MULTI-OBJECTIVE INPUT SIGNAL DESIGN OF MULTI-HARMONIC SIGNALS FOR SYSTEM IDENTIFICATION

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Abstract: The choice of perturbation inputs is critical in the identification and model building exercise. Accurate identification requires that the input be persistently exciting so as to excite all modes or frequencies of interest. However, there are multiple objectives that need to be considered. Multi-harmonic signals are a convenient choice for frequency domain identification. Multi-objective optimization formulations for synthesis of multi-harmonic signals are presented.

Keywords: Identification, Optimal experiment design, Input signals, Multi-objective optimization

1. INTRODUCTION

One of the primary goals of system identification is to ensure that the identified model has good predictive capabilities and the model parameters are accurately estimated. It is common practice to perturb the system with specially tailored inputs and the consequential input output data are used to build the system model. The quality of the model depends strongly on the experiment design and identification and hence the input used for perturbing the system should be carefully selected.

Researchers have paid considerable attention to system identification and input signal design and a number of excellent reviews in these fields are available (Godfrey, 1993; Ljung, 1999; Pintelon and Schoukens, 2001). The theory of statistical experiment design has been applied to system identification to synthesize maximally informative inputs in the time and frequency domains (Goodwin and Payne, 1977; Kalaba and Spingarn, 1982; Mehra, 1981).

Frequency domain synthesis of the optimal input signal involves determination of the input spectrum that minimizes a scalar criterion of the uncertainty of the parameters to be estimated. Multi-frequency binary signals and multi-harmonic (sum of sines) signals are commonly used to realize the signal with the desired power spectrum. There are many advantages to using multi-sine signals with low crest factors (Guillaume *et al.*, 1991; Pintelon and Schoukens, 2001; Solomou *et al.*, 2002).

The concept of plant friendly identification that is relatively less hostile to operating conditions in process plants has received attention amongst members of the process control and identification community recently (Narasimhan *et al.*, 2003; Parker *et al.*, 2001; Rivera *et al.*, 2003). Different measures of plant friendliness have been proposed in literature.

There has been some recent application of multi-objective optimization based methods to identification and control (El-Kady *et al.*, 2003; Johansen, 1996; Johansen, 2000). However, a multi-objective approach to input design has been proposed only recently by the present authors. Illustrative formulations for optimal

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design of input signals that characterize the multi-objective nature of the design problem are presented in this contribution.

2. OBJECTIVES

2.1 Preliminaries

Consider a Linear Time Invariant system described by the Impulse Response Model of the following form

$$y(t) = \int_0^{T_s} u(t-\tau)h(\tau)d\tau + n(t) \quad (1)$$

where $y(t)$, $u(t)$, $n(t)$ are the output, input and noise respectively and $h(\tau)$ is the impulse response of the system. The frequency response function $H(j\omega)$ can be estimated as $Y(j\omega)/U(j\omega)$ where $Y(j\omega)$ and $U(j\omega)$ are the Fourier transforms of the output and input respectively. One of the primary objectives of a good system identification exercise is to accurately estimate the unknown parameters by reducing the bias and the variance of the parameter estimates. For input design purposes, it is convenient to assume that the estimator is efficient and unbiased. This implies that that the covariance of the parameter estimates can be calculated from the Cramer-Rao bound, viz., the inverse of the Fisher information matrix M. Optimal input signal design in the frequency domain thus involves determination of the optimal input spectrum that minimizes a scalar norm of M^{-1}, like $tr(M^{-1})$ or $\det M^{-1}$ or $\lambda_{\max}(M^{-1})$ where λ is an eigenvalue of M^{-1}.

