DANTZIG-WOLFE DECOMPOSITION AND LARGE-SCALE CONSTRAINED MPC PROBLEMS

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Abstract

Model Predictive Control (MPC) strategies are typically implemented in two levels: a steady-state target calculation and a control calculation. The steady-state target calculation consumes excess degrees of freedom within the control problem to provide optimal steady-state performance with respect to some specified objective. In some MPC approaches, the target calculation is formulated as a Linear Program (LP) with a pre-specified objective function and a linear or linearized steady-state model derived from that used in the control calculation. In large-scale problems, centralized MPC schemes find the optimal solution for the plant-wide optimization problem, but may not provide sufficient redundancy or reliability and can require substantial computation. On the other hand, in a decentralized MPC scheme, the target calculations are performed independently by ignoring interactions among units, and as a result will not usually find the optimal operation. In contrast to the centralized MPC approach, a decentralized MPC provides a high degree of redundancy with respect to the failure of an individual MPC. For large-scale process control problems, the desired characteristics for an MPC implementation include: accurate and quick tracking of the changing optimal steady-state operation, a high degree of reliability with respect to failure within the MPC application (i.e., failure of a portion of the control system), and low computational requirements. Fully centralized or monolithic MPC and independent block-wise decentralized MPC represent the two extremes in the "trade-off" among the desired characteristics of an implemented MPC system. In this paper, we propose a coordinated, decentralized approach to the steady-state target calculation problem. Our approach is based on the Dantzig-Wolfe decomposition principle and has been found to be effective at finding the optimal plant operation while providing a high degree of reliability at a reasonable computational load.

Keywords

Model predictive control, Steady-state target calculation, Dantzig-Wolfe decomposition, Coordination, Linear programming.

Introduction

Model predictive control (MPC) strategies have been successful in a wide range of industrial applications. In the current control hierarchy, MPC plays an important role as a connection between local steady-state optimization and basic dynamic control (Kassmann *et al*. 2000). The MPC framework can be further divided into a steady-state calculation and a control calculation. The goal of the steady-state calculation is to calculate the desired targets for both output and input variables at a frequency much higher than those computed from local economic optimizers. The target calculation provides optimal achievable set-points that are passed to the control calculation.

Two approaches to using MPC in large-scale applications are centralized and decentralized MPC. By considering interactions among operating units,

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centralized MPC can offer plant-wide optimal control, including plant-wide target calculations. Unfortunately, the centralized MPC scheme contains a large-scale control calculation with little or no redundancy, implemented within relatively centralized computational environment. Alternatively, in the decentralized MPC scheme, the large-scale control problem is decomposed into a set of separate MPC subproblems being solved independently. Though the completely decentralized MPC target calculation may fail to yield plant-wide optimal solutions, it has higher degree of reliability to subsystem failure and may require less computation. In our work, reliability refers to the possibility that some control subsystems or portions thereof are able to function when other subsystems fail.

For large-scale process control problems, the essential trade-off is between: accurate and quick tracking of the changing optimal operation, reliability with respect to failure within the MPC application (i.e., failure of a portion of the control system), and computational requirements. Therefore, fully centralized or monolithic MPC and independent block-wise decentralized MPC represent the two extremes in the "trade-off" among these desired characteristics. In this paper, we propose an approach to coordinating decentralized steady-state target calculation that can yield the accuracy of centralization and the reliability of decentralization.

In a decentralized MPC scheme, the flowsheet is separated into sections that have separate control systems. In the decentralized approach, target calculation takes the strategy that the upstream units pass their decisions to the downstream units without considering interactions. The downstream units treat the upstream decisions as disturbances. Thus, the decentralized approach usually produces sub-optimal results. If the target calculation could be coordinated as shown in Fig. 1, it may be possible to produce an optimal solution similar to that of

