PROCESS DESIGN FOR REDUCED DISTURBANCE SENSITIVITY OF INTEGRATED PLANTS

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Abstract: The disturbance sensitivity of an integrated plant is to a large extent caused by interactions between the process units, imposed by material and energy recycle flows. In this paper we show how a plug flow (delay) tank integrated in the recycle path can be used to modify these interactions such as to reduce the disturbance sensitivity in a given frequency range. The principle idea is to modify the physical feedback present in the process such that it provides disturbance damping, rather than amplification, in the frequency range where feedback controllers can not be made effective. As we show, the required capacity for a given level of disturbance attenuation with the proposed method can be severalfolds smaller than that required by a traditional cascaded buffer system. The results are illustrated by application to a reactor-separator system with recycle of unconverted material.

1. INTRODUCTION

Storage capacities, in the form of buffer and surge tanks, are extensively used in plants in the process industry. The purpose is partly to facilitate operatios, such as startup, and partly to improve control performance by damping disturbances. Due to both economic considerations and safety aspects, it is however desirable to keep the use of buffer tanks at a minimum.

For attenuation of disturbances, a control system is usually the most effective solution. However, all plants contain inherent properties that limit the achieveable control performance. If these limitations are in conflict with the performance requirements, then acceptable performance can only be achieved by modifying the process design. In this case, adding buffer or surge tanks to dampen disturbances often offers the least expensive solution. To keep the buffer size at a minimum, the buffer system should be designed so that it primarily act as a complement to the control system (Faanes and Skogestad, 2003).

Traditional buffers are typically cascaded next to some process unit with the aim of damping the disturbance magnitude, i.e., acting as low-pass filters. However, the disturbance sensitivity of integrated plants is to a large extent determined by interactions between the various process units. Thus, rather than using a capacity to directly dampen disturbances, it may be more efficient to utilize it to modify the disturbance amplification provided by the process unit interactions. To achieve this, it is necessary to first understand how the interactions affect the disturbance sensitivity, and how the interactions should be modified in order to reduce the disturbance sensitivity.

We start the paper with a brief introduction to controllability analysis, since this forms a basis for designing buffers that complement control systems. We then use linear systems theory to analyze the impact of process unit interactions, in plants with recycle flows, on the overall plant disturbance sensitivity. Based on these results, we show how a plug flow tank integrated in a recycle path can be systematically designed to achieve a desired reduction in the plant disturbance sensitivity. We compare the results with those obtained with traditional cascaded buffer tanks as considered in Faanes and Skogestad (2003) and Mahajanam and Zheng (2002).

2. INPUT-OUTPUT CONTROLLABILITY

Consider a linear dynamic model of a process

$$y(s) = G(s)u(s) + G_d(s)d(s) \qquad (1)$$

where y is the output to be controlled, u is the control input and d is the disturbance, all in deviations from their nominal steady-state values. We

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assume all signals to be scalar, but extensions to multivariable systems is straightforward. We assume the variables are scaled such that acceptable performance corresponds to all variables having magnitude less than 1.

Consider now a frequency response analysis, and define ω_d as the frequency where the scaled disturbance gain is unity, i.e.

$$|G_d(j\omega_d)| = 1 \tag{2}$$

Assume that this frequency is unique. For $\omega < \omega_d$, we then have $|y|/|d| > 1$ and hence the disturbance sensitivity needs to be reduced in the frequency range $\omega \in [0, \omega_d]$. This can be achieved either through feedback control, a modification of the process design, or a combination of the two. Usually, feedback control is the least expensive solution and thus a modification of the process design should only be considered when acceptable performance can not be achieved through feedback control alone.

Applying the feedback control law $u(s) = -C(s)y(s)$, the system can be described as

$$y(s) = S(s)G_d(s)d(s) \tag{3}$$

where $S(s) = 1/(1 + G(s)C(s))$ is the closed-loop sensitivity function. Acceptable disturbance sensitivity is obtained when $|SG_d(j\omega)| < 1,\ \forall\omega$. Define the bandwidth ω_B as the frequency for which $|S(j\omega_B)| = 1$. Then, for acceptable disturbance rejection, we get the bandwidth requirement $\omega_B > \omega_d$, i.e. the control must be effective at least up to the frequency ω_d.

