Robustness Analysis and Tuning for Pressure Control in Managed Pressure Drilling

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Abstract: In this paper, we present a general framework for robustness analysis for pressure control in managed pressure drilling (MPD). In particular, we apply the analysis to the pressure controller proposed in the work Godhavn et al. (2011), based on which we also give an approach to search for controller tuning parameters with the goal of maximizing the robustness of system stability and control performance to various sorts of uncertainties, disturbances and noise. The resulting tuning table can be used for online computation of the controller parameters. The method proves effective in a simulation study.

Keywords: Managed pressure drilling, pressure control, uncertainty quantification, robust tuning

1. INTRODUCTION

In managed pressure drilling, annulus is sealed from the top and the drilling mud is running out of annulus through some choke valves (called MPD chokes or simply chokes in this paper), which provides back pressure in a bid to regulate the downhole pressure at specified depth in the annulus. The drilling mud is pumped from rig pumps into the top of drill string, flowing down through the drill bit and then along the annulus up to the choke. When the mud flow rate into the drill string is small, some pumps may be used to provide additional flow rate through the choke to facilitate the pressure control. These pumps are are usually called back pressure pumps. The drilling mud is not only key to the pressure regulation but also has the function to remove the cuttings produced in the drilling process (The reader is referred to Section 2 in Godhavn et al. (2011) for an illustration of the MPD process).

The pressure at a downhole location depends on the pressure upstream the choke (we call it choke pressure in this paper) and the pressure drop between the downhole location and the choke, which is primarily given by the mud. Despite the fact that PID controllers are popular and well understood (Møgster et al. (2013)), different types of advanced controllers have been recently proposed as improved solutions to the pressure control for MPD (Zhou et al. (2009); Breyholtz et al. (2010); Godhavn et al. (2011); Li et al. (2011); Møgster et al. (2013)). Usually the pressure controllers are designed based on some simple design dynamic models of the process with nominal parameters (e.g. physical properties of the mud and the choke) and have some parameters which can be tuned for good performance in practice. The tuning of the controller parameters, however, can be tedious and difficult as the actual process dynamics may be quite different from the design model and uncertainties, disturbances and noise are inevitably present. To address this issue, in this paper, we first present a framework of robustness analysis for the pressure control in MPD using some well-known tools in robust control theory, by which the consequence of any specific controller parameter tuning on the robustness of the pressure control can be quantitatively evaluated. Furthermore, a numerical guiding rule can be obtained by an optimization procedure based on the results of the evaluation. In this work, we apply this method to the choke pressure controller proposed in Godhavn et al. (2011). But the framework for the robustness analysis and the idea for the guiding rule generation for controller tuning are general.

The layout of the paper is the following: In Section 2 we review the controller structure and the simple plant model used for the controller design. In Section 3 we present the qualitative representation of the model and parameter uncertainties. The robustness analysis is given in Section 4; based on which the approach for robust tuning is shown in Section 5. Finally, some concluding remarks are given in Section 6.

2. PLANT MODEL AND CONTROLLER STRUCTURE

In this section we give a brief review of the simplified plant model and the pressure controller that regulates the choke pressure.

2.1 Simplified plant model

To ease the controller design, the following 3-state simplified well dynamics model is used (Kaasa et al. (2012)):
Physical meaning
- frictional pressure drop from standpipe to choke
- upstream choke pressure
- standpipe flow rate
- volume of drill string
- longitudinal velocity of drill string
- true vertical depth of well
- flow rate of back pressure pump
- standpipe pressure

Table 1. List of notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical meaning</th>
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<tbody>
<tr>
<td>$V_d$</td>
<td>volume of drill string</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>effective bulk modulus of mud in drill string</td>
</tr>
<tr>
<td>$q_{bit}$</td>
<td>bit flow rate</td>
</tr>
<tr>
<td>$q_{pp}$</td>
<td>flow rate of back pressure pump</td>
</tr>
<tr>
<td>$q_c$</td>
<td>choke flow rate</td>
</tr>
<tr>
<td>$A_d$</td>
<td>cross-sectional area of drill string</td>
</tr>
<tr>
<td>$v_d$</td>
<td>longitudinal velocity of drill string</td>
</tr>
<tr>
<td>$q_{err}$</td>
<td>unmodeled flow rate in annulus</td>
</tr>
<tr>
<td>$F(q_{bit}, \omega)$</td>
<td>frictional pressure drop from standpipe to choke</td>
</tr>
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</table>

where

$$
\frac{V_d}{\beta_d} \ddot{p}_p = q_{bit} - q_{pp} - q_c - A_d v_d + \dot{q}_{err} - \frac{V_a}{\beta_a} L_p (\dot{p}_c - p_c), \quad (5)
$$

where $\dot{q}_{err}$ is an estimate of the mud flow rate through the drill bit, which is simply set equal to the standpipe flow rate $q_b$ in this work. $L_p$ and $L_c$ are tunable parameters.

