k) \quad (13)$$

$$y(k) = \left[\, \hat{C}_1 \;\; \hat{C}_2 \,\right] \left[\begin{array}{c} \zeta_1(k) \\ \zeta_2(k) \end{array} \right] \quad (14)$$

Setting $\zeta_2 = 0$ gives the following reduced order model

$$\zeta_1(k+1) = \hat{\Phi}_{11}\zeta_1(k) + \hat{\Gamma}_1 u_c(k) \quad (15)$$

$$\tilde{y}(k) = \hat{C}_1\zeta_1(k) \quad (16)$$

From the above equations, one can extract the reduced order model under normal operation, as well as the reduced order model for each type of fault to be used in FDI design.

3. BASIC FAULT TOLERANT CONTROL SCHEME

The efficacy of the proposed FDI method is assessed by incorporating it in the fault tolerant control scheme developed by Prakash *et al.* (2002). The basic idea of their FTCS is to compensate / adapt the controller and state estimator on-line once a fault is identified. As stated earlier, the soft faults caused by biases in sensors, actuators and abrupt changes in disturbance and operating parameters are considered in the design of FTCS.

In the BFTCS design (figure 1), the FDI module is like an external interface to the existing feedback

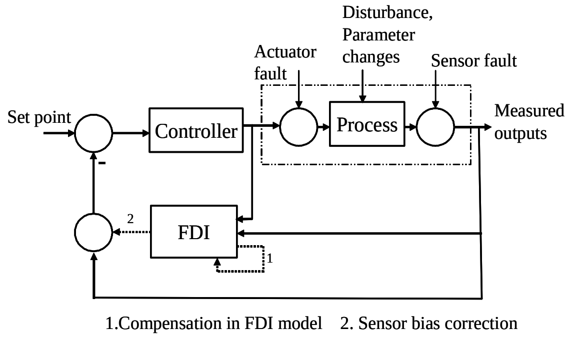

Fig. 1. Schematic of Basic Fault Tolerant Control Scheme

control system. Typically, a conventional feedback controller with integral action can satisfactorily deal with actuator biases, disturbances and process parameter changes in the absence of input constraints. However, in the presence of a sensor bias, it takes corrective action based on the biased value thereby introducing an offset in the true value of the plant output. The BFTCS was developed to give good control performance despite the occurrence of any type of soft fault, by appropriately integrating the FDI method with controller. In case a *sensor bias* is identified, the magnitude of the bias estimated by the FDI method is used to correct the faulty measurement, before being used in the controller. For all types of faults identified, the FDI model is suitably adapted so that any subsequent fault occurring in the process can be detected.

4. SIMULATION STUDIES

The performance of the proposed scheme is tested through simulation of a 20-tray single feed binary distillation column (Luyben, 1990). The following assumptions are made in deriving a nonlinear model from first principles: constant overflows, constant relative volatility, linear liquid flow dynamics, constant pressure, no vapor holdup, total condenser, perfect level control in condenser and reboiler. The only modification made to the column simulation by Luyben (1990) is that the Murphree vapor phase stage efficiency η is considered (assumed the same for all trays) in order to demonstrate parametric faults. The resulting model equations consists of two differential (material balance and component balance) and three algebraic equations (vapor liquid equilibrium, tray efficiency and liquid hydraulic relation) for each stage i. A full-order model of the process derived from first principles (with material and component balances) consists of 42 nonlinear ordinary differential equations. The reflux flow rate (R) and vapor boil-up rate (V) are manipulated to control the top and bottom product compositions (x_d, x_d).

Normal operating data is as reported in table 1. Seven different faults are hypothesized namely, bias in the two concentration measurements (x_d and x_b), bias in the two actuators corresponding to the manipulated variables, reflux R and vapor boil- up V, step change in the two disturbance variables (feed flow rate F and feed composition z_f) and a step change in the tray efficiency parameter (η). We have in total *five* inputs (R, V, F, z_f and η) to be considered for obtaining a reduced order model for fault diagnosis. We next describe the reduction procedure.

