Comparative Analysis Between Two Fuzzy Variants of Harmonic Search Algorithm: Fuzzy Fault Tolerant Control Application

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Abstract: The goal of this research is to improve the harmonic search (HS) algorithm by using type-1 and interval type-2 fuzzy systems to dynamically change one of the evolutionary method's parameters. We have previously used both sorts of fuzzy systems in a variety of benchmark challenges and discovered that using fuzzy logic in conjunction with the harmonic search algorithm produces good results. In some of the experiments, it is clearly demonstrated that our methodology is statistically superior to other algorithms. Using type-1 and interval type-2 fuzzy systems, the harmony memory (HMR) parameter is dynamically changed during the evolution process in this example. The fundamental contribution of this work is the capacity to establish, by experimentation in a benchmark control issue, which of the two types of fuzzy systems employed with the harmonic search method produces better results. This is because there are no previous studies to our idea that employ and compare type-1 and interval type-2 fuzzy systems. Furthermore, three type of uncertainties are employed in the benchmark two-tank level control system to assess the performance of both fuzzy systems, simulating the disturbances that may present in the actual world and therefore allowing statistical validation if there are substantial differences between type-1 and interval type-2 fuzzy systems.

Keywords: Harmonic Search algorithm, Fuzzy controller, Type-2 fuzzy systems, Fuzzy sets

1. INTRODUCTION

Over the last four decades, a slew of novel meta-heuristics have evolved. They have used their strengths to solve crucial optimization challenges in areas including resource allocation, industrial planning, scheduling, medical, engineering, and computer engineering, among others. The objective of the proposed algorithm is one of the key features used to categorise meta-heuristics, and it may be classed based on the judgments of technique presentation. The majority of meta-heuristics are based on physics, biology, and ethology, in which random variables and several parameters are used to attain the target function.Over the last four decades, a slew of novel meta-heuristics have evolved. The objective of the proposed algorithm is one of the key features used to categorise meta-heuristics, and it may be classed based on the judgments of technique presentation.

Natural and physical processes, as well as animal behavioral patterns, are now inspiring algorithm ideas, such as the Genetic Algorithm Patel et al. (2021), Ant Colony Optimization (ASO), Particle Swarm Optimization (PSO), Bee Colony Optimization (BCO) Olivas (2019), Simulated Annealing (SA), and Harmony Search (HS) Algorithm Patel (2022). The Harmony Search Algorithm is one of the most recent meta-heuristic algorithms (HS). This design is based on the idea of music spontaneity, and it keeps polishing its pitches to achieve better harmony. In terms of simplicity, flexibility, adaptability, and scalability, the HS has various advantages Patel (2022). It also features a novel stochastic derivative and requires a simpler mathematical equation to generate new solutions at each iteration, especially when an existing solution is taken into account Patel (2022). When dealing with optimization performance in particular numerical optimization issues to search local optima, adjusting the parameters of the HS method becomes the important task. In the case of PSO and the Differential Evolution Optimization (DEO) method, a similar difficulty arises.

The harmonic memory rate (HMR), pitch adjusting rate (PAR), and range bandwidth (BW) are three factors that have prompted researchers to work on the HS algorithm Patel (2022). Since the inception of HS, much of the work has been devoted to fine-tuning the parameters and their impact on the algorithm's efficiency. Each of these parameters has a role to play in supporting HS in finding the optimal solutions. The HMR parameter, for example, is important for accomplishing a faster convergence rate, PAR is responsible for increasing solution variety, and BW is used to improve the diversity of exact solution at the conclusion of the iteration Patel (2022).

Fuzzy controllers are now optimized using metaheuristics, and these controllers need to be optimised because they often do not attain the best performance possible necessary for real-world applications. Because they employ the original concept of fuzzy sets Zadeh (1965, 1988), these controllers are commonly referred to as type-1 fuzzy logic controllers (FLC). The existing fuzzy logic (type- 1) that was suggested from the inception, type-2 fuzzy logic was later developed with the objective of handling more difficult problems, that is, problems with a higher high degree of uncertainty, than type-1 fuzzy logic can solve Zadeh (1988); Liang and Mendel (2000).

