

Knowledge-based optimal irrigation scheduling of three-dimensional agro-hydrological systems

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Abstract: An agricultural irrigation scheduler determines how much to irrigate and when to irrigate for a field. The accurate and effective scheduler decision for large agricultural fields is still an open research problem. In this work, we address the high dimensionality of the agricultural field and propose a knowledge-based approach to provide optimal irrigation amount and irrigation time for three-dimensional agro-hydrological systems. First, we introduce a structure-preserving model reduction technique to decrease the dimension of the system. Then, based on the reduced model, an optimization-based scheduler is designed. In the design of the scheduler, empirical knowledge from farmers is considered to significantly reduce the computational complexity. The proposed scheduler is designed in the framework of model predictive control. The objective of the proposed scheduler is to maximize crop yield while minimizing irrigation water consumption and the associated electricity usage. The proposed approach is applied to a field to show the effectiveness and superiority of the proposed framework.

Keywords: model reduction; clustering; scheduler; closed-loop; agro-hydrological system.

1. INTRODUCTION

Freshwater scarcity is one of the most critical global risks in the world due to mainly population growth, climate change and, environmental pollution (World Economic Forum, 2015). Almost 70% of the total freshwater (United Nations World Water Assessment Programme, 2017) is consumed in agricultural irrigation every year. Moreover, the water use efficiency of the current irrigation methods is around 50% to 60% (Lozoya et al., 2014), which is not adequate to save water usage significantly. Therefore, increasing water use efficiency is essential for feeding the growing population and managing the water crisis. The common irrigation practice includes open-loop control with no real-time feedback. Moreover, it mostly depends on the farmers' observation and experience about the farm instead of actual field conditions such as soil moisture, which may lead to excessive or insufficient irrigation. The closed-loop approach based on real-time feedback is the key to increasing irrigation efficiency.

There are different control strategies based on closed-loop irrigation. (Goodchild et al., 2015) proposed precision irrigation using the modified PID loop. Model predictive control (MPC) has also been studied to determine the optimal irrigation amount (Park et al., 2009; Mao et al., 2018). These studies focus on a short period of time (minutes or hours). In these works, either a one-dimensional linear model is used, or spatial moisture variability is calculated

using the ordinary kriging method. The one-dimensional models cannot efficiently represent the horizontal flow and horizontal diversity of the soil and crops.

Irrigation scheduling is studied extensively in the past to determine the appropriate time and quantity of water for irrigation (Wardlaw and Barnes, 1999; Hassan-Esfahani et al., 2015; Thorp et al., 2017). However, these studies do not consider current field conditions in the optimization decision and provide the solution only once at the beginning of crop season. Recently, (Nahar et al., 2019) and (Kassing et al., 2020) proposed two-level optimization-based scheduling. The top-level calculates the water allocation decision for the entire crop season, while the bottom layer makes the decision for daily soil moisture regulations. In these works, the simplified linear one-dimensional model is considered. To handle the nonlinearity of the system, (Agyeman et al., 2022) proposed a scheduling approach based on a data-driven model in the framework of MPC with discrete decision variables for one-dimensional agro-hydrological systems.

In this work, we consider the scheduling of irrigation for three-dimensional agro-hydrological systems. The direct application of discretized model is computationally challenging if the states are the decision variables or state constraints are present. Model reduction is one of the efficient techniques to handle the problem by reducing the dimension of states. Therefore, the number of decision variables and the number of state constraints decrease. A few popular classical model reduction techniques are proper orthogonal decomposition (POD), optimal Hankel

* This work is supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) and Alberta Innovates.