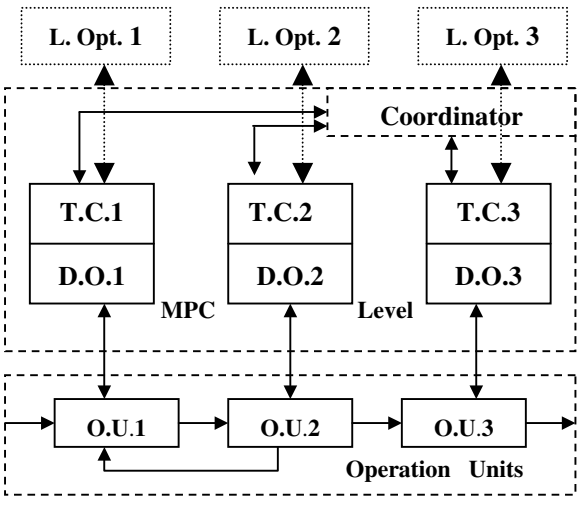

Figure 1.Coordinated, Decentralized MPC Target Calculation

the centralized approach. While Venkat and Rawlings (2003) are developing a communication-based scheme for the control calculations within a decentralized MPC scheme, the focus of our current study is steady-state targets optimization.

This paper discusses one possible approach, by taking advantage of the Dantzig-Wolfe decomposition principle, to coordinating the interactions among decentralized MPC units and provides insight into the information that must be communicated between MPC subsystems and the coordinator to ensure high performance levels. The outline is as follows. Firstly, we introduce the Dantzig-Wolfe decomposition principle. Secondly, we formulate a coordinated LP target calculation problem by introducing a coordination mechanism. Then, we implement the decomposition algorithm to solve the large-scale constrained MPC target calculation problem. An illustrative case study demonstrates the efficacy of the proposed approach.

Dantzig-Wolfe Decomposition Principle

The Dantzig-Wolfe decomposition principle (Dantzig and Thapa, 2002) is illustrated in Fig. 2.

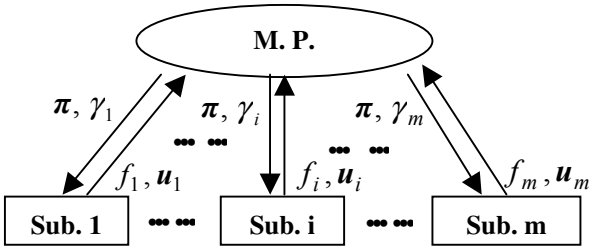

Figure 2. Mechanism of Dantzig-Wolfe Decomposition

A large-scale linear programming problem can be decomposed into independent subproblems, which are coordinated by a *master problem* (MP). The solution to the original large-scale problem can be shown to be equivalent to solving those subproblems and the MP through a finite number of iterations. During each iteration, the MP handles the linking constraints that connect the subproblems and the convexity constraints (which will be discussed later) for each subproblem, using the information supplied by the subproblems [f_i, $\boldsymbol{u}_i$], where f_i and $\boldsymbol{u}_i$ are the objective function value and solution of the i^{th} subproblem, respectively. Then the MP sends sensitivity information [$\boldsymbol{\pi}$, γ_i], corresponding to the linking constraints and convexity constraints respectively, to all the subproblems for updating their objective functions. Consequently, the subproblems with updated objective functions are re-optimized independently. The above procedure is repeated until convergence and the solution is the optimum of the original large-scale problem.

Dantzig-Wolfe decomposition hinges on the theorem of convex combination and the column generation technique. To help understand the Dantzig-Wolfe decomposition principle, the following subsections briefly introduce these two fundamental components of the decomposition principle.

Theorem of Convex Combination

The theorem of convex combination (Lasdon, 2002), or D-W transformation (Dantzig and Thapa, 2002), states that an arbitrary point $\boldsymbol{x}$ in a convex polyhedral set $\boldsymbol{X} = \{\boldsymbol{x} \mid \boldsymbol{Ax} = \boldsymbol{b}, \boldsymbol{x} \geq \boldsymbol{0}\}$ can be written as a convex combination of the extreme points of $\boldsymbol{X}$ plus a nonnegative linear combination of the extreme rays (normalized homogeneous solutions) of $\boldsymbol{X}$, or:

$$\boldsymbol{x} = \sum_{i=1}^{L} \alpha_i \boldsymbol{u}^i + \sum_{j=1}^{M} \beta_j \boldsymbol{v}^j \tag{1}$$

$$\sum_{i=1}^{L} \alpha_i = 1 \qquad \alpha_i, \beta_j \geq 0 \tag{2}$$

where $\{\boldsymbol{u}^i\}$ and $\{\boldsymbol{v}^j\}$ are the finite set of all extreme points and the finite set of all extreme rays, respectively. If the feasible region is bounded, we can reformulate the problem by using the extreme points only.