Because type-2 fuzzy logic systems are a collection of type-1 fuzzy logic systems, their ability to handle uncertainty. The article in Patel and Shah (2021a,b, 2019a,b,c) demonstrate the use of type-2 fuzzy systems to solve a variety of control applications with excellent results.

The type-1 fuzzy systems have previously been optimised with metaheuristic algorithms; for example, the optimization of type-1 fuzzy controllers is discussed in Lagunes et al. (2019), which uses the firefly method to optimise fuzzy controllers of autonomous mobile robots. Galactic Swarm Optimization (GSO) was also utilised to optimise a fuzzy controller for an autonomous robot following a trajectory in Bernal et al. (2019), where the dynamic adjustment of the most critical parameters for the GSO algorithm's operation is described. The GSO algorithm was also employed in the optimization of the liquid level fuzzy controller in Bernal et al. (2020).

There are other studies that use alternative metaheuristic algorithms to optimise fuzzy controllers. In Wagner and Hagras (2007), for example, the genetic algorithm (GA) is used to evolve the framework of a type-2 fuzzy controller in real-world robot navigation. Other authors have used fuzzy controllers to control autonomous robots following a trajectory, as described in Astudillo et al. (2006). There are some more fuzzy controller applications due to their efficiency and performance, as shown in Wu (2012), which compares type-1 and type-2 fuzzy controllers, and Wu and Tan (2004), which uses two fuzzy controllers to control the liquid-level process in a single tank.

The major goal of this research is to provide an optimization approach that uses a metaheuristic algorithm to achieve optimal performance for generating satisfactory outcomes in the control of two-tank interacting level control system subject to faulty circumstances. Because it has been shown that using parameter adjustment in metaheuristic algorithms for the optimization of mathematical functions and control problems produces competitive results, we propose in this paper that we use type-1 fuzzy logic to perform dynamic parameter adjustment and measure the performance of the algorithms used in the optimization of the fuzzy controller.

2. FUZZY LOGIC AND HARMONY SEARCH ALGORITHM

Fuzzy logic is a good methodology to design robust systems which can achieve a satisfactory performance in an environment with uncertainty or ambiguity Zadeh (1965).

The relevant theories and concepts for this study are presented in this section.

2.1 Harmony Search Algorithm

Harmony search algorithm (HSA) is a metaheuristic that was developed during last decade. It imitates the actions of a musician who achieves perfect harmony Z. W. Geem and Loganathan (2001). The following are the main characteristics of HSA: (1) No derivate information is required, (2) just a few control parameters are required for fine tuning, and (3) no initial configuration of decision variables is necessary V. Kumar and Kumar (2012).

2.2 Harmony Search Algorithm

Currently the HS algorithm is one of the most popular metaheuristics used to solve diverse types of problems. HS is based on the musical theory of jazz improvisation for imitating the natural process of a musician, which is translated into mathematical terms and generates the following 5 steps and their respective equations:

Step 1: Initialize the problem and parameters.
Minimize

$$f(x) s.t. x(j)) \in [LB(j), UB(j), j = 1, 2, \cdots, n] \quad (1)$$

Step 2: Initialize the Harmony memory (HM).

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_N^1 & f(x^1) \\ x_1^2 & x_2^2 & \cdots & x_N^2 & f(x^2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_N^{HMS} & f(x^{HMS}) \end{bmatrix} \quad (2)$$

Step 3: Improvise a New Harmony.

$$X_{new}(j) = X_{new}(j) \pm r \times BW \quad (3)$$

Step 4: Update the Harmony Memory.
To update the HM with a new solution vector, x_{new}, the objective function will be used to evaluate them. A comparison is made to find out if the new vector solution is better than the worst historical vector solution and then the worst historical is excluded and substituted with a new one.