The model based controller (MBC) gives out the control reference signal for the opening of the MPD choke. It first computes the desired flow rate through the choke as

$$
q_c^* = \dot{q}_{bit} + q_{pp} - A_d v_d + \dot{q}_{err} + \frac{V_a}{\beta_a} (k_p (p_c - p_c^*) - \dot{p}_c^*). \quad (7)
$$

3. Uncertainty Modeling and Representation

Physical phenomena that occur in drilling process are complex and can hardly be captured accurately by simple models. Therefore an appropriate quantification of uncertainties is necessary to analyze robustness and performance of a controller designed based on simple models.

In this section, we quantify two important uncertainties:

- **Uncertain plant dynamics**: The discrepancy between the simplified plant model (1)-(2) and the actual plant dynamics.
- **Uncertain choke dynamics**: The discrepancy between the choke dynamics model (3)-(4) and the actual choke dynamics.

3.1 Operation points of plant

For our control purposes, we are interested in the responses from input variables to output variables. Even though the actual responses may be complicated and nonlinear, they are in general approximately linear nearby some steady state and thus can be represented by transfer functions.

It is not difficult to see that in a steady state of the plant dynamics (1)-(2), for which all the three derivatives in (1) are zero, the values of $p_p$, $q_{bit}$ and $z_c$ are determined if the values of $q_{pp}$, $q_c$, $q_{err}$, $v_d$ and $q_{err}$ are given. Thus we can define an operation point to be a value of the vector $[q_{pp}, q_c, q_{err}, v_d, q_{err}]$. For simplicity, in this paper we consider operation points with $q_{pp} = v_d = q_{err} = 0$.

3.2 Uncertain plant dynamics

We consider two types of uncertainties in plant dynamics: (1) parameter uncertainty and (2) neglected dynamics.

$^1$ An estimator for $\dot{q}_{bit}$ was presented in Godhavn et al. (2011), it is not considered here for simplicity.
**Parameter uncertainty** Parameter uncertainty refers to the uncertainty of one’s knowledge about the parameters in a nominal or design model. Examples of parameters in the plant model (together with examples of uncertainty ranges with respect to their nominal values) that can vary substantially are listed in the following Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty range %</th>
</tr>
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<tbody>
<tr>
<td>$\beta_a$ bulk modulus in the annulus</td>
<td>$\pm60.10%$</td>
</tr>
<tr>
<td>$\beta_d$ bulk modulus in the drilling string</td>
<td>$\pm10%$</td>
</tr>
<tr>
<td>$F_a$ friction coefficient in the annulus</td>
<td>$\pm50%$</td>
</tr>
<tr>
<td>$F_d$ friction coefficient in the drilling string</td>
<td>$\pm20%$</td>
</tr>
</tbody>
</table>

Using the linear fractional transformation (LFT) (see e.g. Skogestad and Postlethwaite (2005); Balas et al. (2001)), for a given operation point, the 3-state design model with the parameter uncertainties can be represented by the structure illustrated in Figure 1, in which

- the vector $\phi = [q_p, q_{pp}, v_d, q_{err}]$ is considered as part of the input to the closed-loop system;
- the output vector $\xi = [q_p, q_{pp}, v_d, z_c]$ together with $p_c$ are used by the controller block (see Section 2.3) as its input, with $q_p, q_{pp}, v_d$ as feedforward and $z_c, p_c$ as feedback;
- $\Delta_{\theta,p}$ is the frequency-dependent structured perturbation matrix with a block diagonal structure; each diagonal block corresponds to the uncertainty induced by one uncertain parameter and has its largest singular value upper bounded by 1 for all frequencies;
- the entries of the transfer function matrix $G_{nom}$ depend on (i) the 3-state design plant model with a set of nominal values for the parameters and (ii) the bounds (real scalars) of the uncertain parameters;
- the matrix $\Delta_{\theta,p}$ affects the input/output relationship between $z_c$ and $p_c$ in a feedback manner.