The Dantzig-Wolfe decomposition principle is particularly powerful for solving structured linear programs (Chvátal, 1983). Consider a block-wise linear programming problem:

$$\boldsymbol{min} \quad \sum_{i=1}^{m} \boldsymbol{c}_i^T \boldsymbol{x}_i$$

$$\boldsymbol{s.t.} \quad \sum_{j=1}^{n} \boldsymbol{A}^j{}_{\boldsymbol{eq}} \boldsymbol{x}_j = \boldsymbol{b}^0{}_{\boldsymbol{eq}} \qquad n \leq m \tag{3a}$$

$$\boldsymbol{B}^i{}_{\boldsymbol{eq}} \boldsymbol{x}_i = \boldsymbol{b}^i{}_{\boldsymbol{eq}} \tag{3b}$$

$$\boldsymbol{G}^i \boldsymbol{x}_i \leq \boldsymbol{b}^i \qquad \mathrm{i} = 1,2,\ldots\ldots,\mathrm{m} \tag{3c}$$

where the equality constraints in (3a) represent the linking constraints associated with *n* interacting subproblems. The equality constraints in (3b) and the inequality constraints in (3c) are the local constraints of independent subproblems. The master problem (MP) can be formulated as follows using the linking constraints in (3a) and the convex combination as stated in (1), assuming that the feasible regions of the subproblems are bounded.

$$\boldsymbol{min} \quad \sum_{i=1}^{m} \sum_{j=1}^{p(i)} f_{ij} \lambda_{ij}$$

$$\boldsymbol{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{p(i)} \boldsymbol{P}_{\boldsymbol{ij}} \lambda_{ij} = \boldsymbol{b}^0{}_{eq} \tag{4}$$

$$\sum_{j=1}^{p(i)} \lambda_{ij} = 1, \quad \lambda_{ij} \geq 0, i = 1,2,\ldots\ldots,m \tag{5}$$

where $p(i)$ represents the number of extreme points of the feasible region of the i^{th} LP subproblem, and

$$\boldsymbol{x}_i = \sum_{j=1}^{p(i)} \lambda_{ij} \boldsymbol{u}_i^j \tag{6}$$

$$f_{ij} = \boldsymbol{c}_i^T \boldsymbol{u}_i^j \tag{7}$$

$$\boldsymbol{P}_{ij} = \boldsymbol{A}^i \boldsymbol{u}_i^j \tag{8}$$

The resulting MP problem has fewer rows in the coefficient matrix than the original problem. However, the number of columns in the MP is larger due to the increase in the number of variables associated with the extreme points of all the subproblems. The column generation method discussed in next section provides an approach for dealing with the increase in number of columns in the MP.

Column Generation Technique

To solve a large-scale LP problem practically, a column generation technique is applied in solving the MP (Gilmore and Gomory, 1961; Dantzig and Thapa, 2002). If the MP is solved via the Simplex method, we only need a basic set that has the same number of basic variables as the number of rows. Thus we do not need to explicitly know all the extreme points of subproblems. This leads to solving an equivalent problem, the *restricted master problem* (RMP).

Assume that we have a starting basic feasible solution to the RMP and it has a unique optimum, the optimal solution provides us with multiplier vectors [$\boldsymbol{\pi}$, $\boldsymbol{\gamma}$] for the linking constraints in Eq.(4) and convexity constraints in Eq.(5), respectively. Then, subproblems are formulated and solved to find the priced-out column associated with λ_{ij} :

$$f_{ij} = (\boldsymbol{c}_i - \boldsymbol{\pi A}^i)\boldsymbol{x}_i^j - \gamma_i \tag{9}$$

and the i^{th} subproblem is:

$$\boldsymbol{min} \quad z_i^0 = (\boldsymbol{c}_i - \boldsymbol{\pi A}^i)\boldsymbol{x}_i^j$$

$$\boldsymbol{s.t.} \quad \boldsymbol{B}^i{}_{\boldsymbol{eq}} \boldsymbol{x}_i = \boldsymbol{b}^i{}_{\boldsymbol{eq}} \tag{10}$$

$$\boldsymbol{G}^i \boldsymbol{x}_i \leq \boldsymbol{b}^i \qquad \mathrm{i} = 1,2,\ldots\ldots,\mathrm{m}$$