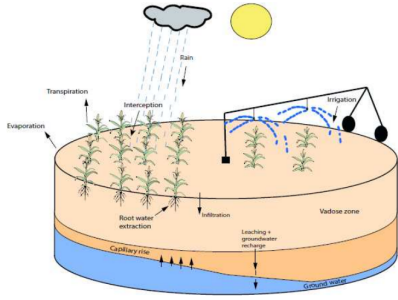


Fig. 1. Schematic of an agro-hydrological system (Agyeman et al., 2021)

norm reduction, balanced truncation methods, etc. (Antoulas, 2005). The state constraints can not be applied in the reduced-order dimension in the classical model reduction techniques because the reduced states do not preserve any structure. There are some recent researches on the structure-preserving model reduction where states preserve the structure (Cheng and Scherpen, 2019; Sahoo et al., 2019, 2020). However, these methods are limited to only linear systems or the projection matrix is constructed using linear dynamics. In this work, we first propose a structure-preserving model reduction method for the nonlinear agro-hydrological system. The proposed model reduction method is based on state trajectories of the system. To further reduce the computational complexity of the proposed scheduler, we design the scheduler based on the empirical knowledge of farmers. By using the knowledge, we can remove integer decision variables from the scheduling optimization problem which reduces the computational complexity significantly. The proposed scheduler is designed in the framework of MPC and maximizes the crop yield while minimizing the water amount of irrigation. The effectiveness of the designed scheduler is investigated using simulations.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

2.1 System description

We consider an agro-hydrological system that characterizes the interaction between the soil, the water, the atmosphere, and the crop. The schematic of the considered agro-hydrological system is shown in Fig 1. The dynamics of the water flow in the agro-hydrological system can be represented by Richards Equation (Richards, 1931) as follows:

$$\frac{\partial \theta}{\partial t} = c(h) \frac{\partial h}{\partial t} = \nabla \cdot (K(h) \nabla (h + z)) + S(h, z) \quad (1)$$

where h [m] is the field water pressure head, θ [m^3m^{-3}] is the field water soil moisture content, $c(h)$ [m^{-1}] is the soil water capacity, $K(h)$ [ms^{-1}] is the hydraulic conductivity, z [m] is the vertical coordinate, $S(h, z)$ [$\text{m}^3\text{m}^{-3}\text{s}^{-1}$] is the source and sink term consists of plant root water extraction.

The details of the nonlinear relationship between hydraulic conductivity $K(h)$, capillary capacity $c(h)$ and soil moisture content (θ) with pressure (h) can be found in (Mualem, 1976; Van Genuchten, 1980; Sahoo et al., 2019).

The sink term $S(h, z)$ in (1) characterizes the root water extraction rate. The total root water uptake depends upon transpiration rate, soil pressure head and root depth. The mathematical formulation of root-water uptake based on the Feddes model (Feddes, 1982) is expressed as follows:

$$S(h, z) = \alpha(h) S_{max}(z) \quad (2)$$

where $S_{max}(z)$ [$\text{m}^3\text{m}^{-3}\text{s}^{-1}$] is the maximum possible water extraction rate under optimal condition. $\alpha(h)$ [-] is the dimensionless water stress factor.

The crop yield model is also important along with the Richards equation. The relationship between the crop yield and soil pressure head is expressed as follows (Doorenbos and Kassam, 1979):

$$\left(1 - \frac{Y_a}{Y_p}\right) = \sum_{k=1}^T K_y(k) \left(1 - \alpha(h)\right) \quad (3)$$

where Y_a is actual yield and Y_p is potential yield. $K_y(k)$ is the crop sensitivity factor at time k . T is the total time for growing seasons.

In this work, we consider that the agricultural field is equipped with a center pivot irrigation system. A center pivot irrigation system rotates across the field around a fixed pivot at the center of the field and irrigates in a circular manner. In order to account for the circular movement of the center pivot irrigation system, the Richards equation in (1) is expressed in the cylindrical coordinates (Agyeman et al., 2021).

The three-dimensional agro-hydrological model is a nonlinear partial differential equation, which renders the problem difficult to solve analytically. In this work, we apply the explicit finite difference method to discretize the Richards equation. Note that spatial discretization of the model is performed, such that a continuous-time state-space model is established as in the following form:

$$\dot{x}(t) = f(x(t), u(t)) \quad (4)$$

where $x(t) \in \mathbb{R}^{N_x}$ denotes the states vector representing the pressure head value at each discretized node of total size N_x and $u \in \mathbb{R}^{N_u}$ represents the input vector containing N_u irrigation values applied at each surface discretized node. As the input (irrigation amount) is applied to each surface node, it is incorporated in the system surface boundary condition. The surface boundary condition is characterized by Neumann boundary condition. The bottom boundary condition is specified as free drainage.

