Critical Manifolds Based-On State Controllability To Design Dynamic Systems

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Abstract: In the present work, the formulation of critical manifolds for state controllability is proposed. These critical manifolds are a boundary in the design parameter space that splits it into two regions, one where the controllability is guaranteed and the other one where the dynamic system loses the controllability. We demonstrate that loss of controllability due to model parameter variations induces a one-parametric nonlinear controllability boundary in the two-dimensional parameter space. A heat exchanger is considered to illustrate the results of this work showing that not all the combinations of design parameters guarantee the controllability property.

Keywords: singular value decomposition; state controllability; linear dynamic systems; critical manifold

1. INTRODUCTION

State controllability is one of the properties that must be guaranteed for designing some technical systems. Taking into account conditions and restrictions on design parameters from the earliest stages, it helps not only to ensure proper operation but also allows analyzing that transitions between states meet restrictions imposed for economic reasons, efficiency, quality, among others (Bahri et al., 1997; Ekawati and Bahri, 2003; Havre, 1998; McAvoy and Braatz, 2003; Seferlis and Grievink, 2001; Vinson and Georgakis, 2000).

To design a dynamic system, Mönnigmann (2004); Gerhard (2010); Muñoz (2015) have developed a methodology to derive critical boundaries to constrain the feasible set where the system can be designed guaranteeing some system properties, the so-called critical manifolds. Initially, this approach was developed to make use of applied bifurcation theory for robust design of nonlinear systems (Mönnigmann and Marquardt, 2002). Lyapunov's indirect method is used to show the local stability of steady states. Stability boundaries correspond to those steady states for which at least one eigenvalue is located on the imaginary axis and the remaining eigenvalues are located in the open left-half complex plane.

We want to point out that it had to be assumed that the variations are slow compared to the time scale of the system, i.e., inputs and disturbances vary only quasi-statically compared to system dynamics. This restriction was remedied by introducing new types of critical boundaries defined for the transient behavior of nonlinear systems (Gerhard et al., 2008), where input and state trajectories are assumed to hit the constraints only tangentially at a single (grazing) point (Gerhard, 2010). These two distinct cases, namely the design for robust asymptotic stability of steady states despite parametric uncertainty and the design for robust feasibility of a transient, were investigated simultaneously by Muñoz et al. (2012), where the formulation of the augmented systems defining the critical manifolds of a transient system considered that the uncertain parameters also influence the steady-state behavior of the nonlinear system.

The notions of critical boundaries, distances to these boundaries, and their use in optimization problems have been extended to other systems and problem classes. Notably, an extension to discrete-time systems, including the case of periodically operating systems that can be treated with the theory for discrete-time systems by virtue of Poincare maps, is given in (Kastsian and Mönnigmann, 2010, 2014). Likewise, the methods have been extended to delay-differential equations which differ from all other system classes in that they are infinite-dimensional (Otten-Weinschenker, 2021).

However, the critical manifold based on controllability has not been considered in the methodology proposed by Mönnig mann and Marquardt (2002). Therefore, in this work, we develop the mathematical condition to guarantee the controllability of a class of dynamic systems using the concept of a critical manifold. To illustrate the results, a heat exchanger is considered for which the design parameter space is split into two regions, one where the controllability is guaranteed and the other one where the dynamic system loses the controllability. These results are very useful to design a dynamic system satisfying one of the most important system properties.

2. CRITICAL MANIFOLDS BASED-ON CONTROLLABILITY

2.1 System class

Let us consider the parametrized linear system defined by

$$\begin{aligned}\frac{dx(t)}{dt} &= A(p)\,x(t) + \sum_{i=1}^{m} u_i(t)\,b_i(p)(x) \\ &= A(p)x(t) + B(p)u(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ is the vector of input variables, $A(p) \in \mathcal{M}_n(\mathbb{R})$ and $B(p) \in \mathcal{M}_{n\times m}(\mathbb{R})$ are the matrices that define the system parameterized by design parameters $p \in \mathbb{R}^p$.

According to Kalman (1960) it is said that a point x_1 in the state space of a system, Eq. (1), is controllable from the state x_0 in $[t_0, t_1]$ if there is an input u defined in the interval $[t_0, t_1]$ such that it transfers the state of the system from x_0 in t_0 to x_1 in t_1. A system is said to be controllable if all points in its state space are controllable. To evaluate this property, the controllability matrix is considered,

$$M_c = \left[B|\ AB|\ \cdots\ |\ A^{n-1}B\right]_{n\times nm}. \tag{2}$$

A linear dynamic system, Eq. (1), is controllable if and only if the controllability matrix, Eq. (2), has rank n. Using this criterion, in the following section, critical manifolds are derived taking into account the design parameters p in an implicit form. These critical manifolds will allow restricting the parameter space during the design stage.

