Anaerobic Digestion Processes Controller Tuning Using Fictitious Reference Iterative Method

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Abstract: In this paper, a new data-based procedure, Fictitious Reference Iterative Tuning, is proposed to control the anaerobic digestion process. In the first phase, the proposed approach uses input-output data from the anaerobic digestion process obtained by using a controller with initial parameters that ensure loop stability. In the second phase, the situation in which the input-output data are obtained in a closed-loop was also analyzed. Therefore, the Fictitious Reference Iterative Tuning method was used to obtain: a PI controller, which was tuned on the basis of an iterative, convergent and monotonous process and a PID controller, which was tuned on the basis of a divergent iterative process. The results obtained confirm the validity of the proposed Fictitious Reference Iterative Tuning method for the control of the anaerobic digestion process.

Keywords: Anaerobic Digestion Process, Anaerobic Digestion Model, Data-Driven Control, Fictitious Reference Iterative Tuning, PI Controller.

1. INTRODUCTION

The water quality is a key issue nowadays. One solution to limit the influence of the human activities on the water quality is to develop performant wastewater treatment technologies (Barbu et al., 2017), (Barbu et al., 2018). The anaerobic digestion process (ADP) is a used technology mainly because considers influents with high concentration and allows to obtain biogas, thus making more economically efficient (Caraman et al., 2015). In the specialized literature were proposed and approached different control structures for the control of the ADP, an extensive review being provided in (Jimenez et al., 2015). Most of the control methods are using a simplified model of the process, instead of using the complex model: Anaerobic Digestion Model No. 1 (Batstone et al., 2002). Controlling the ADP is a real challenge when designing a control strategy with data collected from a virtual ADP simulated based on ADM1. Several contributions and different data-based strategies have been reported in the literature. Specifically, (Condrachi et al. 2019a) proposed a gain scheduling (GS) control procedure for control of anaerobic digestion processes, in which the “local” control laws within GS algorithm are obtained by Virtual Reference Feedback Tuning (VFRFT) approach. Another solution reported by (Condrachi et al. 2019b) provides the use of Model Free Control methods based on a methodology in which the trial and error method has an important weight in determining the parameters of the controller of the ADP. Also the Extremum Seeking Control was proposed with good results in the context of ADP (Caraman et al., 2017), (Barbu et al., 2017b).

Fictitious reference iterative tuning (FRIT), frequently encountered in the literature following the publication in 2004 and 2005 of papers (Soma et al. 2004a) and (Soma et al. 2004b), is a procedure used as an alternative to VFRFT. Within FRIT, a quadratic performance criterion is minimized in which the error defined as the difference between the output variable of the process and the output variable of the reference model, when a “fictitious” reference is applied at its input. The calculation of the latter requires the transfer of the process input quantity through the inverted controller. The optimization of the criterion is done iteratively, through a search method in the controller parameters space. In contrast to previous data-based design method, FRIT uses a set of input-output data from the process when it is in a closed loop (Soma et al. 2004a), (Soma et al. 2004b). According to the literature, the scope of FRIT is wide: power electronics, (Nguyen and Kaneko 2016), process state control (Matsui et al. 2014), tuning of cascade controllers (Nguyen and Kaneko 2017), tuning of fractional order PID (Dif et al. 2017).

In the following, FRIT was approached in the classic version for the ADP control. It uses input-output data from the process, obtained when using a controller with initial parameters that ensures the stability of the loop. In addition, the use of FRIT was also analyzed in case of the input-output data obtained in open circuit.

2. THE ANAEROBIC DIGESTION PROCESS

The anaerobic digester (AD) used in the case study presented in this paper has the volume of liquid $V_l = 3400 \, \text{m}^3$ and the volume of gas $V_g = 300 \, \text{m}^3$. The digester is considered well-mixed and the temperature in the digester is controlled at an optimal value. The ADM1 model that has 35 state variables was implemented in accordance with (Rosen and Jeppsson, 2006). The objective of the control problem is to track the chemical oxygen demand (COD) concentration at a setpoint compatible with the environmental norms. This is defined by the relation $\text{COD} = S_1 + S_2$, where $S_1$ is the sum of the concentrations of the organic substrate components, and $S_2$ is the sum of the volatile fatty acid concentrations. The correspondence between the ADM1 variables and the...
variables $S_1$ and $S_2$ is given in (Hassam et al., 2015). The command variable is the dilution rate $D$ (or the influent flow rate in AD, $Q_{ad} = V_f \cdot D$), and the disturbance variables are the concentration variations in the influent, $S_{1in}$ and $S_{2in}$.