The central pivot rotates in a circular manner so the central pivot can not put water everywhere at the same time. So the nodes which align to the central pivot rotation are non-zero and other surface nodes are zero. Thus this imposes a time-varying constraint on u as follows:

$$\mathcal{U}_{lb}(t) \leq u(t) \leq \mathcal{U}_{ub}(t) \quad (5)$$

A continuous time state-space model with measurements and disturbances is considered as follows:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), d(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (6)$$

where $y(t) \in \mathbb{R}^{N_y}$ denotes the soil pressure head measurements, $d(t) \in \mathbb{R}^{N_d}$ is the weather disturbances.

The objective is to calculate the optimal time and irrigation amount for maximum crop yield and water con-

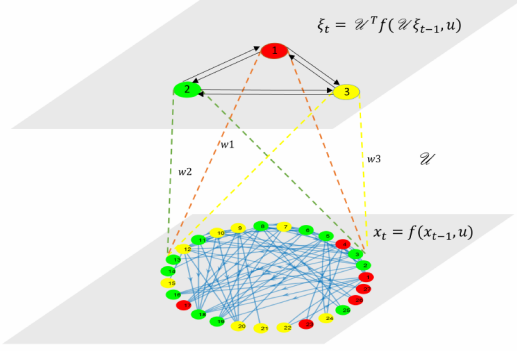


Fig. 2. Illustration of structure-preserving model reduction.

servation for three-dimensional fields with central pivot irrigation systems.

3. PROPOSED MODEL REDUCTION

As discussed in the introduction, the finer discretization results in a large number of states. Thus the use of the state-space model (4) is computationally expensive for the optimization step in the scheduler design where states are the decision variables in multiple shooting and collocation based methods. Further, the optimization cost increases if the state constraints are present. The model reduction can deal with these issues. The states do not preserve the structure in classical model reduction techniques, so applying state constraints in a reduced model is cumbersome. Hence, we propose a structure-preserving model reduction technique. The calculation of a linear model in a large-scale system is also computationally expensive. So in this work, we propose the trajectory-based model reduction techniques.

3.1 Step 1: Snapshot matrix generation

The first step is to generate the state snapshots. Based on the prescribed input from the scheduler, simulate the nonlinear system (4) and generate the state trajectories from the initial time to the final time as follows:

$$\mathcal{X} = [x(t_0) \ x(t_1) \ \dots \ x(t_N)]$$

where $\mathcal{X} \in \mathbb{R}^{n \times N}$ is the snapshot matrix of the actual system, n is the number of states, N is the total number of the sampling intervals.

3.2 Step 2: Calculation of cluster sets

In this step, the cluster matrix sets are generated. The main idea is to create clusters of states having similar dynamics based on the system trajectories. Then the projection matrix is generated using the clustering information and using the projection matrix, the system is projected from a higher dimensional system to a lower-dimensional reduced system. In this work, agglomerative hierarchical clustering (Steinbach et al., 2000) is used. We use the Euclidean distance between trajectories as the distance measure for states. The main reason to choose agglomerative hierarchical clustering is because of the capability to define the distance threshold between the clusters instead of predefining the number of cluster sets. The distance

threshold is a tuning parameter for the accuracy of the reduced model. There are three commonly used linkage methods present in agglomerative hierarchical clustering (e.g. single, average, complete linkage). In this work, we use the average linkage, and it considers the average distance between each point in one cluster to every point in other clusters.

$$d(p, q) = \frac{1}{n_p n_q} \sum_{i=1}^{n_p} \sum_{j=1}^{n_q} d(x_{pi}, x_{qj})$$

where p and q are two clusters, i and j are data points within the clusters, d is the euclidean distance between i and j and n_p, n_q are the size of the clusters of p and q respectively.

Let us consider $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_r\}$ be the collection of clusters after the hierarchical clustering and r is the order of the resulted reduced model. The resulted clusters have following properties: i) $\mathcal{C}_i \cap \mathcal{C}_j = \Phi$ and ii) $\mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_r = N_x$, where N_x is the total number of states.