Calculation of the optimal input spectrum requires *a priori* knowledge of the the noise characteristics and transfer function of the system which is supplied through an initial estimate. An analytical expression for the optimal spectrum can be calculated only for some problems. Iterative algorithms for optimization of the input power spectrum have been proposed (Pintelon and Schoukens, 2001). In general, it is difficult to realize a signal with an arbitrary spectrum and so, it is convenient to restrict the frequency space to a finite, discrete, harmonically related set of frequencies. A multi-harmonic having N_u distinct harmonics $\{\omega_u\}$ (Ref equation 2) can then be generated that has the desired amplitude spectrum.

$$u(t) = \sum_{u=1}^{N_u} a_u \cos(\omega_u t + \alpha_u) \quad (2)$$

where $\omega_u = 2\pi k_u/T$ and k_u are monotonically increasing harmonic numbers ($k_u \in \mathbb{N},\ u = 1,2,\ldots N_u$) and $\{a_u\}$ is the specified amplitude spectrum. The advantage of using a multi-sine or multi-harmonic signal is that it is periodic and easy to generate for any given arbitrary spectrum. The steady state output of the system when subjected to the above input is

$$y(t) = \sum_{u=1}^{N_u} a_u |G(j\omega_u)| \cos(\omega_u t + \alpha_u + \phi_u) \quad (3)$$

where, $|G(j\omega_u)|$ and $\phi_u = \arg(G(j\omega_u))$ are the amplitude ratio and phase shift of the system respectively, evaluated at the frequency $\omega_u = 2\pi k_u/T$.

For a linear q input, p output system, the i^{th} output y_i, $i = 1,2,\ldots p$ is given by

$$y_i(t) = \sum_{l=1}^{q}\sum_{u=1}^{N_u} a_{lu}|G_{li}(j\omega_{lu})|\cos(\omega_{lu}t + \alpha_{lu} + \phi_{li}(j\omega_{lu})) \quad (4)$$

where $|G_{li}(j\omega_{lu})|$ and $\phi_{li}(j\omega_{lu})$ are the gain and phase shift introduced between input l and output i at frequency $2\pi k_{lu}/T$. It can be seen that if the inputs are chosen so that the spectral lines k_{lu} do not coincide for any pair of inputs $l = 1,2,\ldots q$,

$$|Y_i(j\omega_{lu})| = |G_{li}(j\omega_{lu})|a_{lu} \quad (5)$$

Thus, ensuring that the spectral lines of the inputs do not overlap allows the MIMO system to be broken into pq SISO systems.

2.2 Crest factor minimization

Signals with a specified spectral distribution $\{a_u\}$ may be realized in different ways in the time domain. For instance in a multi-sine, the spectral characteristics are described by $\{a_u\}$ alone and the phases do not affect the spectral characteristics. However, the choice of phases can affect the time domain behaviour of the signal. It is possible that an inappropriate choice of phases results in a signal with large magnitude peaks. A more compact or compressed signal would have a small signal amplitude between the peaks. The time domain compression or compactness of a multi-sine signal $x(t)$ with a specified amplitude spectrum is described by the crest factor, (Cf), or the peak factor (Pf) (Godfrey, 1993).

$$Pf = \frac{x_{max} - x_{min}}{2\sqrt{2}x_{rms}} \quad (6)$$

$$Cf = \frac{\max|x|}{x_{rms}} \quad (7)$$

where $x_{min}, x_{max}, x_{rms}$ denote the minimum, maximum and rms values of the signal respectively. The advantage of synthesizing a minimum crest factor input signal is that large peaks are avoided. Further, it has been shown that the number of averages required to measure a signal with a specified accuracy is proportional to the crest factor (Godfrey, 1993). In addition, it has been reported that reducing the crest factor has the added advantage of reducing the effect of distortions due to certain nonlinearities (Solomou *et al.*, 2002). For a specified amplitude spectrum $\{a_u\}$, it is possible to synthesize an input signal with a low crest factor by a suitable choice of phases α_u. Different algorithms for crest factor minimization have been reported, the most efficient and common being the L_∞ algorithm based on a generalization of Polya's algorithm (Guillaume *et al.*, 1991). The crest factor is thus the ratio of the L_∞ norm and the L_2 norm where the general L_p norm of the function $x(t)$ over the interval $[0,T]$ is defined as