2.2 Derivation of critical manifolds based on controllability

Using the criterion of state controllability presented above, the parameter space can be characterized by identifying critical boundaries in which parameters affect the dynamic system showing different behaviors, for instance, the region in the parameter space where the system loses the controllability. To characterize those boundaries, Mönnigmann and Marquardt (2008) proposed a general scheme of derivation for the construction of critical manifolds. According to this scheme of derivation and considering the controllability rank criterion, two different forms to compute the critical manifolds of controllability are used depending on the number of inputs.

For a single-input single-output (SISO) linear dynamic system, i.e., $u \in \mathbb{R}$, the controllability matrix, Eq. (2), becomes a square matrix with dimension $n \times n$ and the rank criterion can be computed using the determinant, i.e., a SISO linear system is controllable if and only if, $\det(M_c(p)) \neq 0$. Thus, the critical manifold (CM) can be formulated as the boundary where the SISO system is not controllable, i.e.,

$$\mathrm{CM}_{\mathrm{SISO}}(p) = \{p \in \mathbb{R}^p|\ \det(M_c(p)) = 0\}. \tag{3}$$

Note that this critical manifold depends implicitly on design parameters p.

For multiple-inputs multiple-outputs (MIMO) linear dynamic system, $u \in \mathbb{R}^m$, the controllability matrix, Eq. (2), becomes a rectangular matrix with dimension $n \times nm$. Although the rank criterion is easy to check when a matrix is constant, a difficulty arises to define a critical manifold mathematically when the matrices A and B are parameterized by design parameters p. Thus, we propose the following procedure to calculate the controllability criterion.

First, let us consider the following Theorem (Friedberg and Spence, 1982), which is useful to calculate the matrix rank.

Theorem 1. Let $C, D \in \mathcal{M}_{n\times nm}(\mathbb{R})$. If $P \in \mathcal{M}_{n\times n}(\mathbb{R})$ and $Q \in \mathcal{M}_{nm\times nm}(\mathbb{R})$ are invertible square matrices such that $C = PDQ$, then $\mathrm{rank}(C) = \mathrm{rank}(PDQ)$.

Note that Theorem 1 establishes that the matrix rank is conservative for a given matrix transformation.

Second, looking for a convenient diagonal shape of D in Theorem 1, the singular value decomposition (SVD) is proposed. Let m, n be positive integers and $M_c(p) \in \mathcal{M}_{n\times nm}(\mathbb{R})$ the controllability matrix, Eq. (2). The singular value decomposition of $M_c(p)$ is the factorization

$$M_c(p) = US(p)V^T \tag{4}$$

where $U \in \mathcal{M}_{n\times n}(\mathbb{R})$ y $V^T \in \mathcal{M}_{nm\times nm}(\mathbb{R})$ are orthogonal matrices and $S(p) \in \mathcal{M}_{n\times nm}(\mathbb{R})$ is a diagonal matrix parameterized by p of the form

$$S(p) = \begin{bmatrix} \sigma_1(p) & 0 & \cdots & & 0 \cdots 0 \\ 0 & \sigma_2(p) & \cdots & & 0 \cdots 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & \sigma_n(p) & 0 \cdots 0 \end{bmatrix}.$$

where $\sigma_1(p) \geq \sigma_2(p) \geq \ldots \geq \sigma_n(p) \geq 0$, are the values of singular values of $M_c(p)$ ordered in descending order and are not negatives.

Third, using Theorem 1 and the SVD defined in Eq. (4), $\mathrm{rank}(M_c(p)) = n$ if $\sigma_i(p) \neq 0, \forall i = 1, 2, \ldots, n$. If any singular value is zero, it follows that the rank of the controllability matrix is smaller, which means that the MIMO linear system is not controllable. Thus, the critical manifold (CM) can be formulated as the boundary where the MIMO system, Eq. (1), loses the controllability as follows

$$\mathrm{CM}_{\mathrm{MIMO}}(p) = \{p \in \mathbb{R}^p|\ \prod \sigma_i(p) = 0\}. \tag{5}$$

Note that this critical manifold also depends implicitly on design parameters p, and the matrix controllability criterion can be checked in a continuous form when design parameters p are modified.