Step 5: Verify if the stopping criteria is met.
The process is repeated until the number of improvisations (NI) is satisfied; otherwise the process is repeated from Steps 3 and 4. Finally, the best solution is achieved and considered as the best result to the problem.

2.3 Fuzzy Harmony Search Algorithm

The main goal of dynamic parameter adaptation using fuzzy systems is to improve the quality of the results obtained by performing a better local and global search than with the original HS method. The metric used in the fuzzy system input is the percentage of elapsed iterations defined by Eq. (4), and the output is the dynamic adjustment of the HMR parameter representing the exploitation of the search space defined by Eq. (5), and to better represent this idea this parameter is converted into a fuzzy parameter. In this method, this value is considered to be fuzzy as it is updated within the FHS progress, and is

Table 1. Rules for the type-1 and interval type-2 fuzzy inference system for (FHS) algorithm

Sr. no.	Iteration (I)	Diversity (D)	HMR
1	Low	Low	Low
2	Low	Moderate	Moderate
3	Low	High	Moderate
4	Moderate	Low	Moderate
5	Moderate	Moderate	Moderate
6	Moderate	High	High
7	High	Low	High
8	High	Moderate	Moderate
9	High	High	High

determined by Eq. (5), where HMR is changing values in the range [0, 1].

$$Iteration(I) = \frac{Current\ Iteration\ (CI)}{Maximun\ of\ iterations\ (MI)} \tag{4}$$

To find out the new parameter values, the fuzzy system uses as input the percentage of transpire iterations and the degree of "Diversity (D)" of individuals from bio-inspired method, and now from these metrics, these parameters are used as an input for the fuzzy system as defined by Equations (5).

$$Diversity(D) = \frac{1}{n_s} \sum_{i=1}^{n_s} \sqrt{\sum_{j=1}^{n_x} \left(X_{ij}(t) - \overline{X}_j(t)\right)^2} \tag{5}$$

Equation (6) considers a percentage of elapsed iterations to find the values of HMR. It initializes with low values of HMR so that the algorithm has diversification and then achieves intensification:

$$HMR = \frac{\sum_{i=1}^{r_{hmr}} \mu_i^{hmr}(hmr_{1i})}{\sum_{i=1}^{r_{hmr}} \mu_i^{hmr}} \tag{6}$$

where HMR is the memory consideration; r_{hmr} is the number of rules of the fuzzy system corresponding to hmr; hmr_{1i} is the output result for rule i corresponding to hmr; μ_i^{hmr} i is the membership function of rule i corresponding to hmr. The pseudo-code for the fuzzy HM optimization algorithm taken from Patel (2022) and implemented.

The rules are designed based on the study of parameters of the algorithm, so that in the initial iterations HS will explore and then in the final iterations it will exploit the search space, and in this case the rules are on an increase fashion. The fuzzy rules are summarized in Table 1.

Table 1 represents the idea of increasing the output of the rules as iterations are progressing.

The fuzzy rules are same for the type-1 and interval type-2 system for parameter adaption of fuzzy HS algorithm.

The HS algorithm parameter adjustment fuzzy system employs the input variable "Iteration (I)" and the output variable the harmony memory (HMR) parameter. As shown in Fig. 1, each variable is made up of three triangle membership functions designated "Low", "Moderate", and "High". As shown in Fig. 1, the fuzzy system HS T1 that performs HS parameter adjustment employs the "Iteration (I)" and "Diversity (D)" variable as an input variable and HMR as an output variable as a parameter, and these variables are made up of three triangular membership functions designated as "Low", "Moderate", and "High".

