

Modeling the Carbon Cycle Dynamics and the Greenhouse Effect

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Abstract: A carbon cycle model is proposed that predicts changes in the atmospheric CO₂ concentration, and provides a global temperature estimate from an empirical correlation of the two variables. The model is validated by simulating the anthropogenic carbon emissions and deforestation since the industrial revolution, and comparing the predicted and measured atmospheric CO₂ concentration and global temperature data. The temperature data are also compared with those predicted by a greenhouse effect model based on the effective emission temperature hypothesis. The result suggests that radiative forcing by CO₂ alone can account for only about half of the measured global warming. The model requires further elaboration for the other half, in order to be applicable to simulation of potential climate control.

Keywords: carbon cycle dynamics, greenhouse effect, global warming, carbon neutrality, climate control

1. INTRODUCTION

About 55.5 million years ago, there was a time period during which the global average temperature rose by 5–8 °C, the so-called Paleocene-Eocene Thermal Maximum (PETM) (Wikipedia, 2022). The carbon based greenhouse gas emissions from the ocean to the atmosphere at that time are estimated to have been 0.3–1.7 PgC/y (Wikipedia, 2022). These values are commonly compared with the current anthropogenic carbon emissions of about 10 PgC/y, leading to concerns that the current global warming may be much faster and more serious than PETM. However, this comparison is tricky, because the old emissions were natural, i.e. an effect of global warming, while the current emissions are from human activities, i.e. a cause of global warming. In order to clarify the cause and effect relationship between global warming and greenhouse gases, a rigorous study of carbon cycle is necessary.

Carbon cycle models have been proposed since long ago (Eriksson and Welander, 1956; Gowdy et al., 1975; Kamiuto 1994). Generally, old models overestimate the rate of CO₂ assimilation by plants, and thus misleadingly encourage fossil fuel usage. A carbon cycle model incorporating a limited CO₂ fertilization effect was proposed by Sallade et al. (2012) as part of the GLOBE (Global Learning and Observations to Benefit the Environment) program (GLOBE, 2022) sponsored by NASA (National Aeronautics and Space Administration), and demonstrated on the internet (GLOBE, 2017). The rate of CO₂ assimilation in this model is proportional to the vegetated land area, and independent of the actual quantity of plants, which excessively discourages fossil fuel usage, and misleadingly encourages forest biomass usage. Model shortcomings were addressed in our previous work, and the new model was termed GLOBE+ (Choi and Manousiouthakis, 2020).

The cause of global warming is still under debate. According to the United Nation's Intergovernmental Panel on Climate Change (IPCC, 2022), humans are solely responsible for recent global warming. However, according to the carbon cycle model proposed by Ollila (2015), the anthropogenic fraction of CO₂ is far less than reported by IPCC. Besides, whatever the reason, the greenhouse effect coded in the GLOBE model suggests that for every 100 ppm increase in the atmospheric CO₂ concentration, the global temperature increases by 1 °C, and that if the global temperature increases by 1 °C, the equilibrium concentration of CO₂ above the ocean surface increases by about 10 ppm. Therefore, the greenhouse effect caused by CO₂ forms a positive feedback loop that should further increase the global temperature.

The global temperature rise also increases the amount of water vapor in the atmosphere, which plays a major role in the greenhouse effect. This makes another positive feedback loop. In this case, however, if the amount of clouds that reflect solar radiation also increases, a negative feedback loop is also formed that regulates the rise in the global temperature. On the other hand, there is a claim that cloud feedback is likely positive (Ceppi and Nowack, 2021). In this case, as clouds grow, the earth receives less solar radiation, but the clouds should absorb more terrestrial radiation, which is unlikely. Nonetheless, according to NASA, the overall feedback effect by water vapor is estimated to more than double the warming by increasing CO₂ alone (Buis, 2022).

2. CARBON CYCLE

The global carbon cycle can be represented as a box model as shown in Fig. 1. In the diagram, the boxes represent carbon reservoirs, and the arrows represent the carbon flows. For simplicity, only the colored boxes and solid arrows are considered in formulating the model equations, as they form an active carbon cycle. The carbon flows represented by

dashed arrows are neglected, as they are considered to be much smaller than the others.