2.3 Computational issues

To construct the controllability conditions defined by Eqs. (3) and (5), the Symbolic Math Toolbox (Matlab®) was used, obtaining parametrized critical manifolds. We want to point out that from a computational point of view, the evaluation of controllability conditions will depend on a threshold from which the equality to zero is satisfied. Therefore, a "zero-interval" must be defined where we accept the controllability loss. Thus, a one-parametric nonlinear controllability boundary in the two-dimensional parameter space can be defined. Note that the "zero-interval" will depend on the magnitudes of variables and parameters of the mathematical model.

3. CASE STUDY: HEAT EXCHANGE

An indirect contact heat exchange is considered, in which fluids do not interact directly but rather through structures such as tubes and shells. The mathematical model and its values are taken from Marin et al. (2004). The following assumptions are considered:

1. The heat transfer coefficients and the fluid and wall material properties of the exchanger are constant.
2. The fluids are incompressible liquids and there is no accumulation of mass along the heat exchanger.
3. There are no heat losses to the environment.
4. The heat exchanger does not perform or receive any kind of work around.
5. Only the change in internal energy is taken into account. Kinetic and potential energy changes are neglected.

The energy balance for the cold fluid is

$$\frac{dQ}{dt} + \overset{\bullet}{v}_c \rho_c \left(\widehat{H_c}\left(T_{c,i}\right) - \widehat{H_c}\left(T_{c,o}\right) \right) = \frac{dE_{T,c}}{dt}, \quad (6)$$

where the subscript c represents the cold fluid, the subscript i represents the inlet, the subscript o represents the outlet, T is the temperature, Q represents heat transferred between the two fluids, $\overset{\bullet}{v}$ is the volumetric flow, ρ is the density, $\widehat{H}$ is the enthalpy per unit mass and E_T is the total energy. To define all the terms in Eq. (6), the following constitutive equations are considered

$$dE_{T,c} = \left(\rho V C_V dT\right)_c, \quad (7)$$

$$\widehat{H_c}\left(T\right) = \left(C_P\left(T - T_R\right)\right)_c, \quad (8)$$

$$\frac{dQ}{dt} = UA \times \Delta T, \quad (9)$$

where V is the volume occupied by the fluid in the heat exchanger, C_P and C_V are the heat capacities at constant pressure and at constant volume, respectively, T_R is the reference temperature, U is the global heat transfer coefficient, A is the transfer area, and ΔT is the temperature difference between the two currents through the entire surface A. ΔT is defined as the logarithmic mean temperature difference ($LMTD$) which takes into account the spatial variations of the temperature difference between the fluids. Countercurrent flow is used since it is carried out more effectively. $LMTD$ is defined as follows

$$\Delta T = LMTD = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left(\frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)}. \quad (10)$$

Replacing Eqs. (7), (8), (9) and (10) in Eq. (6), and using the similar procedure for the hot fluid with subscript h representing the hot fluid, we get the mathematical model for the heat exchanger

$$\frac{dT_{c,o}}{dt} = \left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_c \left(T_{c,i} - T_{c,o}\right) + \frac{UA \times LMTD}{\left(\rho V C_V\right)_c}, \quad (11)$$

$$\frac{dT_{h,o}}{dt} = \left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_h \left(T_{h,i} - T_{h,o}\right) - \frac{UA \times LMTD}{\left(\rho V C_V\right)_h}. \quad (12)$$

The inlet temperatures $T_{c,i}$, $T_{h,i}$ are the manipulated variables, while the temperatures output $T_{c,o}$, $T_{h,o}$ are the states of the system.

3.1 Model linearization and state space formulation

To apply the methodology presented above, a linear dynamic model is required. Thus, we linearize Eqs. (11) and (12) getting the following parametrized linear ordinary differential equations

$$\frac{dT'_{c,o}}{dt} = \alpha T'_{c,o} + \beta T'_{h,o} + \gamma \overset{\bullet}{v}{}'_c + \zeta T'_{c,i} + \eta T'_{h,i}\ , \quad (13)$$

$$\frac{dT'_{h,o}}{dt} = \phi T'_{c,o} + \varphi T'_{h,o} + \psi \overset{\bullet}{v}{}'_h + \vartheta T'_{c,i} + \kappa T'_{h,i}\ , \quad (14)$$