3.3 Step 3: Reduced model construction

In this step, reduced-order system is constructed based on the Petrov-Galerkin projection framework (Antoulas, 2005). For the Petrov-Galerkin projection method, the projection matrix is required. After the construction of state clusters, the projection matrix \mathcal{U} is generated. The projection matrix is defined as $\mathcal{U} \in \mathbb{R}^{n \times r}$, whose elements are expressed as follows:

$$\mathcal{U}_{i,j} = \begin{cases} w_i, & \text{if vertex } i \in \mathcal{C}_j \\ 0, & \text{otherwise} \end{cases}$$

and w_i is determined as follows:

$$w_i = 1/||\alpha_i|| \\ \alpha_i = \mathbb{E}_i^T \alpha$$

where $\alpha = [1, \dots, 1]^T \in \mathbb{R}^n$, $||\alpha_i||$ is the L_2 norm of α_i , $\mathbb{E}_i = e_{\mathcal{C}_i} \in \mathbb{R}^{n \times m}$, e_j is the j^{th} column of the identity matrix of size $\mathbb{R}^{n \times n}$ and m is the cardinality of \mathcal{C}_i set.

The adaptive reduced model of (4) is expressed as:

$$\dot{\xi}(t) = f_r(\xi(t), u(t)) \quad (7)$$

where $f_r(\xi(t), u(t)) = \mathcal{U}^T f(\mathcal{U} \xi(t), u(t))$ and $\xi(t) = \mathcal{U}^T x(t)$. Note that the actual state $x(t)$ can be approximated based on mapping $\tilde{x}(t) = \mathcal{U} \xi$. The discrete model of (7) is expressed as follows:

$$\xi(k+1) = f_{rd}(\xi(k), u(k)) \quad (8)$$

4. PROPOSED CLOSED-LOOP SCHEDULING

This section proposes closed-loop scheduling to calculate the time intervals between irrigation events and the water amount for each event. The primary objective is to maximize the crop yield while reducing the total water use and equipment operating cost. The scheduler considers historical weather and weekly weather forecast, and soil moisture measurements. The main idea is to irrigate the field and calculate the time required to reach the lower stress-free zone.

In this work, the iterative finite-horizon optimization is considered like the classical MPC. However, the length of the horizon is not fixed because the time is also

a decision variable in the optimization problem. The maximum time can be added in the constraint as a higher bound. The optimizer can make a prediction until the higher bound of the time period and decide the optimal time and irrigation amount within that limit. However, the weather forecast until the higher bound of time may not be accurate enough. So the receding horizon strategy is implemented to handle the uncertainty in the weather forecast. The optimization problem is resolved after a few days interval with a more accurate weather forecast and recent measurements.

Each horizon consists of three separate segments. In the first segment, we irrigate the field, and the amount of water to be irrigated is the primary decision variable. In the second segment, the time is a decision variable that calculates the time for the next irrigation event. The time is calculated such that the plants will not experience stress, and the yield will be maximized. The third segment calculates the irrigation amount for the next horizon. The third segment is added to the optimization problem to give the optimizer some flexibility to see a few more day future forecasts and make the irrigation time and amount more robust. In all three segments, the yield and the constraints are considered to keep the pressure head within a stress-free zone. The slack variables are introduced to relax the target zone. Note that the above three-segment design is inspired by the common practice used by farmers. This design can significantly reduce the computational complexity of the scheduler and lead to near-optimal solutions.

For each horizon, the optimization problem is formulated as follows:

$$\min_{\substack{u(j), \bar{\epsilon}_r(j), \\ \epsilon_x(j), T}} Q_y \left(1 - \frac{Y_a}{Y_p}\right)^2 - Q_T \sum_{j=k+N_1}^{N_1+N_2} \frac{T}{T_{ub}} + Q_u \sum_{j=k}^{N-1} u(j) + \sum_{j=k+1}^{k+N} (\bar{Q}_r \bar{\epsilon}_r(j)^2 + Q_x \epsilon_x(j)^2) \quad (9a)$$