A plant always has fundamental limitations which restrict the highest bandwidth ω_B that the feedback control system can achieve, even with the best possible controller. Fundamental limitations come from the process itself, e.g. in the form of time delays, unstable zero dynamics and input constraints. If the achieveable ω_B is smaller than ω_d, acceptable disturbance sensitivity can not be achieved using feedback control alone, and some modification of the process design is required in order to either increase the attainable bandwidth ω_B, or reduce the disturbance sensitivity of the process in the frequency range $\omega \in [\omega_B, \omega_d]$. In this paper we consider the latter option, and base our design modification on addition of capacities, or tanks, to the process.

3. EXAMPLE PROCESS: REACTOR-SEPARATOR PLANT

Chemical reaction followed by separation and recycle of unconverted reactant is a common process arrangement in the process industry. Figure 1 shows the simple reactor-separator system we consider here. The problem considered is that of

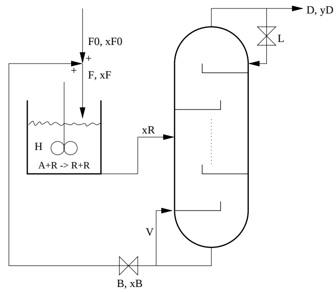

Figure 1. Reactor-separator system.

keeping the product composition y_D within pre-specified bounds in the presence of feed composition disturbances.

The individual model for the reactor is

$$dx_R = \underbrace{\frac{0.42}{2s+1}}_{G_r}(dx_{F0} + dx_B) \tag{4}$$

where x_R is the fraction of the reaction product in the reactor, and x_{F0} and x_B are the corresponding fractions in the two feed flows F_0 and B, respectively. By "individual" we here imply that the input x_B is assumed to be an independent variable when deriving the model, while in the overall plant x_B is determined by the separation unit.

For the separator we employ the individual model

$$dy_D = \underbrace{\frac{0.09}{10s+1}}_{G_{c1}} dx_R; \qquad dx_B = \underbrace{\frac{2}{10s+1}}_{G_{c2}} dx_R \tag{5}$$

where y_D and x_B are the product fractions in the two product flows of the separator.

The recycle flow is assumed to be equal to the fresh feed flow, i.e. $B = F_0$. Combining the two models (4) and (5), i.e. by eliminating x_R and x_B, we obtain the overall transfer-function from the disturbance x_{F0} to the product composition y_D

$$y_D(s) = \frac{0.236}{(73s+1)(1.7s+1)} x_{F0}(s) \tag{6}$$

We assume that the feed composition x_{F0} at most can change by 20% and that acceptable deviations in the product composition y_D is ± 0.005. This implies that the disturbance model (6) should be scaled by a factor 40, such that the scaled disturbance model becomes

$$G_d(s) = \frac{9.45}{(73s+1)(1.7s+1)} \tag{7}$$

Figure 2 shows the frequency response of G_d, from which we find that the disturbance sensitivity exceeds 1, and hence needs to be reduced, up to $\omega_d = 0.125$. Using feedback control only, this requires a corresponding bandwidth $\omega_B = \omega_d$, or a closed-loop time-constant of approximately $1/\omega_B = 8\ min$. If the achievable bandwidth is less than this, then the process design must be modified to reduce the disturbance sensitivity in the range $\omega \in [\omega_B, \omega_d]$.

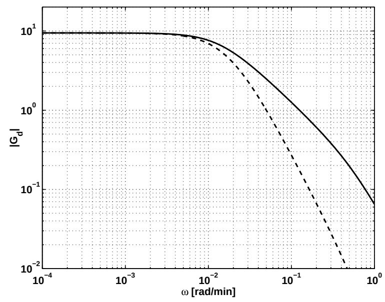

Figure 2. Reactor-separator system. Frequency response of distillate composition y_D to the disturbance feed composition x_{F0}. Dashed line shows response after addition of cascaded buffer with residence time $\tau_B = 45.4\ min$.

Assume that measurement delays and actuator dynamics prevents a bandwidth above $\omega_B = 0.05$, corresponding to a closed-loop time-constant of approximately 20 min. The process disturbance sensitivity must then be reduced by a factor $k_d = 2.5$ at $\omega = 0.05$. The required size of a cascaded mixed buffer system that achieves this can be calculated using the results in Faanes and Skogestad (2003), from which we find a single tank with residence time $\tau_B = 45.4\ min$. The resulting modified disturbance sensitivity is shown in Figure 2.

We next consider the potential improvement obtained by integrating the buffer, so as to modify the process unit interactions.