with

$$\alpha = -\left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_{c,SS} - \frac{UA\Gamma}{\left(\rho V C_V\right)_c}, \quad \beta = -\frac{UA\Delta}{\left(\rho V C_V\right)_c},$$

$$\gamma = \left(\frac{C_P}{C_V}\frac{1}{V}\right)_c \left(T_{c,i} - T_{c,o}\right)_{SS}, \quad \eta = \frac{UA\Gamma}{\left(\rho V C_V\right)_c},$$

$$\zeta = \left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_{c,SS} + \frac{UA\Delta}{\left(\rho V C_V\right)_c}, \quad \phi = \frac{UA\Gamma}{\left(\rho V C_V\right)_h},$$

$$\varphi = \frac{UA\Delta}{\left(\rho V C_V\right)_h} - \left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_{h,SS},$$

$$\psi = \left(\frac{C_P}{C_V}\frac{1}{V}\right)_h \left(T_{h,i} - T_{h,o}\right)_{SS},$$

$$\vartheta = -\frac{UA\Delta}{\left(\rho V C_V\right)_h}, \quad \kappa = \left(\frac{C_P}{C_V}\frac{\overset{\bullet}{v}}{V}\right)_{h,SS} - \frac{UA\Gamma}{\left(\rho V C_V\right)_h},$$

and

$$LMTD' = \Gamma\left(T'_{h,i} - T'_{c,o}\right) + \Delta\left(T'_{c,i} - T'_{h,o}\right), \quad (15)$$

$$\Gamma = \left(\frac{1}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)} - \frac{\left(T_{h,i}-T_{c,o}-T_{h,o}+T_{c,i}\right)}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)^2 \left(T_{h,i}-T_{c,o}\right)}\right)_{SS},$$

$$\Delta = \left(\frac{1}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)} - \frac{\left(T_{h,i}-T_{c,o}-T_{h,o}+T_{c,i}\right)}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)^2 \left(T_{h,o}-T_{c,i}\right)}\right)_{SS},$$

where the subscript SS corresponds to the steady state and ($'$) represents the deviation variable.

Thus, model in the state space is as follows

$$\frac{dx}{dt} = \begin{bmatrix} \alpha & \beta \\ \phi & \varphi \end{bmatrix} x + \begin{bmatrix} \zeta & \eta \\ \vartheta & \kappa \end{bmatrix} u, \quad (16)$$

where x and u are

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} T'_{c,o}(t) \\ T'_{h,o}(t) \end{bmatrix}, \quad (17)$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} T'_{c,i}(t) \\ T'_{h,i}(t) \end{bmatrix}, \tag{18}$$

3.2 Controllability critical manifold for the heat exchanger

For the paremetrized linear dynamic system defined in Eq. (16), the controllability matrix, according to Eq. (2) is given by

$$M_c = \begin{bmatrix} \zeta & \eta & \alpha\zeta + \beta\vartheta & \alpha\eta + \beta\kappa \\ \vartheta & \kappa & \phi\zeta + \varphi\vartheta & \phi\eta + \varphi\kappa \end{bmatrix}_{2\times 4}. \tag{19}$$

Note that Γ, Δ, β, γ, ϕ, ψ, ϑ y η depend on the properties of the fluids, design parameters, and operating conditions, so they take finite values different from zero from a practical point of view.

The singular value decomposition is applied to the controllability matrix (19) obtaining the following equivalence

$$M_c = U \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \end{bmatrix}_{2\times 4} V^T. \tag{20}$$

The state controllability criterion states that the system is controllable if and only if the controllability matrix, in this case (19), has full rank, which is equivalent to singular values σ_1 and σ_2 of Eq. (20) different to zero. Thus, the controllability critical manifold is build according to Eq. (5)

$$\mathrm{CM}_{\mathrm{MIMO}}(p) = \{p \in \mathbb{R}^p | \ \sigma_1(p)\,\sigma_2(p) = 0\}. \tag{21}$$

3.3 Simulation results

To sketch the form of this controllability critical manifold for the heat exchanger, a specific operating point and fluid properties are taken from (Kara and Güraras, 1960), and summarized in Table 1. The heat exchanger tubes and shell internal diameter, $d_{i,t}$ and $d_{i,c}$ respectively, are considered as the design parameters to show the effect to the controllability criterion. These design parameters are related with the volume V of each compartment, namely tubes and shell. We want to point out that singular values σ_i given by the SVD, Eq. (20), depend implicitly on both chosen design parameters, the shell internal diameter $d_{i,c}$ and the tubes internal diameter $d_{i,t}$. Using the parameter values established in Table 1 and considering the mathematical operation to compute the controllability condition (21), the "zero-interval" for this heat exchanger is $(-1 \times 10^{-6}, 1 \times 10^{-6})$.