Therefore, we reach an optimal solution when the following condition is satisfied:

$$\boldsymbol{min}_i \boldsymbol{min}_j f_{ij} = \boldsymbol{min}_i (z_i^0 - \gamma_i) \geq 0 \,, \quad i=1, 2, \ldots\ldots, m \tag{11}$$

Optimality and finite convergence have been proved by Dantzig and Thapa (2002).

Dantzig-Wolfe Decomposition and Coordinated, Decentralized MPC Target Calculations

Typically, in a chemical plant, the process model used for centralized MPC scheme has a block-wise structure. Therefore, most chemical plants are potential candidates for the implementation of Dantzig-Wolfe decomposition to coordinate independent MPC calculations. The coordinator can be designed by considering the interactions among operation units that can be modeled as the linking constraints.

In this work, we focus on the design of coordination mechanism for steady-state target calculations in MPC systems. Target calculation problems can be formulated either as an LP or a QP problem (Kassmann *et al.* 2000; Rao and Rawlings, 1999). In this work, we assume an LP formulation for the steady-state target optimization, with a linear or linearized plant model and a linear objective function.

LP Formulation for Decentralized Target Calculations

For each MPC subsystem, which includes a target calculation and a control calculation, we can formulate an LP problem shown in Eq.(12) for the steady-state target calculation at time *k*.

$$\begin{aligned} \textbf{\textit{min}} \quad & \boldsymbol{c}^T \boldsymbol{x}(k) \\ \textbf{\textit{s.t.}} \quad & \boldsymbol{A}_{eq}\boldsymbol{x}(k) = \boldsymbol{b}_{eq}(k) \\ & \boldsymbol{G}\boldsymbol{x}(k) \le \boldsymbol{b}(k) \end{aligned} \tag{12}$$

where $\boldsymbol{x}(k) = [\boldsymbol{u}_s(k), \boldsymbol{y}_s(k)]$ or $[\Delta\boldsymbol{u}(k), \Delta\boldsymbol{y}(k)]$ is a vector of steady-state input and output variables for the subsystem. The latter corresponds to the velocity form, where:

$$\Delta\boldsymbol{u}(k) = \boldsymbol{u}_s(k) - \boldsymbol{u}_p(k \mid k-1) \tag{13}$$

$$\Delta\boldsymbol{y}(k) = \boldsymbol{y}_s(k) - \boldsymbol{y}_p(k \mid k-1) \tag{14}$$

in which $\boldsymbol{u}_s$ and $\boldsymbol{y}_s$ are the steady-state set-points computed at the k^{th} target calculation, and $\boldsymbol{u}_p$ and $\boldsymbol{y}_p$ are the predicted inputs and outputs at the previous target calculation. Here, the equality constraints are taken from the linear dynamic model:

$$\boldsymbol{Y}(s) = \boldsymbol{G}(s)\boldsymbol{U}(s) + \boldsymbol{G}_d(s)\boldsymbol{D}(s) + \boldsymbol{E}(s) \tag{15}$$

with the steady-state model:

$$\boldsymbol{y} = \boldsymbol{K}\boldsymbol{u} + \boldsymbol{K}_d\boldsymbol{d} + \boldsymbol{e} \tag{16}$$

where $\boldsymbol{d}$ stands for disturbances and $\boldsymbol{e}$ for unmeasured noise. The inequality constraints result from physical limitations on the inputs and outputs, such as actuator limits or production quality. Other MPC target calculation formulations are possible such as including model bias and soft output constraints (Kassmann *et al.* 2000; Lestage, 2002).