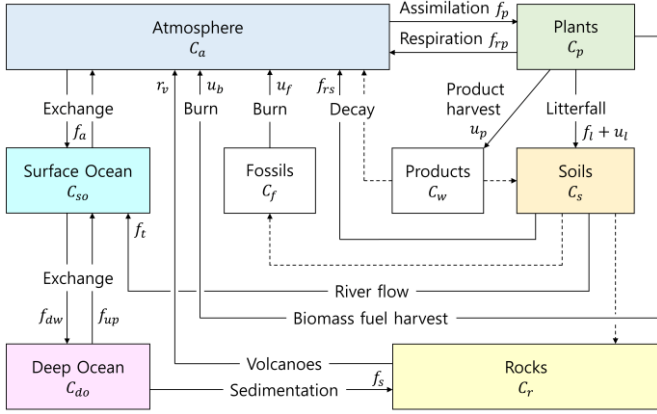


Figure 1. Carbon cycle model (Choi and Manousiouthakis, 2020).

The model equations in function form are as follows:

$$\dot{C}_a = f_{rp}(C_p) + f_{rs}(C_s) - f_p(C_a, C_p, A_v) - f_a(C_a, C_{so}) + r_v + q \quad (1)$$

$$\dot{C}_p = f_p(C_a, C_p, A_v) - f_{rp}(C_p) - f_l(C_p) - h \quad (2)$$

$$\dot{C}_s = f_l(C_p) - f_{rs}(C_s) - f_t(C_s) + u_l \quad (3)$$

$$\dot{C}_{so} = f_a(C_a, C_{so}) + f_t(C_s) + f_{up}(C_{do}) - f_{dw}(C_{so}) \quad (4)$$

$$\dot{C}_{do} = f_{dw}(C_{so}) - f_{up}(C_{do}) - f_s(C_{do}) \quad (5)$$

$$\dot{C}_r = f_s(C_{do}) - r_v \quad (6)$$

$$\dot{A}_v = -f_d(h, s, C_p, A_v) \quad (7)$$

$$q = u_f + u_b \quad (8)$$

$$h = u_b + u_p + u_l \quad (9)$$

where

C_a = mass of carbon in the atmosphere, PgC

C_p = mass of carbon in plants, PgC

C_s = mass of carbon in soils, PgC

C_{so} = mass of carbon in the surface ocean, PgC

C_{do} = mass of carbon in the deep ocean, PgC

C_r = mass of carbon in rocks, PgC

A_v = normalized vegetated land area

f_p = rate of photosynthesis, PgC/y

f_{rp} = rate of plant respiration, PgC/y

f_l = rate of litterfall, PgC/y

f_{rs} = rate of soil respiration, PgC/y

f_t = rate of transfer from soils to the surface ocean by the river flow, PgC/y

f_a = net rate of absorption to the surface ocean, PgC/y

f_{up} = rate of upwelling from the deep ocean, PgC/y

f_{dw} = rate of down-welling from the surface ocean, PgC/y

f_s = rate of sedimentation to the crust, PgC/y

f_d = net rate of deforestation, y^{-1}

r_v = rate of volcano emission, 0.1 PgC/y

q = rate of fuel combustion, PgC/y

h = rate of plant harvesting, PgC/y

s = target rate of biomass production intended when planting seedlings, PgC/y

u_f = rate of fossil fuel combustion, PgC/y

u_b = rate of biomass combustion, PgC/y

u_p = rate of product harvest, PgC/y

u_l = rate of harvest loss, PgC/y

The carbon flow rates and parametric model equations are as follows:

$$f_p(C_a, C_p, A_v) = k_p \eta_C(C_a) \eta_T(C_a) \left(\frac{C_p}{C_p^\circ} \right)^\alpha \left(\frac{A_v}{A_v^\circ} \right)^{1-\alpha} \quad (10)$$

$$k_p = 110 \text{ PgC/y}, C_p^\circ = 560 \text{ PgC}, A_v^\circ = 1, 0 < \alpha < 1$$