Continuously dependency of design parameter with the mathematical condition, $\sigma_1(p)\,\sigma_2(p)$, used to formulate the controllability critical manifold, is shown in Figure 1. Note that there exist values of the design parameters where the product $\sigma_1(p)\,\sigma_2(p) \neq 0$ guaranteeing the heat exchanger controllability. Controllability critical manifold, Eq. (21), splits the design parameter space in one side where the heat exchanger is controllable and on the other side where the heat exchanger loses the controllability, as shown in Figure 2.

Table 1. Fluid properties and operating point of the heat exchanger

	Hot Fluid	Cold Fluid
Fluid	Water	Water
Mass flow $\left[\frac{kg}{s}\right]$	13.88	8.33
Inlet temperature $[K]$	340.15	290.15
Outlet temperature $[K]$	326.36	313.15
Density $\left[\frac{kg}{m^3}\right]$	979.4	999.0
Heat capacity $\left[\frac{kJ}{kgK}\right]$	4.188	4.184
Viscosity $\left[\frac{Ns}{m^2}\right] \times 10^6$	420	1080
Thermal conductivity $\left[\frac{W}{mK}\right] \times 10^3$	660	598
Restrictions	Maximum allowed pressure drop $= 12000Pa$	
Materials used: Carbon steel	Thermal conductivity $= 60W/mK$	
	Tube thickness $= 0.002m$	

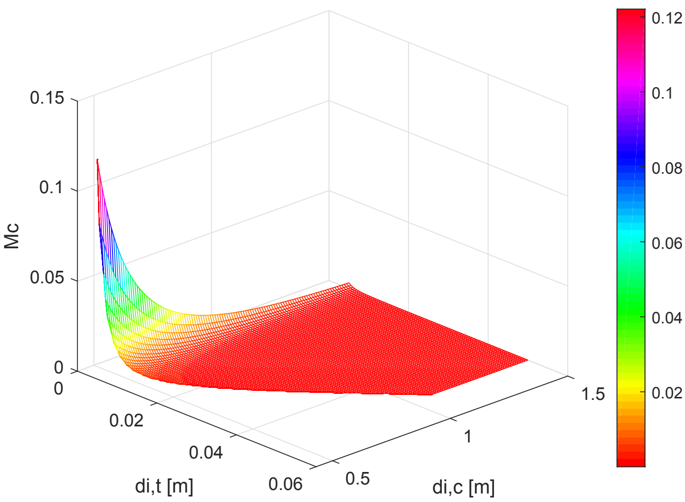

Fig. 1. Continuously dependency of design parameter with the mathematical condition, $\sigma_1(p)\,\sigma_2(p)$, for the heat exchanger.

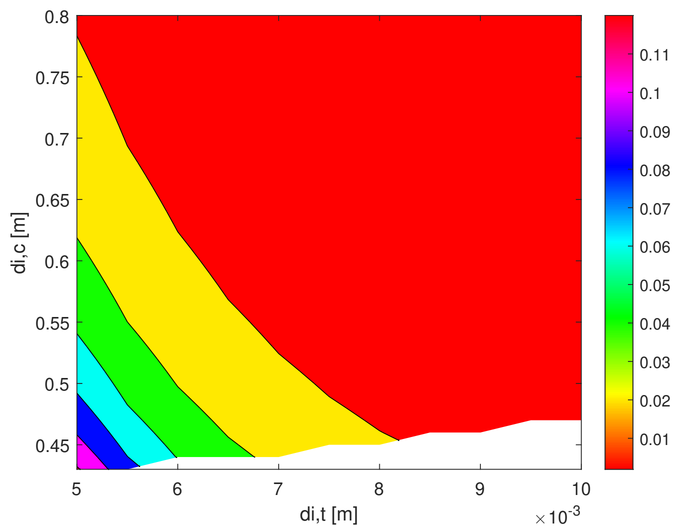

Fig. 2. Controllability critical manifold for the heat exchanger. The red region represents the combination of design parameters where the dynamic system loses the controllability. (Red region) "zero-interval": $(-1\times 10^{-6}, 1 \times 10^{-6})$