$$L_p(x) = \left[\frac{1}{T}\int_0^T |x(t)|^p dt\right]^{(1/p)} \tag{8}$$

and the L_∞ norm is $\max|x(t)|$. The crest factor of the output $y(t)$ can be defined in a similar manner. The output $y(t)$ is a multi-harmonic signal represented by Equation 3 for ease of calculation since the steady state output is a phase and amplitude modulated version of the multi-harmonic input signal. Thus the L_∞ algorithm can be applied to minimize the crest factor of the output if the system transfer function or an estimate is available.

2.3 Plant friendly identification

Multi-variable model based control strategies are commonly used in chemical process industries. Plant friendly identification has received the attention of researchers in recent times (Narasimhan *et al.*, 2003; Parker *et al.*, 2001; Rivera *et al.*, 2003). Identification experiments in process industries are carried out on running plants. While a persistently rich excitation with high signal to noise ratio is theoretically preferred, operational, safety, environmental and economic considerations have to be taken into account during identification.

- An input requiring aggressive and frequent movement of valves and actuators is not desirable as this can lead to equipment wear and tear.
- Identification experimentation time has to be kept to a minimum so as to minimize off-spec products and consumption of utilities. Tests using the popular Pseudo-Random Binary Signals (PRBS) usually require days to conduct (Smith, 2003).
- Output deviations should be reduced to ensure that the product quality differs as little as possible from the set point

The input spectrum is chosen either by optimization with respect to the criteria discussed above or based on the user's experience. Given an input spectrum, a convenient measure of plant friendliness (both the input and output) is the crest factor defined above. It must be noted that a minimum crest factor input signal does not necessarily imply a minimum crest factor output signal. Thus, from a practical consideration, it is necessary to use an input signal that has a low **Input Crest factor** and results in an **Output with low crest factor**.

2.4 Distortion due to nonlinearities

The linear convolution integral can be generalized to describe a general, causal, time invariant stable nonlinear system in the following manner

$$y(t) = \sum_{i=1}^{\infty}\int h_i(\tau_1,\ldots,\tau_i)\prod_{j=1}^{i} u(t-\tau_j)d\tau_j \tag{9}$$

where $h_n(\tau_1,\ldots,\tau_n)$ is the n dimensional n^{th} order Volterra kernel and can be viewed as an n dimensional impulse response.

The corresponding frequency domain representation of an nth order system is

$$Y(s_1,s_2,\ldots,s_n) = \sum_{i=1}^{n} H_i(s_1,\ldots,s_i)\prod_{j=1}^{i} U(s_j) \tag{10}$$

where $H_n(s_1,\ldots,s_n)$ is the corresponding n dimensional Laplace transform of $h_n(\tau_1,\tau_2,\ldots,\tau_n)$. When such a system is subjected to a multi-sine as described in equation 2, the output will contain frequencies other than the test or input frequencies. The frequency contributions due to the nonlinearities can be classified as Type I (arising from pairs of equal positive and negative frequencies) and Type II (which are not accounted by Type I) contributions. It is possible to suppress even order nonlinearities by considering only odd harmonics, as the nonlinear contributions of the even order effects will fall at even harmonics (Solomou *et al.*, 2002) and hence will be distinct from the linear and higher odd order contributions. Hence, for illustrative purposes, a simple cubic static nonlinearity with unit gain is considered. The nonlinear contribution of the output corresponding to a multi-sine input can be expressed in the frequency domain as

$$Y_{nl}(j\omega) = \sum_{\substack{m=-N_u\\ m\neq 0}}^{N_u}\sum_{\substack{n=-N_u\\ n\neq 0}}^{N_u}\sum_{\substack{o=-N_u\\ o\neq 0}}^{N_u} a_m a_n a_o \exp(j(\alpha_m+\alpha_n+\alpha_o)) \times\delta(\omega-(k_m+k_n+k_o)2\pi/T) \tag{11}$$

The extent of distortion due to nonlinearity at the test frequencies can be quantified by

$$E = \frac{1}{N_u}\sum_{u=1}^{N_u} |Y_{nl}(j\frac{2\pi k_u}{T})| \tag{12}$$

If the primary aim of the identification experiment is to identify the linear kernels accurately, it is possible to reduce the contribution of the nonlinearities by a suitable choice of phases.