Table 1. Normal operating values

Distillate concentration x_d	0.904
Bottom concentration x_b	0.272
Reflux R	124.08 (mol/min)
Vapor boil-up V	178.01 (mol/min)
Feed flow rate F	100.00 (mol/min)
Feed composition z_f	0.5
Tray efficiency parameter η	0.7

4.1 Model reduction and FTCS development

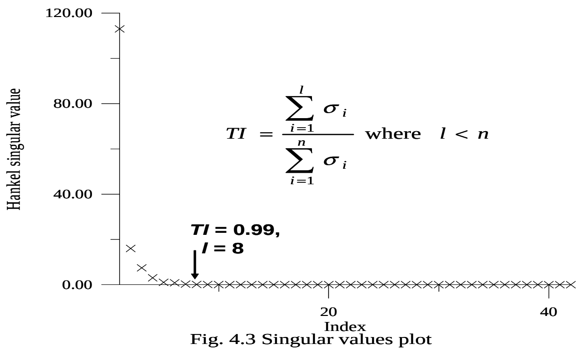

Fig. 2. Hankel singular values plot

A linearized continuous state space model is obtained by linearising the nonlinear equations around the normal operating point (table 1). The state space matrices and the fault coupling matrices are obtained by numerically evaluating the Jacobians of the differential equations with respect to the state, input and fault variables. The MATLAB function '*sysbal*' is used for computing the balancing transformation and '*strunc*' for obtaining the reduced dimensional state space subsystem by truncation. A truncation index (TI) of 0.99 was chosen to decide upon the model order of 8 as explained in the plot of Hankel singular values *vs* state indices (figure 2).

Figure 3 compares step responses of the distillate product composition obtained with respect to different normal and fault inputs for the full-scale and reduced order model. As can be seen from the Figures 3(a)-(e), the reduced order model is able to capture the system dynamics reasonably well for all the inputs considered. Similar kind of close match is obtained for responses of the bottom

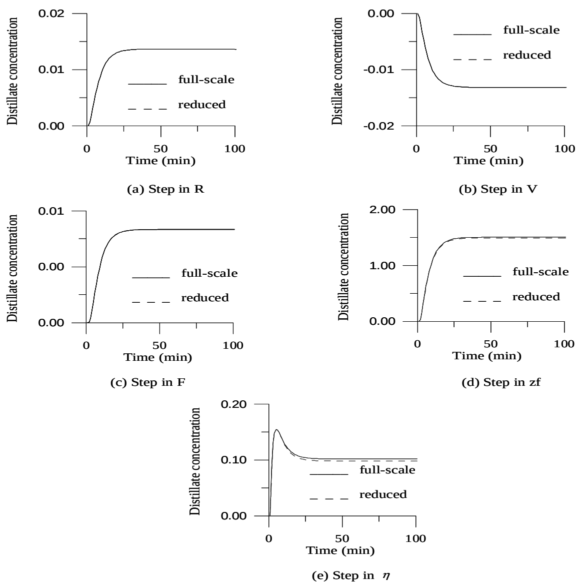

Fig. 3. Step responses of distillate concentration x_d with respect to different fault inputs

product composition. A point to note is that the MATLAB function for balancing is written for the continuous system description. Since the balancing theory is equivalent for continuous and discrete systems, the model reduction is carried out first in the continuous domain and discretised later. An 8^{th} reduced order continuous model so obtained is discretised with a sampling time of 1 min for use in FDI and controller development.

One can develop a reduced order model for FDI and controller independently since the FDI is like an external interface in the developed FTC scheme. However, for the sake of consistency and ease in maintainability, the controller is designed using the reduced order normal model of the process obtained from Eq. (15). An unconstrained DMC control law is developed with tuning parameters as given in Table 2. The reduced model based FDI strategy is incorporated in the BFTCS design with the tuning parameters as given in Table 3. To decide upon the parameter values, certain guidelines as given by Prakash *et al.* (2002) are used. In all simulation runs, 50 simulation trials (N_t) were performed with each trial lasting for 1000 sampling instants. State noise is simulated with random fluctuations in disturbance inputs (feed flow rate F and feed composition z_f) around the mean value as Gaussian zero-mean white noise sequences with standard deviation as 5% and 1% of their nominal values respectively. Measurement noise is simulated as Gaussian zero-mean white noise sequence with standard deviation as 1% of their respective nominal values. The Kalman filter for FDI development is designed using reduced order normal description of the process with these noise characteristics. For all the simulation cases reported, the fault is introduced at the 1^{st} sampling instant. The magnitude of the sensor and

Table 2. Parameter values for DMC

Parameter	value
Prediction horizon (p)	20
Control horizon (q)	1
Error weighting matrix W_e	diag[1 37]

Table 3. Parameter values for FDI

Parameter	value
Window length (N)	60
Level of significance for FDT (α_{FDT})	0.75
Level of significance for FCT (α_{FCT})	0.01

actuator bias introduced was in proportion to the standard deviation of the output or input, as the case may be under normal operation.