To provide certain validation for proposed method, a simulation result with the combination of design parameters in both controllable and non-controllable region are considered. The initial condition and two set of diameters are established as follows,

$$\mathbf{x}(0) = \left[T'_{c,o}(0), T'_{h,o}(0)\right] = [20, -15],$$
$$P_1 = (d_{i,t,1}, d_{i,c,1}) = (0.01188, 0.489),$$
$$P_2 = (d_{i,t,2}, d_{i,c,2}) = (0.035, 1).$$

P_1 is located on the controllable region of Fig. 2 and corresponds to the optimal values obtained in (Kara and Güraras, 1960). P_2 was selected into the non-controllable region. For both P_1, P_2, the open-loop system is stable because all the eigenvalues of the matrix $A(p)$, Eq. (16), are located on the left-hand side of the complex plane. The open-loop transient behavior of the heat exchanger for both parameter sets P_1 and P_2 are shown in Figs. 3 and 4, respectively. Note that for the parameter set P_1 the open-loop behavior is better than for the parameter set P_2. Although the heat exchanger is stable for both parameter sets and the steady state is reached in both cases, for the parameter set P_2 the system required more time to reach the steady state, which is undesirable to design the closed-loop system.

At the design stage, these results are very useful because the values of the parameters should be chosen in such a way that they do not cause the system to reach values that are not allowed for reasons of safety, product quality, among others. Additionally, with this mathematical formulation, the normal vectors approach developed in (Mönnigmann and Marquardt, 2002; Gerhard, 2010; Muñoz, 2015) can be applied to the robust design guaranteeing state controllability of dynamic systems.

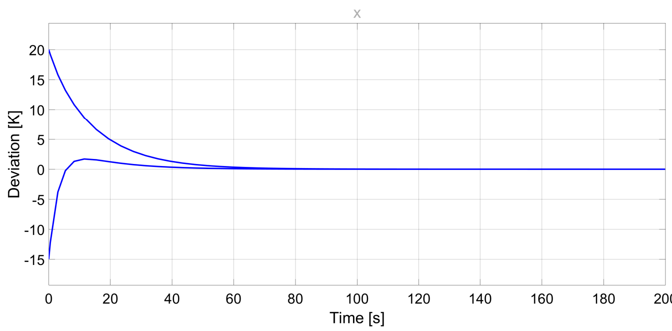

Fig. 3. $T'_{c,o}$ and $T'_{h,o}$ profiles of the heat exchanger using the set of parameters $P_1 = (d_{i,t,1}, d_{i,c,1}) = (0.01188, 0.489)$ which are located on the controllable region of Fig. 2.

4. CONCLUSIONS

Through the full-rank criterion for the state controllability of linear dynamic systems proposed by Kalman (1960), in the present work, the mathematical representation of controllability critical manifolds depending on design parameters of the system was obtained using the derivation scheme proposed in (Mönnigmann, 2004; Gerhard, 2010; Muñoz, 2015). These critical boundaries split the design parameter space in two regions, one where the controllability is guaranteed and the other one where the dynamic system loses the controllability.

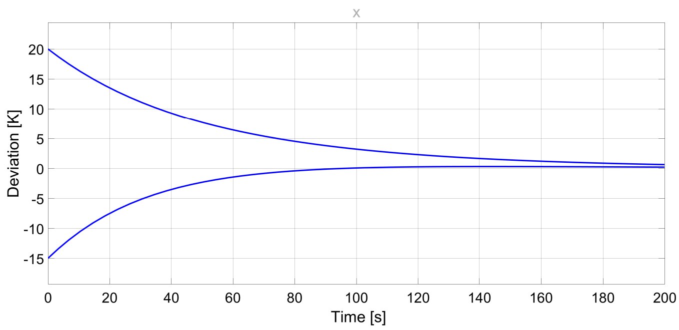

Fig. 4. $T'_{c,o}$ and $T'_{h,o}$ profiles of the heat exchanger using the set of parameters $P_2 = (d_{i,t,2}, d_{i,c,2}) = (0.035, 1)$ which are located on the non-controllable region of Fig. 2.

A heat exchanger was considered to illustrate the results, showing that there exist values of the design parameters where the controllability is not guaranteed. These controllability boundaries are useful for the simultaneous design of a system and its control, the so-called integrated design approach. Future work will be the comparison between the use of state controllability criteria for parametrized linear systems and the state controllability criteria for nonlinear systems.

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