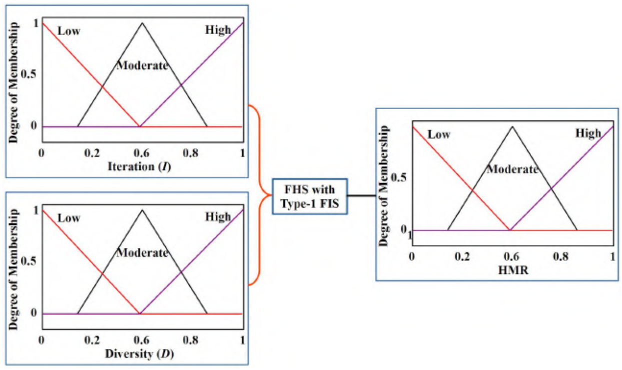

Fig. 1. Type-1 fuzzy system for the FHS

Same way, IT2FIS design for parameter adaption of FHS algorithm and the input and output membership function presented in figure. 2.

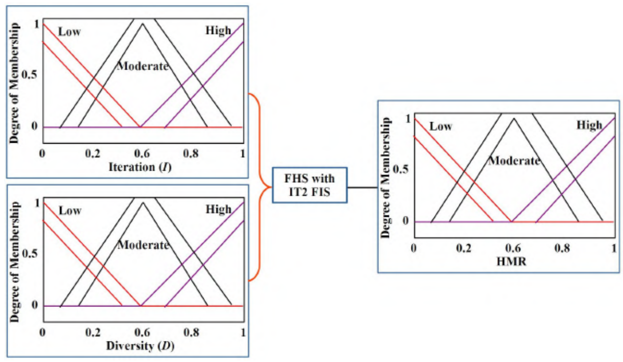

Fig. 2. Interval Type-2 fuzzy system for the FHS

The logic behind fuzzy system rules is that when the algorithms are in their first iterations, they can explore, and when they are in their final iterations, they can utilize.

To measure the performance of the algorithms for the optimization of the fuzzy controller, the mean squared error (MSE) is used. Its equation is described below:

$$MSE = \frac{1}{n} \sum_{I=1}^{n} (X_i - Y_i)^2, \tag{7}$$

where X_i is the reference value at time I; the reference values are given in Martínez et al. (2009); Y_i is the value produced by the system at time I, and n is the number of samples considered in the test.

3. SIMULATION RESULTS

In this section, the results obtained from the fuzzy controller optimization of the two-tank level control system are presented. The methodology consists on using a metaheuristic algorithm to generate a vector of the necessary parameters to form the membership functions of the type-1 fuzzy controller that is optimized. For this specific case, the metaheuristics are the harmonic search optimization and their variants with dynamic adaptation of parameters using type-1 and interval type-2 fuzzy systems. Table 2 shows the parameters used in harmonic search optimization algorithm to perform the optimization of the type-1 fuzzy controller for two-tank level control process.

Table 2. Parameters of the proposed fuzzy HS algorithms

Parameter	FHS
PA_{rate}	0.01-0.99
BW	0.01-0.07
Iteration	600
HMR	Dynamic

Figure 3 shows the performed simulation to obtain the best optimized fuzzy system with the interval Type-2 fuzzy harmonic search algorithm, where it can be observed that the best error found at 30^{th} simulation.