$$\text{s.t.} \quad \left(1 - \frac{Y_a}{Y_p}\right) = \sum_{j=k+1}^{k+N} K_y(j)(1 - K_s(y_r)) \quad (9b)$$

$$\tilde{\xi}(j+1) = \mathcal{U}^T f(\mathcal{U} \tilde{\xi}(j), u(j), d), \quad j = k, \dots, N-1 \quad (9c)$$

$$y_r(j) = C_r \tilde{\xi}(j) \quad (9d)$$

$$u_{lb} < u(j) < u_{ub}, \quad j = k, \dots, k+N_1 \text{ and } k+N_1+N_2, \dots, k+N \quad (9e)$$

$$u(j) = 0, \quad j = k+N_1, \dots, k+N_1+N_2 \quad (9f)$$

$$\tilde{\xi}(j) \in Z, \quad j = k, \dots, k+N-1 \quad (9g)$$

$$\underline{V} - \epsilon_x(j) < y_r(j) < \bar{V} + \bar{\epsilon}_r(j) \quad (9h)$$

$$\bar{\epsilon}_r(j) \geq 0, \epsilon_x(j) \geq 0 \quad (9i)$$

$$T_{lb} < T < T_{ub} \quad (9j)$$

$$N_1 + N_2 + N_3 = N \quad (9k)$$

where (9a) defines the cost function to be minimized and the input (u), time (T) and slack variables ($\bar{\epsilon}_r, \epsilon_x$) are the decision variables. In (9a), the first term is the crop yield deficiency cost, the second term denotes the normalized time cost which is active only in second segment, the third

term considers the irrigation water cost. The last term in (9a) is the cost term of non-negative slack variables ($\bar{\epsilon}_r, \epsilon_x$) which is introduced to relax the bounds of target zones \bar{V}, \underline{V} in (9h). $Q_y, Q_u, Q_T, Q_x, \bar{Q}_r$ are the positive weighting factors. Equation (9b) is the model used to evaluate the yield deficiency. Equations (9c,9d) represent the discrete time reduced-order model and the output function. $\tilde{\xi}(k)$ denotes current state estimates at time k . In this work, we assume all the states can be estimated. N_1 is the number of sampling time for first segment, N_2 is the sampling time for second segment ($\Delta T_2 = T/N_2$). Things to note that in segment 2, the time is unknown so the number of sampling points (N_2) is fixed and it is chosen based on upper bound of T_{ub} such that the model does not experience numerical issues. N_3 is the number of sampling time for third segment. Equation (9k) shows the total sampling time is N . Equation (9e) provides the bounds of input for first segment and third segment. Equation (9f) defines the input amount is zero for second segment. Equation (9h) imposes the zone constraints with the slack variables and Equation (9i) implies the slack variables are non-negative. Equation (9j) defines the constraints for the lower and upper bound of time.

As discussed before, the receding horizon strategy is implemented to handle the weather uncertainty and use the scheduler as a closed-loop system. The day (T_s) till which we can predict the accurate weather prediction is selected. In general, we predict the weather for 7 days. So if the scheduler predicts the next irrigation event to more than 7 days then, we reevaluate the scheduler optimization after 7 days again with the current field condition as initial condition and future 7 days weather prediction. If the scheduler predicts the irrigation event less than 7 days, we solve the optimization problem for the next horizon using the recent day field condition as the initial condition. The algorithms for the receding horizon is as follows:

- (1) At the current time (k) solve the optimization problem (9), with initial condition $\xi(k)$ and obtain the optimum input (u) and time (T).
- (2) If the optimum time (T) is greater than T_s ($T > T_s$) then resolve the optimization problem (9) with initial condition $\xi(k + T_s)$ and obtain the optimum input and time.
- (3) Else ($T < T_s$) solve the optimization problem (9) with initial condition $\xi(k + T)$ and obtain the input and time.

5. RESULTS

In this section, the proposed algorithms are applied to demonstrate the performance of the reduced-order model and the scheduler. A field of radius 50 m and depth 30 cm is considered. The field is equipped with a central pivot irrigation system. The model of the farm is constructed using finite difference discretization of the Richards equation. The entire system is discretized into 1920 nodes with 5 in radial, 64 in azimuthal, and 6 in the axial direction. Each node corresponds to the states of the system. The central pivot takes around 8 hours to irrigate the whole field. Different types of crops and soil types are considered in different scenarios.