**Neglected dynamics** Even with the values of the parameters exactly known, the 3-state design plant model presented in Section 2.1 captures the actual plant dynamics accurately only at low frequency (Aarsnes et al. (2012)).

By neglected dynamics in plant model we mean, for fixed plant parameters, the discrepancy between the transfer function from the choke opening $z_c$ to choke pressure $p_c$ given by the 3-state design plant model and that given by the actual plant.

Utilizing the so called multiplicative uncertainty model (see e.g. Skogestad and Postlethwaite (2005); Balas et al. (2001)), for a given operation point, the plant model with both the uncertain parameters and the neglected dynamics can be represented in the structure shown in Figure 2, in which

- the system in the dashed box is simply the one shown in Figure 1;
- $w_{p,c}$ is a frequency-dependent multiplicative uncertainty weighting function;
- $\delta_{p,c}$ is a frequency-dependent function whose magnitude is upper bounded by 1 for all frequencies;
- the product $w_{p,c}\delta_{p,c}$ quantifies the neglected dynamics from the input $z_c$ to the output $p_c$.

Fig. 2. Plant model with mixed uncertainty

Now, let us denote the transfer functions from $z_c$ to $p_c$ in the actual plant as $P_{z_c,p_c}(j\omega; x, \theta)$, where $x$ represents the operation point and $\theta$ stands for the actual parameter vector. Exploiting the multiplicative uncertainty model, we wish to establish the following relationship: for any $\theta \in \Theta$,

$$P_{z_c,p_c}(j\omega; x, \theta) = (1 + w_{p,c}(j\omega)\delta_{p,c}(j\omega))G_{z_c,p_c}(j\omega; x, \theta)$$

(The requirement on that (8) should hold for any $\theta \in \Theta$ is because the only thing we know about the plant parameter $\theta$ is that it belongs to the set $\Theta$.)

To achieve this we may choose the weighting function $w_{p,c}$ as

$$w_{p,c}(j\omega) \geq \max_{\theta \in \Theta} \left| \frac{P_{z_c,p_c}(j\omega; x, \theta) - G_{z_c,p_c}(j\omega; x, \theta)}{G_{z_c,p_c}(j\omega; x, \theta)} \right|, \quad (9)$$

because then a function $\delta_{p,c}(j\omega)$ can be chosen to satisfy (8) with $|\delta_{p,c}(j\omega)| < 1$ for any frequency $\omega$.

In addition, for practical reasons:

- when computing the weighting function, we replace $P_{z_c,p_c}(j\omega; x, \theta)$, which is literally unknown, with the transfer function identified from a high-fidelity drilling simulator (The reader is referred to Ljung (1999) for some system approaches for this purpose);
- the weighting function may be computed only for a frequency range of interest;
- the weighting function is preferred to be the transfer function of a low-order SISO stable linear system.
Uncertain choke dynamics The uncertainties in the choke dynamics can be represented in the same manner as those in the plant dynamics. The uncertain parameters and examples of the ranges of uncertainties are given in Table 3 (Note that here the uncertainty in the function $G_c$ is converted approximately to that of the parameter $K_c$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty range %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c$ choke constant</td>
<td>±10%</td>
</tr>
<tr>
<td>$\omega_c$ choke natural frequency</td>
<td>±10%</td>
</tr>
</tbody>
</table>

The choke model with mixed uncertainty of parameters and neglected dynamics can be represented by the structure shown in Figure 3, in which $z_c^*$ is the choke opening reference given by the controller.

Fig. 3. Choke model with mixed uncertainty

4. ROBUST STABILITY AND PERFORMANCE ANALYSIS

4.1 Closed-loop system diagram

For a given operation point the (linearized) closed-loop system can be formed by connecting the uncertain plant model, uncertain choke model (described in Section 3), and the controller (described in Section 2). The system diagram is shown in Figure 4, in which

- $n$ denotes the vector of sensor noises on $p_c$ as well as the signals in $\xi$;
- $e$ is the actual tracking error, and $\tilde{e}$ is called the weighted tracking error (it is also called the weighted system output);
- $w_{ref}$ is ideal model (transfer function) for the reference tracking, $w_{\phi}, w_n, w_c$ are called weighting (transfer) functions which are used to specify the desired performance in $H_\infty$ framework (see Section 4.3 for the details); note that $w_{ref}, w_{\phi}, w_n$ and $w_c$ are diagonal matrices such that each scalar signal in $\tilde{p}_c^*, \tilde{\phi}, \tilde{n}$ and $\tilde{e}$ has an individual scalar weighting function.
- $\tilde{p}_c^*, \tilde{\phi}$ and $\tilde{n}$ are called normalized choke pressure reference, exogenous inputs and noise respectively.