Coordinated, Decentralized Target Calculations

The solution from the decentralized MPC target calculation may not be optimal because of plant/model mismatch resulting from ignoring the interactions between operation units. Since the interactions can be modeled as the linking constraints, we can find the plant-wide optimum by designing a coordinator to coordinate independent MPC target calculations in the decentralized MPC scheme. Here, we propose a coordinated, decentralized approach to find the plant-wide optimum in MPC target calculations by applying the Dantzig-Wolfe decomposition.

The interactions between two operating units can always be modeled by equating the output variables from the upstream unit and the input variables to the downstream unit. In formulating the subproblems, those streams connecting different operating units are torn and consistency relationships can be used to model the interactions between different units. Assume that we have *m* separate operating units. Each of them is controlled by one MPC subsystem. If *n* units have interactions, by introducing interprocess stream consistency as the linking constraints in Eq.(3a), we can formulate a large-scale LP problem that has a form of Eq.(3). This problem has a block-wise structure which can be solved efficiently by Dantzig-Wolfe decomposition.

At each sampling instant of the plant-wide MPC target calculation, a coordinated LP problem will be formulated and solved. Then all the calculated steady-state targets both for inputs and outputs, including interprocess stream variables, are passed to MPC control calculation.

Illustrative Case-Study

The system under consideration in this paper is shown in Fig. 3. The normalized gains for the system are given in Eq.(17a) through Eq.(17c). An identity matrix is chosen for $\boldsymbol{K_d}$ in Eq.(16) assuming that the disturbances influence the outputs directly. The locations where the disturbances entering the plant are shown as dashed lines in Fig. 3.

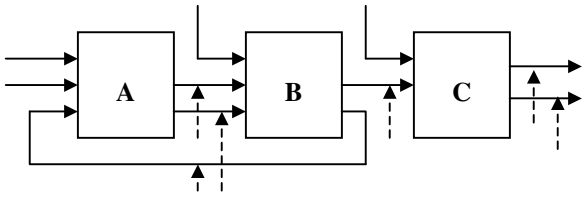

Figure 3. Interacting MIMO Operating Units

$$\boldsymbol{K}_A = \boldsymbol{G}_A(0) = \begin{bmatrix} 0.4 & 0.6 & 0.1 \\ 0.5 & 0.4 & 0.1 \end{bmatrix} \tag{17a}$$

$$\boldsymbol{K}_B = \boldsymbol{G}_B(0) = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.1 \end{bmatrix} \tag{17b}$$

$$\boldsymbol{K}_C = \boldsymbol{G}_C(0) = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.5 \end{bmatrix} \tag{17c}$$

Each operating unit has its own objective, which is a subset of information used by plant-wide optimizers, and the profit function cost coefficients are:

$$\boldsymbol{c}_A^T = \begin{bmatrix} -1 & -1 & -1 & 3 & 3 \end{bmatrix} \tag{18a}$$

$$\boldsymbol{c}_B^T = \begin{bmatrix} -2 & -2 & -0.5 & 1 & 1 \end{bmatrix} \tag{18b}$$

$$\boldsymbol{c}_C^T = \begin{bmatrix} -1 & -2 & 5 & 5 \end{bmatrix} \tag{18c}$$

So for each operating unit, by tearing the interprocess stream, a linear program at the k^{th} target calculation is:

$$\begin{aligned} \textbf{\textit{min}} \quad & \boldsymbol{c}_j{}^T \boldsymbol{x}_j(k) \\ \textbf{\textit{s.t.}} \quad & \boldsymbol{K}_j \boldsymbol{x}_j(k) = \boldsymbol{b}^j{}_{eq}(k) \\ & \boldsymbol{G}_j \boldsymbol{x}_j(k) \le \boldsymbol{b}^j(k) \quad j = A, B, C \end{aligned} \tag{19}$$

where $\boldsymbol{G_j}$ stands for the coefficient matrix associated with all the inequality constraints. The R.H.S. of the equality constraints $\boldsymbol{b}^j{}_{eq}(k)$ represents the updated model bias at each target calculation execution. The R.H.S. of the inequality constraints $\boldsymbol{b}^j(k)$ contains the lower bounds $\boldsymbol{lb}$ and upper bounds $\boldsymbol{ub}$ of the variables in the operation units. The bounds on the variables in this case study are shown in Eq.(20).