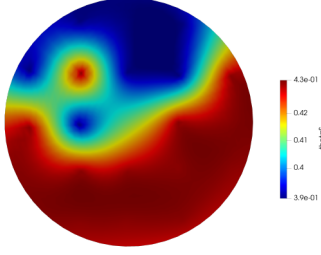


Fig. 3. One selected soil parameter θ_s for the field

5.1 Results: model reduction

In this subsection, the proposed model reduction discussed in section 3 is applied to the system. First, the effect of reduced model order on the mean square error (MSE) of the reduced model is discussed. Then the robustness of the reduced model is discussed.

In this simulation, we consider the real soil properties of the field located at Lethbridge, Canada. In summer 2019, we collected soil samples at 20 points in the field and estimated the soil types in the soil lab. We found three different soil types present in the field: loam, sandy clay loam, and clay loam. The kriging method is applied to get the soil properties of all other nodes of the field. Fig. 3 shows one selected parameter (θ_s) of the surface nodes. The other parameters also follow the same trend.

Fig. 4 shows the MSE values of the reduced model with the number of reduced states. The number of reduced states is obtained by changing the threshold values of the agglomerative hierarchical clustering method starting from 0.3 to 3.5 by increasing 0.2. As discussed in section 2, we can specify the threshold values in hierarchical clustering instead of the number of reduced models. The MSE values are calculated between the actual model and the reduced-order model. From Fig. 4 it can be observed that after 56 reduced states, the error values are minimal. For this simulation, we consider 30 reduced states.

Next, we study the robustness of the reduced model. In optimization-based controller design, the input amount is one of the important decision variables. So the reduced model should be robust enough to handle different input amounts. First, the projection matrix is calculated using the initial condition -4.0 m and input amount $2e-06$ m/s. Then using the same projection matrix, we simulate the reduced-order model starting from different initial condition (-3.0 m) and different input amounts ($1e-06$ m/s, $0.5e-06$ m/s, $0.1e-06$ m/s, $0.05e-06$ m/s). Fig. 5 shows the state trajectories of the actual system and the reduced-order system randomly chosen state 77 (surface) and 1897 (depth 25 cm) for different input amounts. It can be observed that reduced order trajectories are very close to the actual order trajectories, which shows the robustness of the proposed model reduction method.

5.2 Result: scheduler

In this subsection, the performance of the proposed scheduler design is demonstrated under uniform soil, lettuce crop type, variable ET and rain.

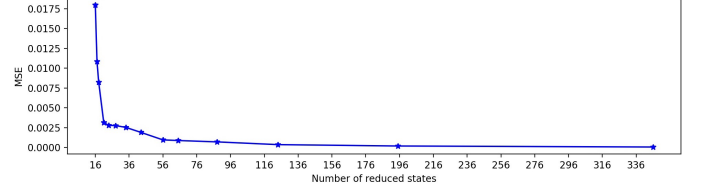


Fig. 4. Values of MSE with different reduced order

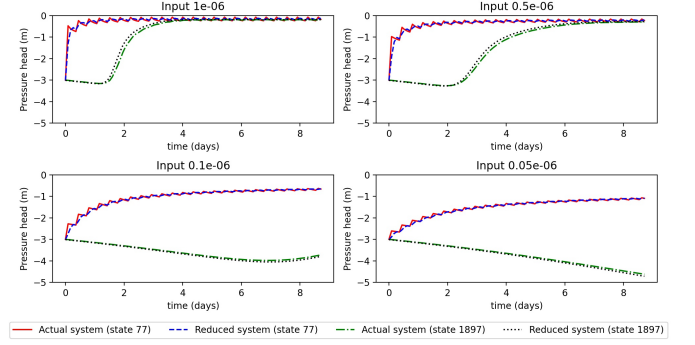


Fig. 5. Selected state trajectories of the actual system and reduced system for four different inputs