Thus, from an optimization point of view, one could require that the phases of the input multi-sine signal be chosen so that the input crest factor is minimized and the nonlinear contributions at the test frequencies are also minimized. Again, it must be noted that minimum crest factor signals do not necessarily minimize the nonlinear contributions.

In the previous sections, two design scenarios for input signal design, each with multiple objectives were described. Since, both objectives need not be attainted simultaneously, the problem is inherently multi-objective in nature and should be treated as such. In the succeeding sections, some common techniques for solution of a multi-objective optimization problem are described.

3. MULTI-OBJECTIVE OPTIMIZATION

In traditional single objective optimization problems, the aim is to find a globally optimal solution, if it exists. Unlike single objective optimization problems, in optimization with possibly conflicting objectives, there is no unique optimal solution. System and real world design usually involves tradeoffs between different objectives and more than one decision maker. A fair amount of subjectivity and user influenced decision making are characteristics of multi-objective problems. There are several possible approaches for solving a multi-objective optimization problem (Miettinen, 1998).

Single weighted cost function: One simple and common approach to multi-objective optimization is to formulate a single weighted objective function from the individual costs J_i.

$$J = \alpha_1 J_1 + \ldots + \alpha_n J_n \tag{13}$$

such that the weights $\alpha_i \geq 0$. The resulting problem can be solved by standard methods of optimization.

Goal programming: In goal programming, a goal or aspiration value, γ_i is associated with each objective function and the weighted deviation from the target is minimized. These goals are not constraints, but only aspiration levels that may or may not be satisfied. Given an objective function J_i and goal γ_i, the deviation variable δ_i can be written as the difference of δ_i^- (negative deviation or underachievement) and δ_i^+ (positive deviation or overachievement) . It is necessary only to minimize the weighted sum of positive deviation variables δ_i^+ for minimization problems where δ_i^+ are the individual weights.

$$\text{Minimize}\{\sum w_i^+ \delta_i^+\} \;\; \texttt{s.t} \begin{cases} J_i + \delta_i^- - \delta_i^+ = \gamma_i & \forall i \\ L_j \leq 0 & \forall j \\ E_k = 0 & \forall k \\ \delta_i^+, \;\; \delta_i^- \geq 0 & \forall i \end{cases} \tag{14}$$

where E_k and L_j are equality and inequality constraints respectively.

Pareto solutions: The above techniques essentially convert a multi-objective optimization problem into a single objective problem. Unlike single objective optimization, in optimization with conflicting objectives, there is no single optimal solution. The interaction among different objectives gives rise to a set of solutions, called the Pareto optimal solutions (Ref Fig. 1). Solutions A, D, B form a Pareto optimal front and no one solution in this set can be said to be better than another in pure quantitative terms. However, solution C is dominated by solution D as solution D is better than C in both objectives. A set is called a global Pareto-optimal set, if no solution in the search space dominates any member in it. The optimization algorithm should attain two goals :- search for the global Pareto-optimal front, and maintain population diversity in the optimal front so that no bias towards any particular objective function exists. The final solution that is chosen for implementation is based on some other criteria. Evolutionary algorithms have been particularly useful in solving multi-objective optimization problems (Deb, 1999).