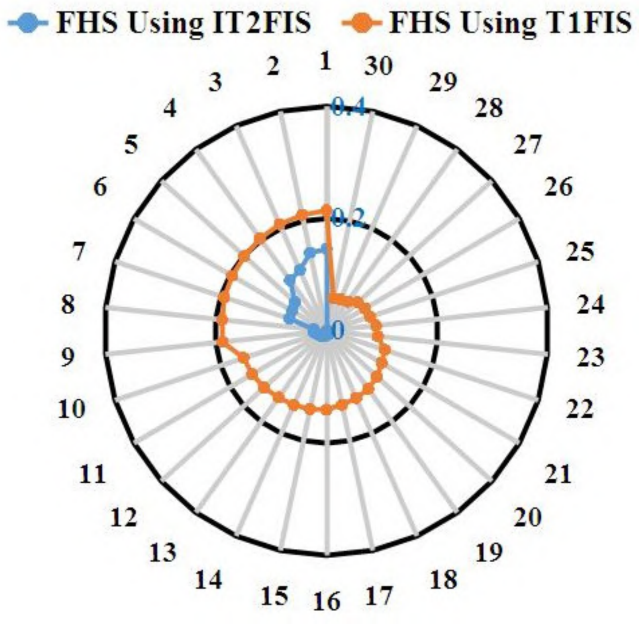

Fig. 3. Results for the fuzzy HS using type-1 and interval Type-2 fuzzy system

3.1 Two-tank level control system mathematical model and fuzzy controller

$$A_1\frac{\mathrm{dh}_1}{\mathrm{dt}} = (\alpha \times q_1) - q_{o1} - q_{12} \text{ \& } A_2\frac{\mathrm{dh}_2}{\mathrm{dt}} = q_{12} - q_2 - q_l \quad (8)$$

$$q_{o1} = \gamma_1\sqrt{h_1},\ q_2 = \gamma_2\sqrt{h_2}, \&\ q_{12} = \gamma_{12}\sqrt{h_1 - h_2} \quad (9)$$

In figure 4 the input and output membership functions (MFs) for fuzzy controller are presented for TTLCS process. The linguistic variable for input and output are taken triangular and trapezoidal MFs.

3.2 Regulatory response with and without faults

In order to validate the proposed fuzzy algorithm the fuzzy controller optimized using type-1 FHS and IT2FHS, the figure 5, 6, 7, and 8 presented the regulatory response of

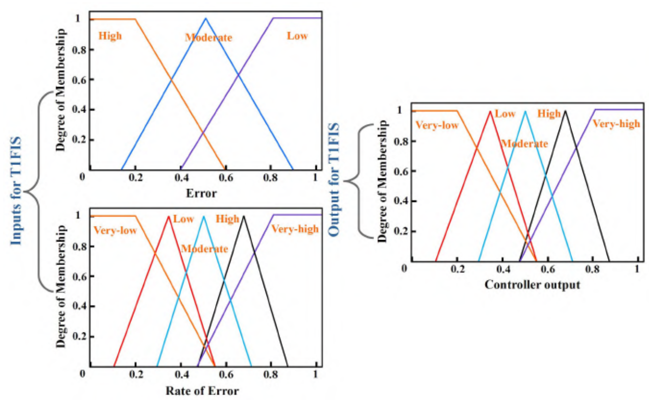

Fig. 4. Input and output MFs of Type-1 Fuzzy controller for two-tank level control process

the two-tank level control process. The proposed algorithm is capable to handle uncertainty like faults effectively because of using interval type-2 fuzzy system. In summary, we can state that there is sufficient statistical evidence to say that the harmonic search optimization (and its interval type-2 fuzzy variant) outperforms the harmonic search optimization using type-1 fuzzy variation.