In the simulations, the variable ET and rain uncertainties are considered. The uniform soil type of loam and crop type of lettuce is chosen for the simulation. This scenario is shown to check the efficacy of the scheduler in the presence of weather disturbances and crop growth stages. As discussed in the crop modeling, 85% root water is extracted from the top 30 cm soil. For the high yield and the crop not to experience any stress, keeping the top 30 cm in the stress-free zone is required. In this scenario, the objective of the scheduler is to keep all the layers in the zone. The values of the upper and lower bound for the actual zone are -0.25 m and -3.1 m. The upper and lower bound for the conservative zone is considered as -0.5 m and -2.3 m. The values of the tuning parameters $Q_y, Q_u, Q_T, \overline{Q_r}, \overline{Q_r}$ are 1, 100, 1, 100, 0.01, 1 respectively. Things to note are that the tuning parameter values can be adjusted depending on the root growth with time. The lower and upper bounds of the time are 30 mins and 12 days. The upper and lower bound of the input are 0 m/s and $4e-07$ m/s. In general, seven days rain predictions can be 80% accurate. So for one horizon in the scheduler, the accurate weather prediction of 7 days is used, and for the rest of the days, the long-term prediction value is used. The values of accurate weather prediction and long-term weather prediction considered for this simulation are shown in Fig. 6(a). Similarly, the reference ET value for accurate and long-term weather prediction is shown in Fig. 6(b). The crop coefficient (K_c) for lettuce crop type for all the growing season is shown in 6(c).

Fig. 7(a) shows some selected state trajectories at different depths. We can observe that for all the depths, the states are in the stress-free zone. We can also see that around 50 days, state 60 (bottom node) goes outside of the conservative zone. This happens because around day 50, the crop is at the mature stage, and the ET values are high 6(c), and there is some delay between the water to reach the bottom node. That is why we choose a conservative zone of -2.3 m, So even if the bottom nodes go outside of

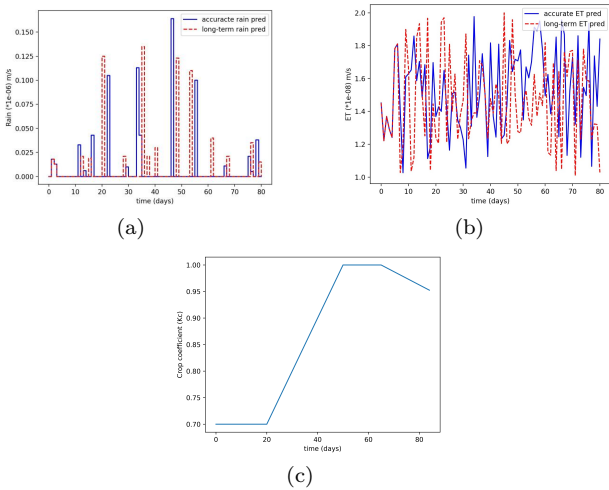


Fig. 6. (a) Accurate rain prediction and long term rain prediction; (b) Accurate ET prediction and long-term ET prediction, (c) Crop coefficient for total growing season for lettuce

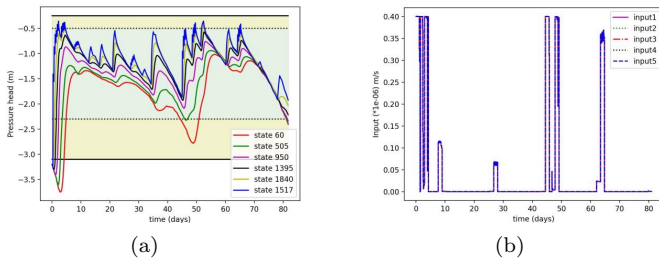


Fig. 7. (a) Selected state trajectories for all layers; (b) Irrigation amount for 5 different sprinklers.

the conservative zone, it may have less chance to go outside of the actual stress zone. 7(b) shows the input amount for all five sprinklers. We can observe that there is frequent irrigation at the beginning because the states are outside of the zone. Moreover, around day 45-50, there is frequent irrigation because of high ET values, and the crop needs more water at that stage. For other days, because of the rain, the crops don't need much water.

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