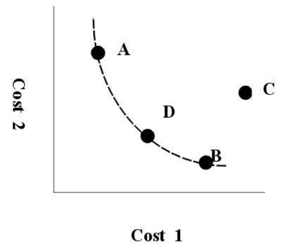

Fig. 1. Pareto optimal sets

Thus, in input signal design, there are multiple objectives, some of which cannot be satisfied simultaneously. In the subsequent section, two Pareto solutions corresponding to different cost functions are presented.

4. GENERATION OF PARETO FRONTS

4.1 Input-output crest factor minimization

Consider a 15 harmonic multisine described by equation 2 with a flat amplitude spectrum $a_u = 1, \; u = 1, 2, \ldots, 15$ and $k_u = 1, 2, \ldots, 15$, with a fundamental harmonic equal to 0.0625 rad/sec. It is required to minimize the crest factors of the input and output where the system transfer function is

$$H(s) = \frac{s}{s^3 + 2s^2 + 2s + 1} \tag{15}$$

The phases of the respective harmonics are chosen so as to minimize the input and output crest factors. As mentioned previously, use of a minimum input crest factor signal does not necessarily result in a minimum output crest factor. The L_∞ algorithm can be applied to compress the input and outputs simultaneously by defining a common norm of the input $u(t)$ and output $y(t)$ (Guillaume *et al.*, 1991). By suitably weighting the inputs and outputs, signals with different (not necessarily minimum) crest factors can be synthesized. However, the above method is not guaranteed to generate the complete Pareto front, since the solutions obtained by solving the weighted problem are weakly Pareto optimal. Another disadvantage is that it is necessary to solve a single valued objective optimization problem several times by varying the weights.

Since Genetic Algorithms (GA) work with a population of points, it is natural to use GAs for obtaining the set of non-dominated solutions simultaneously. GAs

Table 1. GA Parameters

Parameter	Crest factor	Nonlinear error
Mutation probability	0.05	0.07
Crossover probability	0.9	0.9
Population size	500	500
Number of generations	350	500
Crossover Distritution index	50	60
Mutation distribution index	50	60

do not use gradient based techniques and so, the non-differentiable nature of the L_∞ norm does not pose any analytical problems. For a specified amplitude spectrum and given system transfer function, the L_2 norm is invariant with respect to the phases, and so minimization of the crest factor is equivalent to minimizing the respective L_∞ norms. The L_∞ norm of the function $f(t), t \in [a,b]$ is approximated by determining the maximum of $|f(t)|$ over a discrete number of points in the interval $[a,b]$. The real coded variable version of the Nondominated Sorting Genetic Algorithm (NSGA-II) (Deb *et al.*, 2002) is used to generate the Pareto front of input and output crest factors. The Pareto front for the input and output crest factors at the end of 350 generations is shown in Fig. 2 (A). Parameters used for the solution are tabulated in Table 1. In order to verify that the solution is close to the true Pareto optimal solution, the following metric is defined

$$d = \sum_{i=1}^{N_p} (f_{1i}^2 + f_{2i}^2) \tag{16}$$

where $[f_{1i}, f_{2i}]^t$ is the i^{th} non-dominated objective vector and N_p is the number of non-dominated solutions in the current population. In this particular example, they are the input and output crest factors respectively. Fig. 3(A) shows the evolution of the distance metric d evaluated for all the non-dominated solutions in every generation. It is seen that the metric d does not increase with subsequent generations, thus confirming that the solutions are reasonably close to the true Pareto optimal solution. It must be noted that while the NSGA -II algorithm aims to preserve diversity among the population, it is possible that the front obtained is a subset of the whole front, since the true Pareto front is not known.

4.2 Reducing the effects of nonlinear distortions

Consider a muti-sine subjected to a simple nonlinear element

$$y(t) = u(t) + u(t)^3 \tag{17}$$

Thus, the nonlinear distortion at the test frequencies is caused solely by the cubic element and can be quantified by equation 12. The objectives are to choose the phases in a 15 harmonic multi-sine with a flat amplitude spectrum $a_u = 2,\ u = 1,2,\ldots,15$ and $k_u = 1,2,\ldots,15$ so as to minimize the input crest factor and the nonlinear distortions. As above, the multi-objective optimization problem is solved using the NSGA-II algorithm (Deb *et al.*, 2002) with the parameters mentioned in Table 1. The corresponding Pareto front and the evolution of the metric d is presented in Fig. 2 (B) and Fig. 3 (B).