$$\boldsymbol{lb} = [0.3 \;\; 0.3 \;\; 0.15 \;\; 0.45 \;\; 0.4 \;\; 0.45 \;\; 0.4 \;\; 0.3 \;\; 0.45 \;\; 0.15 \;\; 0.45 \;\; 0.3 \;\; 0.45 \;\; 0.5] \quad (20a)$$

$$\boldsymbol{ub} = [0.5 \;\; 0.5 \;\; 0.25 \;\; 0.55 \;\; 0.5 \;\; 0.55 \;\; 0.5 \;\; 0.5 \;\; 0.55 \;\; 0.25 \;\; 0.55 \;\; 0.5 \;\; 0.55 \;\; 0.6] \quad (20b)$$

Three different MPC strategies, centralized MPC, decentralized MPC and coordinated, decentralized MPC, are implemented to evaluate their abilities to track the changing optimum in steady-state target calculations. For the centralized MPC target calculation, a direct LP problem is formulated treating all the inputs and outputs, including interprocess interactions, as decision variables. For the decentralized MPC scheme, separate LP problems are formulated by passing the upstream decisions to downstream units as disturbances. Finally, the coordinated MPC target calculation incorporates the linking constraints in modeling the interactions and solves the RMP and independent subproblems iteratively.

In our case study, unknown disturbances are generated by filtering random series of uniformly distributed variates in order to restrict these disturbances within the interval ±0.05. These unknown disturbances are directly imposed on the outputs when the optimized targets are implemented in our simulation. By using the autoregressive models in Eq.(21) as simplified disturbance models, we predict one-step ahead disturbances based on the past information of estimated disturbances. The estimated disturbances used to update the disturbance model in (21) are calculated by comparing the measured outputs and model predictions at every control execution. At the current control calculation, the parameters, c_1 and c_2, in the disturbance model are estimated using the estimated disturbances in the past 10 control execution periods. The one-step ahead disturbances are predicted using the estimated c_1 and c_2. The process models are updated using the predicted disturbances. The steady-state targets are then calculated by using the updated process models.

$$d(t) = \frac{1}{1 + c_1 q^{-1} + c_2 q^{-2}} e(t) \quad (21)$$

The following accumulated profit function is defined for performance comparison:

$$P = \sum Z(k) * T_s - \sum V(k) * T_s \quad (22)$$

where $Z(k)$ is the actual profit per unit time from the k^{th} target calculation; $V(k)$ represents the penalty for constraint violations when we implement the calculated targets; and T_s is the sampling period between two target calculations.

A benchmark has to be defined for comparing the performances of different MPC steady-state target calculation strategies. The benchmark used for comparison is defined as the maximum profit achieved when the plant is operated at the true optimum, which is calculated using the perfect process model and exact knowledge of disturbances. Although this maximum profit is not achievable, it is a useful basis for performance comparison.

Fig. 4 shows the profit achieved as a function of control execution using different MPC steady-state target calculation strategies. The coordinated target calculation gives the same achievable optimum as the centralized MPC scheme does (because the same interactions are considered in both approaches), while the decentralized scheme yields lower performance as interactions are ignored in the calculation. Note that, in the figure, two curves for the centralized scheme and coordinated scheme overlap each other and are very close to the true optimum.

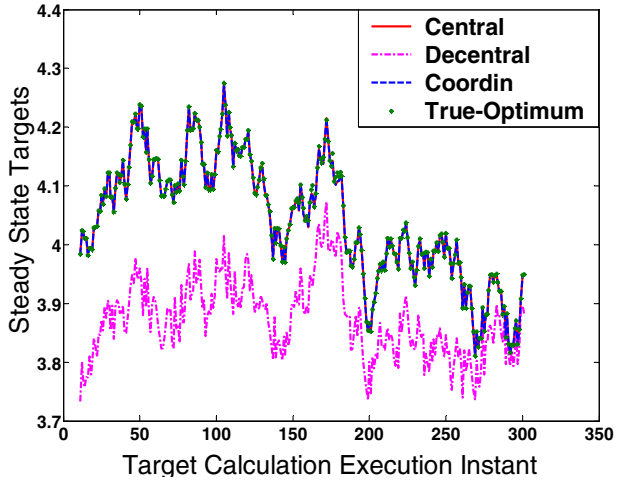

Figure 4. Calculated Targets Using Different Approaches

Table 1 compares the performance of different MPC strategies for a simulation of 290 target calculation executions. From Table 1, we can see that the centralized and the coordinated, decentralized target calculation give the same best achievable profit and achievable ratio to the true optimum, while the fully decentralized target calculation only captures around 95.7 % of the maximum profit.