4. DISTURBANCE SENSITIVITY IN PLANTS WITH RECYCLE

A block diagram representation of a typical recycle process is shown in Figure 3. Note that we for simplicity assume all variables to be scalar and that the disturbances act on the output via variables in the recycle loop only. To relax these assumptions, see Carlemalm (2003). Here $G_f(s)$

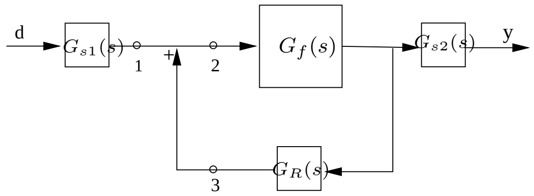

Figure 3. Block diagram representation of a typical recycle system. The numbers 1, 2 and 3 denote possible locations for a buffer tank.

is the transfer function of the forward path of the recycle loop, $G_r(s)$ represents the recycle path transfer function, and $G_{s1}(s)$, $G_{s2}(s)$ are transfer functions representing process units placed outside the recycle loop. For instance, for the reactor-separator in Figure 1, with x_{F0} as input and y_D as output, the reactor is in the forward loop, the separator bottoms in the recycle path, while the effect of reactor composition x_R on the top of the column y_D is external to the loop. Note that we here include variable scaling, as discussed above, in $G_{s1}(s)$ and $G_{s2}(s)$, such that $|d| \leq 1$, $|y| \leq 1$ correspond to acceptable performance. Also note that all transfer-functions in Figure 3 are derived for the unit operations individually, i.e. with the recycle flow replaced by an equivalent fixed flow.

Introducing

$$G_0 = G_f G_R \ ; \quad G_k = \frac{G_{s1}G_{s2}}{G_R} \tag{8}$$

the overall scaled disturbance sensitivity becomes

$$G_d(s) = \frac{y(s)}{d(s)} = \frac{G_k(s)G_0(s)}{1 - G_0(s)} \tag{9}$$

Based on (9) we are in a position to divide the overall disturbance sensitivity into one part caused by the individual units, i.e. the forward part $G_k(s)G_0(s)$, and one part caused by the unit interactions, i.e. the feedback part

$$S_p(s) = \frac{1}{1 - G_0(s)} \tag{10}$$

The function S_p, which describes the feedback amplification of disturbances, is known as the *sensitivity function* in feedback control theory, and we here correspondingly label it the *process sensitivity function* to emphasize that it describes the relative change in disturbance sensitivity due to the physical feedback present in the process itself.

In feedback control systems, the sensitivity is typically made small at low frequencies by making the loop-gain $|G_0|$ large and the phase $\angle G_0$ close to $-180°$ (negative feedback). However, a typical process unit has a positive steady-state gain, and hence $\angle G_0(0) = 0$. If we also assume stability of the individual process units as well as of the integrated plant, then it follows from Bodes stability theorem that $G_0(0) < 1$. With these assumptions we get from (10) that

$$S_p(0) > 1 \tag{11}$$

Thus, for most processes the physical feedback imposed by recycling will serve to increase the steady-state disturbance sensitivity. This is a well known fact, see e.g. (Gilliland *et al.*, 1964).

However, for higher frequencies the gain $|G_0(j\omega)|$ and phase lag $\angle G_0(j\omega)$ of the process units will change and the hence the feedback properties will also change with frequency. For feedback systems which are open-loop stable, i.e. individual process units stable, and for which there are at least two more poles than zeros in $G_0(s)$, the Bode Sensitivity Integral applies

$$\int_0^\infty ln|S_p(j\omega)|d\omega = 0 \tag{12}$$

See e.g. (Skogestad and Postlethwaite, 1996). According to (12), if $|S_p| > 1$ for some frequencies, then we must have $|S_p| < 1$ for some other frequencies. Thus, while recycling serves to increase the disturbance sensitivity at some frequencies, it will effectively reduce the disturbance sensitivity at other frequencies.

Figure 4 shows the process sensitivity function

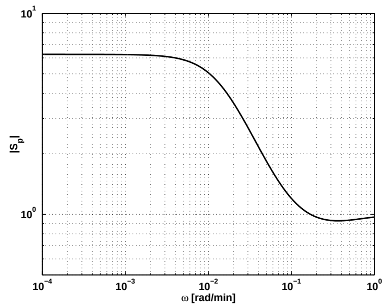

Figure 4. Process sensitivity function S_p as a function of frequency for reactor-separator.

$|S_p|$ for the reactor-separator process. As can be seen, the recycle serves to increase the disturbance sensitivity by a factor more than 6 at low frequencies, while it serves to slightly attenuate disturbances for frequencies above 0.15 rad/s.