Thus, it is clear from the above figures that there exists a trade-off between the different objectives. It is possible to sacrifice the input friendliness to achieve a lower value of the output crest factor. Likewise, for a relatively small change in the nonlinear error, it is possible to reduce the input crest factor. Standard techniques of input signal design do not allow the user the flexibility to tailor an input signal that satisfies several objectives or consider the trade-offs. Solution of the appropriate multi-objective optimization problem results in the Pareto set of non-dominated solutions. The final choice of the input signal is made by the user. It must be noted that computing the output crest factor requires knowledge of the system transfer function. In a practical system identification setting, this is unknown or only an estimate is available. The effect of uncertainty of the transfer function on the output crest factor needs to be investigated. The effect of dynamic nonlinearities and a multi-optimization formulation to reduce the distortion of non-linearities also needs to be investigated.

5. CONCLUSION

Input signal design for system identification typically requires that multiple objectives be satisfied. Multiple objectives that arise in the design of multi-harmonic input signals were discussed and appropriate problems were formulated. A Genetic Algorithm based approach was used to solve for the Pareto front of non-dominated solutions.

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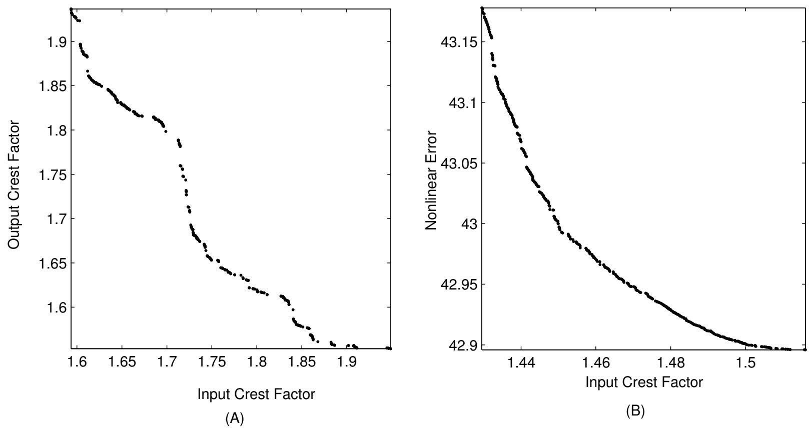

Fig. 2. Pareto optimal fronts

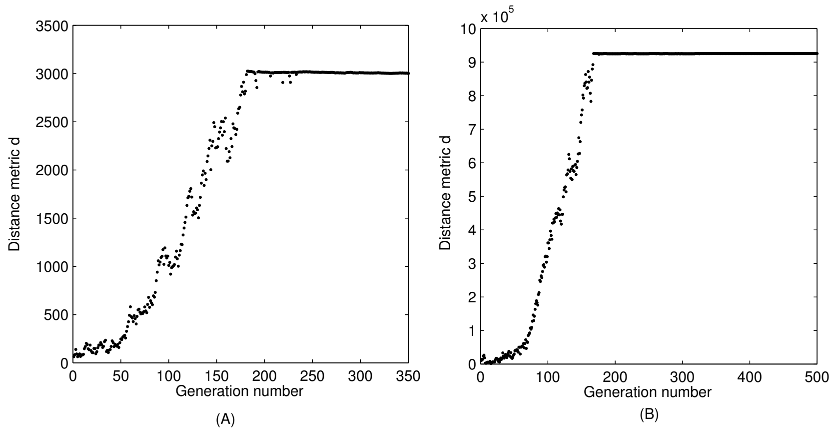

Fig. 3. Distance metric d

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