$$\eta_C(C_a) = 1.5 \frac{p_a(C_a) - 40}{p_a(C_a) + 80} \quad (11)$$

$$p_a(C_a) = \frac{280 \text{ ppm}}{750 \text{ PgC}} C_a \quad (12)$$

$$\eta_T(C_a) = \frac{[60 - T_g(C_a)][T_g(C_a) + 15]}{1350} \quad (13)$$

$$T_g(C_a) = 15 + 0.01[p_a(C_a) - 280] \quad (14)$$

$$f_{rp}(C_p) = k_{rp} \frac{C_p}{C_p^\circ}, k_{rp} = 55 \text{ PgC/y} \quad (15)$$

$$f_l(C_p) = k_l \frac{C_p}{C_p^\circ}, k_l = 55 \text{ PgC/y} \quad (16)$$

$$f_{rs}(C_s) = k_{rs} \frac{C_s}{C_s^\circ}, k_{rs} = 55 \text{ PgC/y}, C_s^\circ = 1,500 \text{ PgC} \quad (17)$$

$$f_t(C_s) = k_t \frac{C_s}{C_s^\circ}, k_t = 0.8 \text{ PgC/y} \quad (18)$$

$$f_a(C_a, C_{so}) = k_{ao} [p_a(C_a) - p_a^*(C_a, C_{so})] \quad (19)$$

$$k_{ao} = 0.278 \text{ PgC/(y ppm)}$$

$$p_a^*(C_a, C_{so}) = 280 \text{ (ppm/mM)} K_{CO_2} \frac{[HCO_3^-]^2}{[CO_3^{2-}]} \quad (20)$$

$$K_{CO_2} = 0.0255 + 0.0019 T_g(C_a) \quad (21)$$

$$[HCO_3^-] = \frac{[CO_2]_i - \sqrt{[CO_2]_i^2 - A_T(2[CO_2]_i - A_T)(1 - 4K_{CO_3^{2-}})}}}{1 - 4K_{CO_3^{2-}}} \quad (22)$$

$$K_{CO_3^{2-}} = 0.000545 + 0.000006 T_g(C_a) \quad (23)$$

$$[\text{CO}_2]_i = \frac{C_{so}}{(12 \text{ gC/mol})(36.2 \text{ PkL})} \quad (24)$$

$$[\text{CO}_3^{2-}] = \frac{A_T - [\text{HCO}_3^-]}{2} \quad (25)$$

$$f_{dw}(C_{so}) = k_{dw} \frac{C_{so}}{C_{so}^\circ} \quad (26)$$

$$k_{dw} = 90.1 \text{ PgC/y}, C_{so}^\circ = 890 \text{ PgC}$$

$$f_{up}(C_{do}) = k_{up} \frac{C_{do}}{C_{do}^\circ} \quad (27)$$

$$k_{up} = 90 \text{ PgC/y}, C_{do}^\circ = 38,000 \text{ PgC}$$

$$f_s(C_{do}) = k_s \frac{C_{do}}{C_{do}^\circ}, k_s = 0.1 \text{ PgC/y} \quad (28)$$

$$f_d(h, s, C_p, A_v) = \left[h(t) - \int_0^t s(t-\tau) \dot{g}(\tau) d\tau \right] \frac{A_v}{C_p} \quad (29)$$

$$g(t) = (1 - e^{-kt})^p \quad (30)$$

$$p = \frac{1}{1 - \alpha} \quad (31)$$

$$k = \frac{k_p}{p C_p} \quad (32)$$

where

η_C = factor of CO₂ effect on photosynthesis

η_T = factor of temperature effect on photosynthesis

p_a = concentration of CO₂ in the atmosphere, ppm

p_a^* = equilibrium concentration of CO₂ in the atmosphere, ppm

T_g = global temperature, °C

A_T = total alkalinity of seawater, 2.222 mM (Sauvage et al., 2014)

g = Chapman-Richards growth function (Pommerening and Muszta, 2016)

Among the above equations, (11)-(14), (17), (19)-(28) are from the GLOBE model (Sallade et al., 2012; GLOBE, 2017), (10), (18), (30)-(32) are from the GLOBE+ model (Choi and Manousiouthakis, 2020), and (15), (16), (29) were proposed later (Manousiouthakis and Choi, 2021). The reference values denoted by the degree symbol and the rate constants associated with them are from the GLOBE model (Sallade et al., 2012; GLOBE, 2017), and correspond to estimates for around 1750, i.e. just before the industrial revolution (Choi and Manousiouthakis, 2020). The carbon cycle dynamics can be predicted by integrating (1)-(7) from given initial values. The model is based on mass balances only, and simply estimates the global temperature using the GLOBE model's correlation with the atmospheric CO₂ concentration, provided in equation (14). As a numerical analysis tool, MATLAB is used in this work. The above initial value problem is solved by ode45, and the integral in (29) is evaluated by the MATLAB built-in integral function.