Thus, we can conclude that while recycling typically provides disturbances amplification at low frequencies, it will always provide disturbance attenuation in some frequency range. This suggests that a design modification aimed at reduced disturbance sensitivity should, at least partially, aim at modifying the properties of the loop units such that the recycling provides disturbance attenuation, rather than amplification, in the frequency range where feedback controllers can not be made effective, i.e. around the bandwidth ω_B.

To better understand the conditions for when the feedback imposed by recycling will provide disturbance damping, consider Figure 5. According

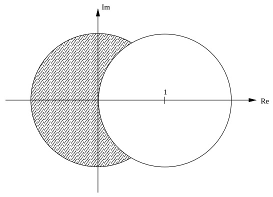

Figure 5. Region (scratched) of complex plane where the frequency response of the recycle loop $G_0(j\omega)$ should be for recycle feedback to provide disturbance damping.

to (10) the feedback will provide sensitivity reduction when $|1-G_0(j\omega)| > 1$, i.e. when the distance from the point 1 to the frequency response $G_0(j\omega)$ in the complex plane exceeds unity. Thus, $|S_p| < 1$ when $G_0(j\omega)$ is outside a circle with centre at 1 and radius 1. See Figure 5. If we also assume that $|G_0(j\omega)| < 1 \ \forall\omega$, corresponding to assuming that the process units has low-pass properties and that recycling does not affect stability, which applies to most process units, then we get that the feedback imposed by recycling will provide disturbance damping when the frequency response $G_0(j\omega)$ is within the scratched region of Figure 5.

The polar form of the open loop frequency response is

$$G_0(j\omega) = re^{j\theta} \tag{13}$$

where $r = |G_0(j\omega)|$ and $\theta = \angle G_0(j\omega)$. Then, the region outside the white circle in Figure 5, i.e. for which $|1 - G_0| > 1$ corresponding to disturbance damping from recycling, can be shown to correspond to

$$cos(\theta) < \frac{r}{2} \tag{14}$$

With the additional condition $0 < r < 1$, a sufficient condition for $|S_p| < 1$ is that $\theta \in [-3\pi/2, -\pi/2]$ while $\theta \in [-\pi/3, \pi/3]$ is a sufficient condition for $|S_p| > 1$. For other values of θ the conclusion will depend on the size of r. The minimum sensitivity is achieved for $\theta = -\pi$ and $r = 1$ for which $|S_p| = 0.5$.

By integrating a buffer in the recycle loop, we will modify both the loop-gain r as well as the phase-lag θ and can therefore expect to significantly modify the feedback properties. In particular, from the above discussion we see that the phase lag of the loop is crucial to obtain reduced disturbance sensitivity.

4.1 *Integrated Mixed Buffer Tank*

We consider adding a mixed buffer with transfer-function

$$G_B(s) = \frac{1}{\tau_B s + 1} \tag{15}$$

inside the recycle loop of a process system, with the aim of modifying the recycle feedback properties. We consider placement in the recycle path only. The disturbance sensitivity of the modified process becomes

$$G_{d1}(s) = \frac{y(s)}{d(s)} = \frac{G_k(s)G_0(s)}{1 - G_0(s)G_B(s)} \tag{16}$$

where the residence time $\tau_B = V/q_3$ and q_3 is the recycle flow, i.e. the flow at position 3 in Fig. 3.

The aim of the buffer design is to determine τ_B such that

$$|G_{d1}(\tau_B, j\omega_B)| = 1 \tag{17}$$

On polar form

$$G_0(j\omega_B) = re^{j\theta} \ ; \quad \frac{1}{G_B}(j\omega_B) = \tau_B\omega_B i + 1 \tag{18}$$

The required buffer residence time can now be determined by inserting the frequency responses into (16) and solving for $|G_{d1}(j\omega_B)| = 1$. This gives the required residence time of the buffer as

$$\tau_B = \frac{r sin\theta \pm \sqrt{r^2 sin^2\theta + (k_d^2 - 1)(1 - k^2 r^2)(kr/k_d)^2}}{(1 - k^2 r^2)\,\omega_B} \quad (19)$$

where $k = |G_k(j\omega_B)|$.

Note that all the values of τ_B, as computed from (19), may be physically unrealizable, being either complex or negative, in which case the required buffering effect can not be achieved with a mixed buffer placed in the recycle path. When there are two positive real roots τ_{B1} and τ_{B2} of (19), the smallest value gives the optimal residence time.