3. THE FICTITIOUS REFERENCE ITERATIVE TUNING METHOD

The design of the command law for the ADP is made under conditions of the existence of the following initial data:

1. the transfer function of the reference model, $M(s)$, by which the characteristics of the static and dynamic closed-loop system are imposed;
2. the structure of the controller with an invertible transfer function, $C(s, \rho)$, where $\rho$ is the parameters vector;
3. the initial value of this vector, $\rho^0$, for which the closed-loop system is stable;
4. the $[u(t), y(t)]_{t=1,N}$ data set collected in a closed loop, around the operating point provided by the control loop reference.

Remark: Although in all papers dealing with FRIT it is considered that the data set is obtained in closed-loop operation, the case when these data, denoted by $\{u(t, \rho^0), y(t, \rho^0)\}_{t=1,N}$, are obtained in open-loop will be analyzed first.

The problem of tuning the controller starts from the requirement to minimize the control error. The optimal parameter, $\rho^*$, is:

$$ \rho^* = \arg \min \sum_{t=1}^{N} \| e(t, \rho) \|_2^2 $$

in which $e(t, \rho) = y(t, \rho) - M(s) \cdot r(t)$

The minimization of criterion (3) is done by classical numerical methods. When using the Gauss-Newton method, the parameter adjustment algorithm:

$$ \rho^{i+1} = \rho^i - \gamma R_i^{-1} \frac{\partial J(\rho)}{\partial \rho} \bigg|_{\rho^i} $$

involves the calculation of the criterion gradient, as well as of $R_i$, which represents the approximation of the Hessian matrix. The gradient and the approximation of the Hessian matrix are:

$$ \frac{\partial J(\rho)}{\partial \rho} = \sum_{t=1}^{N} \left( \frac{\partial e(t, \rho)}{\partial \rho} \right) \frac{\partial e(t, \rho)}{\partial \rho} \bigg|_{\rho^i} $$

respectively:

$$ R = \sum_{t=1}^{N} \left( \frac{\partial e(t, \rho)}{\partial \rho} \right)^T \left( \frac{\partial e(t, \rho)}{\partial \rho} \right) $$

For the detail of the calculation procedure, the expression of the error in (4), (5) and (6) is deduced:

$$ e(t, \rho) = y(t) - M(s) \cdot \left( C^{-1}(s, \rho^j) \cdot u(t) + y(t) \right) $$

where $y_1(t)$ and $y_2(t)$ do not depend on $\rho$ in the form $y(t, \rho)^0$:

$$ \left( \frac{\partial e(t, \rho)}{\partial \rho} \right) \bigg|_{\rho^i} = -\left( \frac{\partial C^{-1}(s, \rho)}{\partial \rho} \right) \bigg|_{\rho^i} \cdot y(t) $$

In what follows it is considered that the transfer function of the controller is given by zeros-poles distribution:

$$ C(s, \rho) = \frac{1}{P_0} \frac{(s + \rho_{11})(s + \rho_{22})...(s + \rho_{mn})}{(s + \rho_{p1})(s + \rho_{p2})...(s + \rho_{pn})} $$

and define the vector of parameters:

$$ \rho^i = [\rho_{01} \rho_{11} \rho_{21} \ldots \rho_{mn} \rho_{p1} \rho_{p2} \ldots \rho_{pn} ]^T $$