4.2 Performance measures

In order to assess the performance of the FTCS, the proposed FTCS as well as conventional control scheme are implemented, and the following performance measures are evaluated.

- *Performance index (PI)*

$$PI = \frac{ISE_i(\text{FTCS})}{ISE_i(\text{Conventional control})} \tag{17}$$

  where ISE_i is the sum squared difference between the true value of a controlled variable and the corresponding setpoint over the simulation trial, averaged over all the trials in a run.
- *Percentage of successful trials (PST)*

$$PST = \frac{N_S}{N_T} \times 100 \tag{18}$$

  where N_S is the number of simulation trials in which the fault is correctly identified at least once.
- *False alarm index (FAI)*

$$FAI = \frac{N_F}{N_T \times \frac{L}{N}} \tag{19}$$

  where N_F is the total number of faults falsely identified in the simulation run.

5. RESULTS AND DISCUSSIONS

A comparative study of FDI and BFTCS control performance with different degrees of plant model mismatch is made for the following three *cases* namely,

- *FDI and process simulation using full-scale linear model (FS-L)*
- *FDI based on reduced order linear model, process simulated using full-scale linear model (RO-L)*

- *FDI based on reduced order linear model, process simulated using full-scale nonlinear model (RO-NL)*

As can be seen in tables 4 and 5, for sensor and parametric faults with linear process simulation ($FS - L, RO - L$), the FDI and BFTCS control performance obtained in terms of number of successful detections, false alarms committed and the PI values is almost identical with the two different accuracy models used for FDI. Further, in the presence of a sensor bias, the conventional control scheme produces an offset between the true value and the setpoint whereas the fault tolerant scheme corrects for the bias and therefore improves upon the control performance as indicated by the lower PI values. With parametric faults, the BFTCS does not further improves the control performance as the DMC conventional controller can itself handle such small magnitude process parameter changes.

Table 4. Comparison of FDI and BFTCS control performance for a sensor bias in $x_d(3\sigma_{x_d} = 0.027)$

Model	$\hat{b}$	PST	FAI	$(PI)_{x_d}$	$(PI)_{x_b}$
FS-L	0.0265	100	0.008	0.08	1.007
RO-L	0.0266	100	0.006	0.076	1.006
RO-NL	0.0239	100	0.014	0.09	0.964

Table 5. Comparison of FDI and BFTCS control performance for a parametric fault in $\eta(\sim 10\% = -0.075)$

Model	$\hat{b}$	PST	FAI	$(PI)_{x_d}$	$(PI)_{x_b}$
FS-L	-0.076	100	0.014	1.028	1.001
RO-L	-0.077	100	0.007	1.004	1.0002
RO-NL	-0.10	100	0.002	1.013	1.0002

With full-scale model, for actuator and disturbance faults, there is a significant deterioration in FDI performance in terms of increasing false alarms and in control performance because of subsequent fault accommodation. In fact these are quite unacceptable results from control performance point of view. However, the reduced dimensional model gives acceptable performance for all the fault cases and the model approximation introduced via reduction does not deteriorates the FDI performance. On the contrary, Kalman filter design based on an observable system model improves state estimation which leads to a better diagnostic performance.

Table 6. Comparison of FDI and BFTCS control performance for an actuator bias in $R(-1\sigma = -5)$

Model	$\hat{b}$	PST	FAI	$(PI)_{x_d}$	$(PI)_{x_b}$
FS-L	-4.87	66	0.096	1.36	1.01
RO-L	-5.07	74	0.03	1.013	1.004
RO-NL	-5.33	96	0.03	1.029	1.006

Table 7. Comparison of FDI and BFTCS control performance for a disturbance step change in $F(10\% = 10)$

Model	$\hat{b}$	PST	FAI	$(PI)_{x_d}$	$(PI)_{x_b}$
FS-L	9.9	100	0.14	1.79	1.01
RO-L	10.001	100	0.07	1.007	1.06
RO-NL	10.18	100	0.056	1.032	1.001

In all the cases, the nonlinear process simulation deteriorates the FDI and BFTCS control performance when compared to the linear case because of the resulting plant model mismatch. However, the PI values indicate that the resulting FTCS control performance is superior to conventional control in the presence of a sensor bias and is as good as the conventional control for the other input faults that are considered.

6. CONCLUSIONS

The proposed reduced order model based FDI scheme enables efficient real-time FTCS implementation for large dimensional processes. Furthermore, state estimation using reduced order observable models improves the diagnostic performance considerably when compared to the full-scale implementation.

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