Example revisited: for the reactor-separator problem we have

$$G_0(s) = G_r(s)G_{c2}(s)\ ; \quad G_k(s) = 40\frac{G_{c1}(s)}{G_{c2}(s)} \quad (20)$$

From (19) we find that the smallest frequency for which the integrated buffer can reduce the disturbance sensitivity $|G_{d1}(j\omega)|$ to 1 is $\omega = 0.09$, with $k_d = 1.4$. For this bandwidth an integrated buffer with residence time $\tau_B = 8.3\ min$ is sufficient, while a cascaded buffer cascaded requires $\tau_B = 10.9\ min$. If we assume that the control system bandwidth is limited to $\omega_B = 0.05$, then an integrated buffer is not sufficient. However, Figure 6 shows the overall disturbance sensitivity as a function of the buffer residence time at $\omega = 0.05$, with an integrated and cascaded buffer, respectively. As can be seen, the integrated buffer is significantly more effective in reducing the disturbance sensitivity at low τ_B, but is not able to reduce the disturbance sensitivity below 1.25. and in fact increases the sensitivity if τ_B is increased further. This latter effect is explained by the fact that increasing the buffer size further will start to increase the process sensitivity function $|S_p|$ due to phase-lag effects.

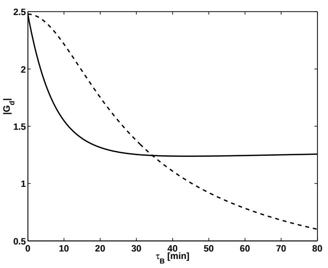

Figure 6. Reactor-separator system. Disturbance sensitivity at $\omega_B = 0.05\ rad/min$ as a function of buffer residence time for integrated mixed buffer (solid) and cascaded mixed buffer (dashed), respectively.

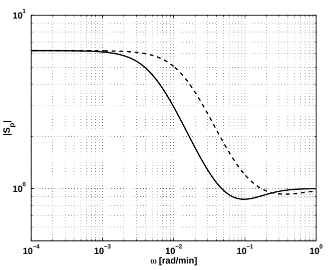

Figure 7. Reactor-separator system. Process sensitivity function S_p prior to (dashed) and after (solid) addition of a mixed buffer with residence time $\tau_B = 20\ min$ in the recycle path.

The effect of an integrated buffer on the process sensitivity is illustrated in Figure 7, showing the process sensitivity of the reactor-separator process before and after addition of a buffer with residence time $\tau_B = 20\,\mathrm{min}$. As can be seen, the buffer significantly reduces the sensitivity in the frequency range $\omega \in [0.002, 0.2]$, while it increases the sensitivity slightly for $\omega > 0.2$.

4.2 *Integrated Delay Tank*

According to (10), the effect of process unit interactions on the disturbance sensitivity is reduced by increasing $|1 - G_0 G_B(j\omega_B)|$. The geometrical interpretation is that the buffer should be used to increase the distance of the loop-gain $G_0 G_B(j\omega_B)$ from the point 1 in the complex plane. See Fig. 5.

Consider the integrated buffer that minimizes $|S_p|$ at ω_B. On polar form

$$G_0(j\omega_B) = re^{j\theta}\ ; \quad G_B(j\omega_B) = r_B e^{j\theta_B} \quad (21)$$

As before we assume $r < 1$ and $r_B \leq 1$. Then, the maximum of $|1 - r r_B e^{j(\theta+\theta_B)}|$ is achieved for $r_B = 1$ and $\theta_B = -\theta - (2m+1)\pi$, with m an integer, resulting in

$$\min_{G_B} |S_p(j\omega_B)| = \frac{1}{1+r} \quad (22)$$

Thus, the buffer $G_B(s)$ which yields the minimum process sensitivity function has unit magnitude and a non-zero phase-lag. This corresponds to a pure delay

$$G_B(s) = e^{-\tau_B s} \quad (23)$$

A delay process can be realized by a plug-flow tank with residence time τ_B.

The optimal solution to the buffer design problem, i.e. to make the disturbance sensitivity $|G_{d1}(j\omega_B)| = 1$ for the smallest total buffer volume, does not necessarily correspond to making $|S_p(j\omega_B)|$ minimal. However, we find that for most values of $G_0(j\omega_B)$, the distance $|1 - G_0 G_B(j\omega_B)|$ is indeed maximized for a given τ_B by employing $G_B(s) = e^{-\tau_B s}$, i.e. a pure delay.