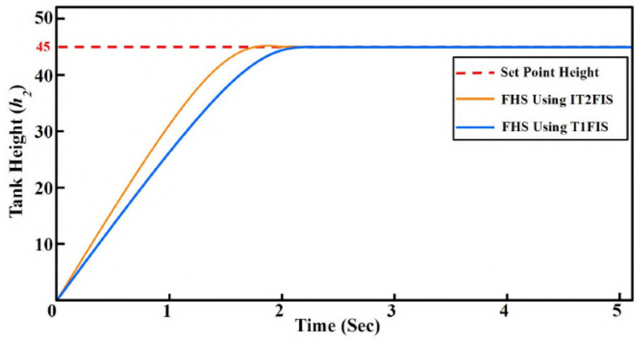

Fig. 5. Regulatory response of two-tank level control process without fault

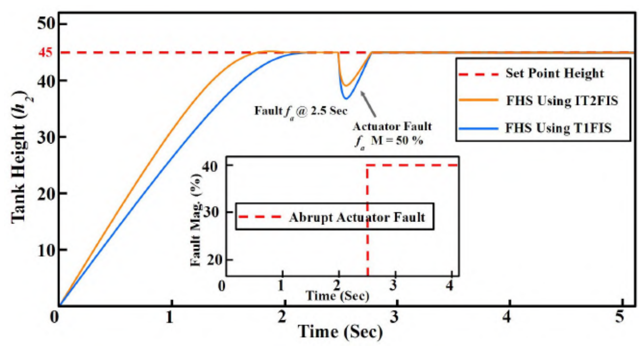

Fig. 6. Regulatory response of two-tank level control process with actuator fault M=50% in (CV_1)

Table 3 shows the fault recovery performance for the design optimized fuzzy controller for two-tank level control system under actuator fault, and results clear demonstrate that the FHS using IT2FIS gives superior fault recovery response as compared to FHS using T1FIS.

Figure 9, depicts the best type-1 FLS that the fuzzy HS algorithm finds using IT2FIS for the benchmark two-tank level control system.

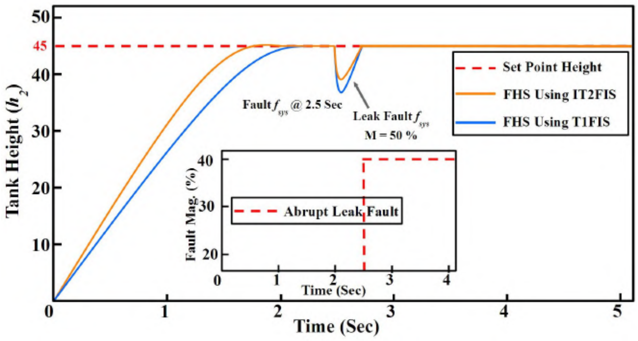

Fig. 7. Regulatory response of two-tank level control process with system component (leak) fault M=50% in (CV_1)

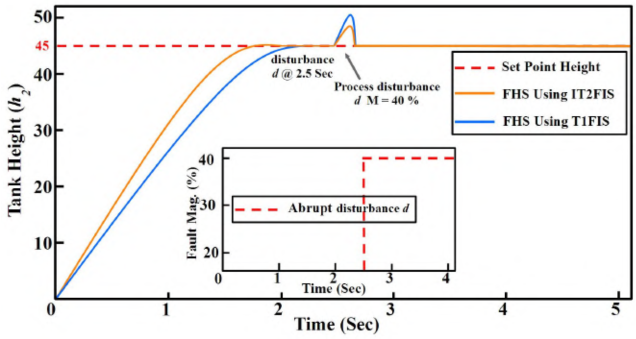

Fig. 8. Regulatory response of two-tank level control process with external process disturbances M=40% in (q_2)

Table 3. Fault recovery time comparison with uncertainty

Fuzzy Algorithm	Uncertainty Magnitude	T_{fr}
FHS Using IT2FIS	Actuator Fault	**0.53 Sec**
FHS Using T1FIS	M=50%	0.64 Sec
FHS Using IT2FIS	Leak Fault	**0.42 Sec**
FHS Using T1FIS	M=50%	0.57 Sec
FHS Using IT2FIS	Process Disturbances	**0.37 Sec**
FHS Using T1FIS	M=40%	0.46 Sec

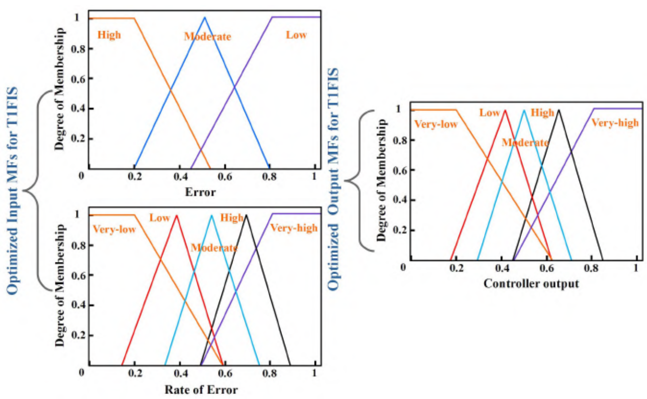

Fig. 9. Optimized MFs for type-1 fuzzy controller using IT2FHS for the benchmark two-tank level control process