Table 1. Performance Comparison

Strategy	Profit	Achiev. Ratio (%)	Comp.* Effort (s)	Problem Dimension
Central	1176.8	99.98	19.37	46×42
Decentral.	1126.8	95.73	34.14/3	15×15×3
Coordin.	1176.8	99.98	184.9/4	7×7 + 15×15×3
True-Opt.	1177.1	100	NA	NA

* All the simulations were performed in Matlab 6.5 on a Pentium III 1.0G Hz, 512M RAM machine.

Table 1 also reports the problem sizes for different steady-state target calculation strategies. The problem size is defined as the size of the coefficient matrix in the LP standard form used in the Simplex method. Therefore, slack and excess variables are added to convert the inequality constraints to equality constraints, and the columns of the coefficient matrices correspond to the process variables as well as the slack and excess variables. We can see that the centralized scheme has the largest problem size, which will grow significantly when the

dimension of separate problems and the number of operation units in the flowsheet increase. The problem size for the decentralized MPC is reported as the dimension of the largest subproblem multiplied by the number of units. For the coordinated scheme, the problem size is expressed as the addition of two components, the dimension of RMP (the coordinator) and the problem dimension in the decentralized scheme (the subproblems).

The computational effort is also reported in Table 1. All the simulations were performed using one single-processor computer. Therefore, the computation times for the decentralized and coordinated, decentralized strategies reported are divided by the number of processors required for parallel computation, assuming a decentralized computational environment. As shown in Table 1, the decentralized scheme requires the least computational effort, because the large plant-wide optimization problem is broken down into smaller independent optimization problems being solved using multiple processors. Table 1 also shows that the coordinated approach requires approximately twice the computation time as compared to the centralized approach, even after accounting for the distributed computing environment that would normally be used for coordinated target calculations. To our best knowledge, little is known regarding the complexity or scaling of the Dantzig-Wolfe decomposition technique and since our example represents a single problem instance, no general conclusions can be drawn as to which approach is computationally "faster". What must be noted is that the coordinated approach does provide a level of subsystem failure tolerance, which the centralized approach cannot. Such failure tolerance is an important feature for industrial automation systems.

Further, in our example, no attempts were made to develop efficient codes for the decomposition algorithm such as those reported by Martinson and Tind (1999) and Dantzig and Thapa, (2002) in our simulations. Further studies are required to investigate the potential reduction in computation time using the coordinated, decentralized strategy. A key point that should be drawn from our study is that the proposed approach does yield acceptable computational effort when implemented in parallel. As such, it provides an approach to plant-wide control that does not require a centralized computing environment.

Conclusions and Future Work

In this paper, a new approach to solving plant-wide MPC target calculation by coordinating independent subproblems has been proposed. The coordinator is designed by considering the interactions between operation units, which are not considered in the existing decentralized MPC calculation scheme. In our study, the coordinated, decentralized MPC target calculation can provide the same performance as does the centralized scheme, while it can utilize parallel computation to ensure acceptable real-time calculation speeds. This scheme offers a higher reliability than the centralized scheme. In fact, the proposed scheme can provide the same reliability as the decentralized control scheme. The proposed approach only requires a minor modification in the existing decentralized MPC structure by applying Dantzig-Wolfe decomposition.

A number of challenges remain in the decomposition of the MPC target calculations. Since subproblems are solved independently, this property ought to be naturally employed for parallel computing, which may have strong potential to enhance the solution efficiency. Furthermore, a full understanding of the complexity and scaling behavior of Dantzig-Wolfe decomposition algorithm is required to determine the feasibility of the proposed approach on industrial-scale problems.

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