By integrating a delay $G_B = e^{-\tau_B s}$ in the recycle path, the transfer-function for the modified disturbance sensitivity becomes

$$G_{d2} = \frac{G_k G_0}{1 - G_0 e^{-\tau_B s}} \qquad (24)$$

and the frequency response at $\omega = \omega_B$

$$|G_{d2}(j\omega_B)| = \frac{kr}{|1 - re^{j(\theta - \tau_B \omega_B)}|} \qquad (25)$$

where r and θ are defined by (21) and $k = |G_k(j\omega_B)|$. The requirement $|G_{d2}(j\omega_B)| = 1$ then yields

$$\tau_B = (\theta + cos^{-1}(\underbrace{\frac{1 + (1-k^2)r^2}{2r}}_{E_1}))/\omega_B \qquad (26)$$

Note that the addition of a delay will not always be sufficient to reduce the overall disturbance sensitivity $|G_{d2}|$ to 1 at ω_B. This will occur when the minimum process sensitivity function $|S_p| = 1/(1 + r)$ is not sufficient. in which case (26) does not have a solution.

Reactor-separator: With $\omega_B = 0.05$ we find from (26) that a delay tank with residence time $\tau_B = 23.5$ min will reduce the overall disturbance sensitivity to 1 at ω_B. The resulting disturbance and process sensitivity are shown in Figure 8. As can be seen, the delay reduces the process sensitivity function at ω_B from about 2 to 0.7, i.e., almost by a factor 3. Note that the delay introduces a resonance in the system such that the sensitivity is increased at higher frequencies. However, the total disturbance sensitivity $|G_{d2}|$ stays less than 1 for all frequencies above the bandwidth. The required delay tank size is almost half of the optimal mixed cascaded buffer which requires a residence time of $\tau_B = 45.4$ min. The effective filtering effect

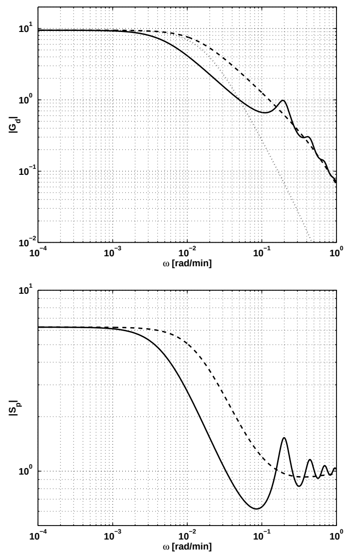

Figure 8. Reactor-separator system with integrated delay tank, $\tau_B = 23.5$ min. Top: disturbance sensitivity G_d. Bottom: process sensitivity function S_p. Solid line: delay buffer. Dashed line: original process. dotted line: cascaded buffer with $\tau_B = 45.4$ min.

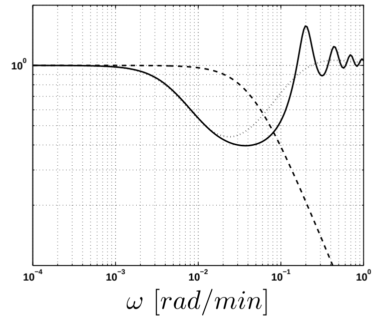

Figure 9. Reactor-separator system. Effective disturbance filtering of integrated delay (solid), cascaded (dashed) and integrated (dotted) mixed tank, $\tau_B = 23.5$ min.

obtained by an integrated delay with residence time $\tau_B = 23.5$ and a corresponding mixed buffer cascaded and integrated, respectively, is shown in Figure 9. As can be seen, the filtering effect of the delay tank is more than twice of the cascaded mixed tank, and about 30% more than the integrated mixed tank, at the frequency $\omega = 0.05$. As noted above, integrating a delay in the recycle path can give rise to resonance peaks in the disturbance sensitivity. Due to space limitations we refer to Carlemalm (2003) for results on how to deal with such resonances.

5. CONCLUSIONS

The disturbance sensitivity of an integrated plant is to a large extent determined by feedback interactions between the process units, induced by recycling of material and energy. We considered the use of storage capacities to modify the interactions such as to obtain favorable feedback properties in the frequency range where disturbance attenuation is needed, but infeasible with feedback control. As shown, adding a delay in the recycle path is the most effective means of reducing the process sensitivity in a given frequency range. We only considered monovariable recycle here, but extensions to multivariable cases are discussed in Carlemalm (2003).

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