4.2 Robust stability

To apply the robust stability theory (see e.g. Skogestad and Postlethwaite (2005)), we first transfer the closed system into the more compact representation as shown in Figure 5. In the figure,

- $\Delta = \text{diag}[\Delta_{\phi, e}, \Delta_{\phi, p}, \Delta_{p, e}]$ is a block diagonal perturbation; the inputs to the block $\Delta$ include those to its component diagonal blocks that can be easily identified in Figure 4;
- $\tilde{d}$ represents the vector containing $\tilde{p}_c^*$ and all elements in the vectors $\tilde{\phi}$ and $\tilde{n}$; it is thus regarded as the normalized system input (vector).

Fig. 5. A compact form of the closed-loop system diagram

The robust stability criterion says that the closed-loop system in Figure 5 is robustly stable against all possible uncertainties described in Section 3 if and only if the nominal closed-loop system is stable and $\mu_\Delta(N_{11}(j\omega)) \leq 1$, $\forall \omega$, where the $\mu_\Delta$ is the structured singular value, and $N_{11}$ is the upper left block of the transfer function matrix $N$ (i.e., it is the transfer function matrix from the output to the input of the block $\Delta$). Normally, we can define the real number $1/\max_\omega \mu_\Delta(N_{11}(j\omega))$ as the robust stability margin. Also note that the generalized input signals do not affect the stability of the closed-loop system.

4.3 Robust performance

Input/output relationship is central to performance study. In Figure 5 it is the lower right block in the transfer function matrix $N$, denoted here by $N_{22}$, that relates the generalized system input $\tilde{d}$ to the weighted tracking error $\tilde{e}$. In the framework of $H_\infty$ control, the control objective, besides the robust stability stated in Section 4.2, is to make the norm $\|N_{22}\|_\infty \leq 1$ for all frequencies; and the use of the weighting functions in the diagonal matrices $w_{ref}, w_{\phi}, w_n$ and $w_c$ is to scale the inputs and outputs such that desired closed-loop performance is achieved if the norm condition above on $N_{22}$ is met. One theoretical guideline to choose the weighting function is the following: Let $w_i(j\omega)$ be the weighting function for the $i$th element in the normalized input vector ($\tilde{d}$), and denote the actual input vector by $d = [p_c^*, \phi, n]$. Then $w_i(j\omega)$ may be chosen such that the inequality $\sum_i \alpha_i^2(\omega) |w_i(j\omega)|^2 \leq 1$ holds in the frequency band of interest, where $\alpha_i(\omega)$ represents the magnitude of the frequency component of the signal $d_i$ (i.e. the $i$th component of the vector $d$) around frequency $\omega$.

The robust performance is guaranteed if $\mu_\Delta(N(j\omega)) \leq 1$, $\forall \omega$, where the perturbation $\hat{\Delta} = \text{diag}(\Delta_{\text{perf}}(j\omega))$ with $\Delta_{\text{perf}}(j\omega)$ being any possible complex matrix with the dimension $n_d \times n_c$ which satisfies $\|\Delta_{\text{perf}}\|_\infty \leq 1$. The number $1/\max_\omega \mu_\Delta(N(j\omega))$ may be called the robust performance margin.

5. A ROBUST TUNING APPROACH

5.1 An approach searching for robust tuning parameters

As expected, different operation points correspond to different linearized closed-loop system. For each operation point the ideal controller tuning would be finding the controller parameters $k_p, L_p, L_s$ such that both robust stability and robust performance are satisfied with all allowed uncertainties (for stability) and the selected weighting
functions (for performance), i.e., we have both robust stability margin and robust performance margin larger than 1. However, this may not be always achievable. If this is the case, we impose priority on robust stability over robust performance; or in other words we are willing to sacrifice robust performance while pursing robust stability.