With these notations, it results the following calculus relation necessary for the calculation of the gradient (13):

$$ \frac{\partial e(t, \rho)}{\partial \rho_{pi}} \bigg|_{\rho^i} = \frac{1}{(s + \rho_{pi})} y_1(t, \rho) \bigg|_{i=1,n} $$

and

$$ \frac{\partial C^{-1}(s, \rho)}{\partial \rho} \bigg|_{\rho^i} = \frac{y_2(t, \rho)}{P_0} $$


\[
\frac{\partial \mathcal{C}^{-1}(s, \rho)}{\partial \rho_i} \cdot y_1(t) = -\rho_i \frac{(s + \rho_{p1})(s + \rho_{m1})}{(s + \rho_{z1})... (s + \rho_{zm})} \cdot y_1(t) = \frac{1}{(s + \rho_{z1})} \cdot y_{v1}(t, \rho) \quad i = 1,m
\]

where:
\[
y_{v1}(t, \rho) = \mathcal{C}^{-1}(s, \rho) \cdot y_1(t)
\]

3. USING FRIT TO OBTAIN A PI CONTROLLER

Consider the input-output data signals \( \{u(t, \rho^0), y(t, \rho^0)\}_{t=1}^{N} \), obtained by the open-loop simulation of the ADM1 model around an imposed operating point. These signals are represented in Figure 1.

![Figure 1. Process command (blue) and COD (blue)](image)

In the problem of ADP control, a reference model of the following form was considered:
\[
M(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

where \( \omega_n = 2.5 \) [rad/d] and \( \zeta = 1 \).

It was imposed the objective of tuning a PI controller with the transfer function:
\[
C(s, \rho) = \frac{s + \rho_2}{\rho_1 s}
\]

where:
\[
\rho = [\rho_1 \quad \rho_2]
\]

and the initial parameters are: \( \rho_1^* = 80; \rho_2^* = 3 \).

The following relations were used in the calculus of the \( J \) gradient and the approximation of the Hessian matrix.
\[
\frac{\partial \mathcal{C}^{-1}(s, \rho)}{\partial \rho_1} \cdot y_1(t) = \frac{s}{s + \rho_2} \cdot \frac{y_{v1}(t, s)}{\rho_1}
\]

where:
\[
y_{v1}(t, \rho) = -\rho_1 s \cdot y_1(t)
\]

\[
\frac{\partial \mathcal{C}^{-1}(s, \rho)}{\partial \rho_2} \cdot y_1(t) = -\rho_1 s \frac{y_1(t)}{(s + \rho_2)^2}
\]

With the notation:
\[
y_{v2}(t, s) = -\frac{1}{s + \rho_2} \cdot y_{v1}(t, s)
\]

the error gradient is:
\[
\frac{\partial \hat{e}(t, \rho)}{\partial \rho} = \begin{bmatrix} y_{v1}(t, s) \\
 y_{v2}(t, s) \end{bmatrix}
\]

Using \( \gamma = 0.1 \), the criterion minimization algorithm is convergent, and the evolution of \( J \) in the tuning process is rapid, as noted in Figure 2.

![Figure 2. Evolution of the criterion in the tuning process](image)

The final parameters of the controller obtained by tuning are: \( \rho_1^* = 10.47; \rho_2^* = 1.351 \).

A preliminary test of the controller can be done in an approach that involves linearizing the process at the considered operating point. Using the available input-output data set, the identification of the fourth order linear model by the least squares method led to relatively modest performance (Figure 3), due to important nonlinearities in the process. Using the identified linear model and the tuned controller, the results illustrated in Figure 4 were obtained, where the variations of the reference and the output variable are given.

![Figure 3. COD evolutions from process (blue) and identified model (red)](image)

![Figure 4. COD evolutions from process (blue) and identified model (red)](image)
A more conclusive validation is obtained using the ADM1 model in the control loop. Step variations of the reference around the operating point of the process were considered and, in addition, a step variation of the disturbance $COD_{init}$. The systems response is depicted in Figure 5 and Figure 6.

As mentioned in the previous section, in the literature (Kaneko et al, 2011), (Vilanova, 2012) it is mentioned that the input-output data set must be collected when the process is in closed circuit, with a controller having a vector of the parameters $\rho^0$ which ensures the stable operation of the loop. We will now impose this condition, considering the same operating regime of the loop, its reference being $COD_{ref} = 1.5 \text{ [g/m}^3]\text{]$, and the same initial vectors of the parameters: $\rho^0 = [80 \ 3]^T$. In this case, the input-output variables of the process are illustrated in Figure 7. In Figure 8 is given the evolution of the $J$ criterion in the tuning process using data in open and closed loop, respectively. It is found that in the second case the value of the criterion at the first tuning step is significantly lower, but after 40 tuning steps, the difference between the values of the criterion are negligible. With the parameters of the controller $\rho_1^* = 17.62; \rho_2^* = 2.21$, obtained with the new input-output data, the control loop that includes the ADM1 model achieves similar performances to the previous case, the evolutions of the controlled and command variables being practically identical to those depicted in Figure 5 and Figure 6.