3.3 *Statistical Results and Comparison*

To validate the performance of the proposed fuzzy optimization algorithm to find out the optimal MFs parameters of the two-tank level fuzzy controller, a statistical comparison is made to find evidence that the algorithms have performed well in the case study of the two-tank level control plant.

In Table 5, the null hypothesis (H_o) states that the average MSE error for the HS with IT2FLSs algorithm (μ_1) is greater than or equal to the average MSE error for HS with T1FLSs (μ_2). The alternative hypothesis (H_a) states that the average MSE error for the HS with IT2FLSs algorithm (μ_1) is less than the average MSE error for HS with T1FLSs (μ_2).

The statistical test utilized is the Z-test, which is based on Equation (10), and the parameters for this test are an α of 0.05 and a 95 % level of confidence.

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{(\sigma_1 - \sigma_2)} \tag{10}$$

The statistical z-test results in table 4 show that the proposal employed for FHS algorithms is good in most circumstances; however, the main focus of this research is a comparison of the same HS algorithm using T1FIS and IT2FIS for uncertain nonlinear level control systems.

4. CONCLUSION

First, we may emphasise and underline that using generic type-2 fuzzy logic is preferable for larger levels of uncertainty. The following is a comparison of the outcomes of the two types of fuzzy systems, T1 FHS and IT2 FHS. To begin, the best simulations without defects were 1.1×10^{04} and 5.94×10^{02}, respectively, which are considerably far. The same can be observed with the average of the MSE error where the results were 2.99×10^{02} and 1.39×10^{01}, respectively.

Overall, the research carried out positive results when comparing the two types of fuzzy systems, as well as when comparing them to high-speed interval type 2 fuzzy systems integrated with the HS algorithm. We can appreciate that we were able to accomplish superior outcomes with our proposed IT2FHS methodology because the generic type-2 fuzzy systems greatly aid the harmony search algorithm in achieving better results.

The fundamental contribution of this study can be described as a proposal of the harmony search mixed with type-1 and interval type 2 fuzzy systems to dynamically adjust a parameter of the HS algorithm, which has not been done previously in the literature. The results of the experiment can be used by other researchers as a guide to the good outcomes that can be produced when employing interval type 2 fuzzy systems under high levels of uncertainty.

As future work, we envision that the proposed method could be also applied in other problems in areas such as pattern recognition, time series prediction, and medical diagnosis among others Rubio (2017); Olivas (2019). Another important idea is to be able to perform experimentation using the two kinds of fuzzy systems to dynamically adapt the pitch adjustment rate (PAR), parameter in some other control problems, to be able to validate with

Table 4. Results for the statistical Z-test with fuzzy harmony search algorithm

System/Process	μ_1	μ_2	Z-Value
Two-Tank Level Control System Benchmark Process	Level System without Fault with FHS Using IT2FIS	Level System without Fault with FHS Using T1FIS	-4.1237
	Level System with Process Disturbance with FHS Using IT2FIS	Level System with Process Disturbance with FHS Using T1FIS	-2.0948
	Level System with Actuator Fault with FHS Using IT2FIS	Level System with Actuator Fault with FHS Using T1FIS	-1.7135
	Level System with Leak Fault with FHS Using IT2FIS	Level System with Leak Fault with FHS Using T1FIS	-1.8614

Table 5. Statistical z-test parameters

Parameter	Value
H_o	$\mu_1 \geq \mu_2$
H_a	$\mu_1 < \mu_2$ (Claim)
Level of significance	95 %
A	0.05
Critical value	-1.645

which parameter of the harmony search algorithm the best results are obtained.

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