3. MODEL VALIDATION

Let us consider fossil fuel usage, deforestation, atmospheric CO₂, and temperature data since the industrial revolution. The CO₂ emission data (Ritchie et al., 2020), total forest area estimates (Ritchie, 2021), atmospheric CO₂ concentration data (Ritchie et al., 2020) are normalized and plotted as black, green, and red dots respectively in Fig. 2, while the temperature data are represented by orange dots in Fig. 2, and were obtained by adjusting, using a single temperature value, the original anomaly data since 1880 (NOAA National Centers for Environmental Information, 2021) so as to match an estimated increase of 0.3 °C from 1750 to 1900 (Arctic News, 2021). The CO₂ emission data and total forest area estimates will be applied as input to the proposed carbon cycle model to predict the atmospheric CO₂ concentration and the global average temperature.

Let us simulate deforestation using the proposed carbon cycle model. As the net rate of deforestation can be attributed to the net rate of harvesting, the following equations are suggested:

$$h = \frac{f_d C_p}{A_v}, s = 0 \quad (33)$$

Furthermore, as suggested in the GLOBE model (Sallade et al., 2012), it is assumed that the harvested carbon moves to air and land, half and half as follows:

$$u_b = u_l = \frac{h}{2}, u_p = 0 \quad (34)$$

The simulation results are represented by solid ($\alpha = 2/3$), dashed ($\alpha = 0$), and dash-dotted ($\alpha = 1$) lines in Fig. 2, where $\alpha = 0$ corresponds to the original GLOBE model (Sallade et al., 2012), and $\alpha = 1$ conceptually corresponds to the old model (Eriksson and Welander, 1956). As the blue solid line matches the red dots, the GLOBE+ model with $\alpha = 2/3$ is validated.

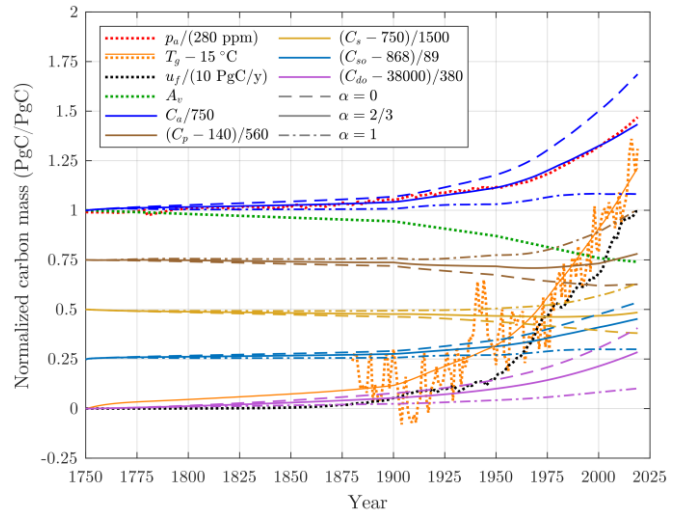


Figure 2. GLOBE+ model validation with measured data.

4. GREENHOUSE EFFECT

The greenhouse effect is caused by radiative forcing by greenhouse gases in the atmosphere. For CO₂, it can be approximated as follows (Myhre et al., 1998):

$$\Delta F = 5.35 \text{ (W/m}^2\text{)} \ln \frac{p_a}{p_a^\circ} \quad (35)$$

where p_a° represents a reference concentration.