4. USING FRIT TO OBTAIN A PID CONTROLLER

The controller with the following transfer function was considered:

$$C(s, \rho) = \frac{(s + \rho_2)(s + \rho_3)}{\rho_1 s(s + \rho_4)}$$

(28)
In this case, the error gradient is:

\[
\begin{bmatrix}
    \frac{\partial e(t, \rho)}{\partial \rho_1} & \frac{\partial e(t, \rho)}{\partial \rho_2} & \frac{\partial e(t, \rho)}{\partial \rho_3} & \frac{\partial e(t, \rho)}{\partial \rho_4}
\end{bmatrix}^T
\]

where: \( y_{v1}(t,s) \) and \( y_{v2}(t,s) \) are given by (24) and (26), respectively, and:

\[
y_{v3}(t,s) = \frac{1}{s + \rho_3} \cdot y_{v1}(t,s)
\]

\[
y_{v4}(t,s) = \frac{1}{s + \rho_4} \cdot y_{v1}(t,s)
\]

When using open-loop data, the criterion minimization algorithm using \( \gamma = 0.1 \) is divergent. Since the algorithm used is unrestricted, the parameter exceeds the stable operating range, in which the condition \( \rho_i > 0, i = 1, 4 \). In Figure 9 is given the evolution of \( J \) when the parameters of the controller remain positive.

![Figure 9. Evolution of the criterion for tuning the PID controller with open-loop data](image)

In the case of using closed-loop data, the vector of the initial parameters was considered: \( \rho^0 = [13 \ 3.7 \ 16 \ 100]^T \), for which the Bode characteristics is represented in blue in Figure 10. When adjusting the parameters of the controller, \( \gamma = 0.02 \) step was used, resulting in the evolution of the \( J \) from Figure 11. After, 140 adjustments steps, the parameters were obtained: \( \rho^* = [17.36 \ 1.646 \ 87.45 \ 20.12]^T \) to which corresponds the Bode characteristics represented in green in Figure 10.

The \( J \) evolution in the tuning process is not monotonous. If only 80 adjustment steps are used, up to which the \( J \) criterion decreases monotonously (but within very wide limits), the Bode plot of the obtained controller, represented in red in Figure 10, is basically of the PI type. Further adjustment of the controller parameters (refining the tuning process) somewhat alters the Bode characteristics, without finally obtaining a typical form for a PID command.

![Figure 10. Bode plot of the initial PID controller (blue), after 80 adjustments steps (red) and after 140 adjustments steps (green)](image)

![Figure 11. Evolution of the criterion for tuning the PID controller with closed-loop data](image)

In Figure 12 are given the results of the validation of the controllers obtained after 80 and 140 steps of adjustment of the parameters.

![Figure 12. Evolution of the reference (blue-dash), disturbance (black) and the controlled variable after 80 and 140 steps of adjustment of the parameters (green and red)](image)
6. CONCLUSIONS

The tuning of the PI controller for ADP is based on a convergent and monotonous iterative process, including the use of open-loop data. Obviously, it is preferable for the data to be obtained in closed-loop.

In the case of the PID controller, the iterative open-loop data tuning process is divergent. With the usual use of FRIT, with closed-loop data, iterative tuning can lead to a PI-type control law, and by refining the criterion minimization process, a controller is obtained whose Bode characteristic is closer to that of a PI controller, than a PID.

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REFERENCES


Barbu, M., Ceangă, E., Vilanova, R., Caraman, S., Ifrim, G.,(2017b) Extremum-Seeking control approach based on the influential variability for anaerobic digestion optimization, IFAC World Congress, Toulouse, France.


C. Rosen, U. Jeppsson, Aspects on ADM1 implementation within the BSM2 framework, Lund University, 2006.