Specifically, for each operation point \( x \) in a selected set \( X \), we first compute the robust stability margins and the robust performance margins for a fixed grid of combinations of \( k_p \) and \( L_i \), which is defined by the set

\[
\Gamma = \left\{ (k_p, L_i) : k_p = n \frac{K_p^{\text{max}}}{N}, L_i = m \frac{L_i^{\text{max}}}{M}, n = 0, 1, \ldots, N, m = 0, 1, \ldots, M \right\}. \tag{10}
\]

where \( K_p^{\text{max}} > 0 \) and \( L_i^{\text{max}} > 0 \) are the upper bounds of the ranges for the values of \( k_p \) and \( L_i \), and the positive integers \( N \) and \( M \) define the resolution of the parameter grid. For each point in \( \gamma \in \Gamma \) let us denote the resultant robust stability margin and robust performance margin as \( s_x(\gamma) \) and \( p_x(\gamma) \) respectively, with the subscript \( x \) indicating the dependence on the operation point. Let us suppose now that the set \( \Gamma \) is ordered and with its elements indexed as \( \gamma_i \), with \( i = 1, 2, \ldots, |\Gamma| \) (Here \(| \cdot |\) denotes the size of a set). Then we may use the following Algorithm 1 to pick the best tuning point in \( \Gamma \) for each operation point.

**Algorithm 1** Searching for the best tuning parameters

1: procedure **SearchTuning**\((\Gamma, s_x, p_x)\)
2: for \( i \leftarrow 1 \) to \( |\Gamma| \) do
3: if \( i = 1 \) then
4: \( \gamma_i^* \leftarrow \gamma_1 \)
5: \( p_x^* \leftarrow p_x(\gamma_1), s_x^* \leftarrow s_x(\gamma_1) \)
6: else if \( (p_x(\gamma_i) \geq p_x^* \) and \( s_x(\gamma_i) \geq s_x^*) \) or \( (p_x(\gamma_i) > p_x^* \) and \( s_x(\gamma_i) \geq a) \) then
7: \( \gamma_i^* \leftarrow \gamma_i \)
8: \( p_x^* \leftarrow p_x(\gamma_i), s_x^* \leftarrow s_x(\gamma_i) \)
9: end if
10: end for
11: return \( \gamma_x^*, p_x^*, s_x^* \)
12: end procedure

In Algorithm 1, the number \( a \) should be set to 1 to reflect strict requirement on robust stability unless we see the resulting tuning is too conservative, in which case we may lower the value of \( a \) to accept tuning with less robust stability margin (An alternative may be sticking to \( a = 1 \) while adjusting the uncertainty weighting functions).

Note that, by the searching procedure, we will have the best tuning parameter \( \gamma_x^* \) (with corresponding robust stability margin \( s_x^* \) and robust performance margin \( p_x^* \)) for each operation point \( x \in X \). Thus a *tuning table* for all operation points is indeed created. Tuning parameters for operation points which are not in \( X \) can be obtained by simple interpolation, which is fast enough to be run with real-time applications. We should also point out that the computation of robust margins and the tuning parameter searching algorithm are in general time-consuming and therefore should be run offline.

5.2 Simulation results

Now we show some simulation results that compare the robust tuning with some nominal tuning which gives good performance with the simple design model. Firstly, in Figure 6a we show that the tuning parameter \( K_p = 1.5, L_i = 4 \) and \( L_p = 25 \) gives very good pressure tracking performance with the simple 3-state design plant model. In Figure 6b, however, we see that this tuning makes the system unstable when simulating with a high-fidelity model. On the other hand, the robust tuning can achieve acceptable tracking performance in this case, which is shown in Figure 6c.

Next we show the effectiveness of the robust tuning with low flow rate (300 lpm) and very low frictional pressure loss in the drill string and annulus. In Figure 7a, the controller is tested with \( K_p = 0.8, L_i = 4 \) and \( L_p = 25 \) which stabilizes the system with normal frictional pressure loss at 300 lpm. It is clearly seen that this tuning cannot stabilize system with low friction. On the other hand, with some initial fluctuation, the system is stabilized by the robust tuning, which can be seen in Figure 7b.

6. CONCLUSION

In this paper, we have presented an approach that yields a guide rule for the tuning of the pressure control for managed pressure drilling. The key to our approach is the robustness analysis via a qualitative representation of the model and parameter uncertainties. It allows us to evaluate the robustness of the stability and performance of the closed-loop control system with any particular tuning of the controller. The guide rule for tuning is simply made to somehow maximize the robustness such that the closed-loop system can be stabilized with acceptable control performance in presence of expected uncertainties.
Pressure tracking with nominal tuning using the simple design plant model
Pressure tracking with nominal tuning using a high-fidelity plant model
Pressure tracking with robust tuning using a high-fidelity plant model

Fig. 6. Comparison of pressure tracking performance at high flow rate and normal friction

Fig. 7. Comparison of pressure tracking performance with low flow rate and low friction

REFERENCES