A simple, energy balance based, model for the global temperature increase can then be put forward using the effective radiation temperature hypothesis (Berger and Tricot, 1992) as follows:

$$F = F_{a0} + F_{c0} + F_{s0} \quad (36)$$

$$F_{a0} = (1 - \omega_c - \omega_s) \varepsilon_a \sigma T_{a0}^4 \quad (37)$$

$$F_{c0} = \omega_c \varepsilon_c \sigma T_{c0}^4 \quad (38)$$

$$F_{s0} = \omega_s \varepsilon_s \sigma T_{s0}^4 \quad (39)$$

$$F_{a1} = F_{a0} - \Delta F \quad (40)$$

$$T_{a1} = T_{a0} \left(\frac{F_{a1}}{F_{a0}} \right)^{1/4} \quad (41)$$

$$T_{a2} = T_{a1} + \Delta T \quad (42)$$

$$T_{s1} = T_{s0} + \Delta T \quad (43)$$

$$F_{a2} = F_{a1} \left(\frac{T_{a2}}{T_{a1}} \right)^4 \quad (44)$$

$$F_{s1} = F_{s0} \left(\frac{T_{s1}}{T_{s0}} \right)^4 \quad (45)$$

$$F = F_{a2} + F_{c0} + F_{s1} \quad (46)$$

where

F = solar radiation absorbed by earth, W/m²

F_a = radiation from the atmosphere, W/m²

F_c = radiation from clouds, W/m²

F_s = radiation from the surface, W/m²

ΔF = radiative forcing, W/m²

T_a = effective radiation temperature of the atmosphere, K

T_c = cloud temperature, K

T_s = surface temperature, K

ΔT = temperature increase, K

ε_a = emissivity of the atmosphere

ε_c = emissivity of clouds

ε_s = emissivity of the surface

ω_c = fraction of radiation from clouds

ω_s = fraction of radiation from the surface

σ = Stefan-Boltzmann constant, 5.670×10^{-8} W/(m² K⁴)

Let us assume that $\varepsilon_a \approx \varepsilon_c \approx \varepsilon_s \approx 1$ provisionally, and consider preindustrial initial conditions, $p_a^\circ = 280$ ppm and $T_{s0} = 14$ °C. The radiation fluxes are set to $F = 239$ W/m², $F_{c0} = 30$ W/m² as reported by Trenberth and Fasullo (2012), and $F_{a0} = 189$ W/m², $F_{s0} = 20$ W/m² as suggested by Costa and Shine (2012). Then, from (39), $\omega_s = 0.052$ is obtained, and from (37) and (38), if $T_{a0} \approx T_{c0}$ is assumed, $\omega_c = 0.130$ and $T_{a0} = 252.63$ K are obtained. Let us now assume that the atmospheric CO₂ concentration is increased to $p_a = 420$ ppm, which is approximately the present level. Then, if F_c is assumed to be constant, i.e. $F_c = F_{c0}$, the above equations give $\Delta T = 0.64$ °C. The values $\Delta T \approx 1.3$ °C in Fig. 2, obtained using the data from NOAA National Centers for Environmental Information (2021) and Arctic News (2021), and the prediction $\Delta T = 1.4$ °C by the GLOBE model equation (14), are about double the above prediction $\Delta T = 0.64$ °C obtained using the effective radiation temperature hypothesis for the greenhouse effect caused by CO₂ alone.

Lacis et al. (2010) at NASA reported that only 25% of the greenhouse effect is due to radiative forcing by CO₂ and other noncondensing greenhouse gases, and 75% is due to positive feedback effects of water vapor and clouds, hence proposing a feedback factor of 4. Under this assumption, the overall ΔF in (40) is estimated to be over 4 times the value of ΔF given by (35). In this case, $\Delta T > 2.6$ °C is obtained. If NASA's latest estimate for the feedback factor, i.e. more than double the warming by increasing CO₂ alone (Buis, 2022), is applied, $\Delta T > 1.3$ °C is obtained, which is near the aforementioned value.

5. CONCLUSIONS

It has been shown that the GLOBE+ carbon cycle model can predict changes in the atmospheric CO₂ concentration and the global average temperature with moderate accuracy. This model is expected to be applicable to carbon cycle impact assessment of carbon neutrality policies and technologies. On the other hand, the greenhouse effect model requires further elaboration on water vapor, cloud cover, and ice cover, before it can be applied to global impact assessment of climate control as anticipated by von Neumann (1955).

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