Proceedings of the 9th International Symposium on
Dynamics and Control of Process Systems (DYCOPS 2010),
Leuven, Belgium, July 5-7, 2010
Mayuresh Kothare, Moses Tade, Alain Vande Wouwer, Ilse Smets (Eds.)

Damping Feedback Stabilization of a Wastewater Treatment Plant

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Abstract: This paper considers the problem of stabilizing a wastewater plant using damping feedback controller. The nonlinear model used is the AM1 anaerobic digestion model. The proposed approach consists in using a damping state feedback controller that renders a desired equilibrium stable through feedback. We first construct an approximate dissipative potential function centered at the desired equilibrium for the drift system. Then, a damping feedback controller is computed based on the Jurdjevic–Quinn approach. Through numerical simulations, we show that the resulting controller stabilizes the desired equilibrium, and keeps the pollutant at the desired level when input disturbances are applied to the system.

Keywords: Wastewater treatment, AM1 anaerobic digestion model, Nonlinear feedback stabilization, Dissipative control, Disturbances rejection.

1. INTRODUCTION

The design of smooth stabilizing controllers for wastewater plants is a challenging task: the model structure and parameters are often not known precisely, the measurements are limited and the inherent nonlinearities of the plant are not polynomials (see for example Bastin and Dochain (1990) and Olsson and Newell (1999) for an overview of the problem in the context of control). In this paper, we address the problem of stabilizing a plant modeled by the AM1 model as described, for example, in (Bernard et al., 2001) and (Grognard and Bernard, 2006). More exhaustive models are considered in the literature for (optimal) control, for example, the ASM1 model was studied extensively by Smets et al. (2003). Here, we consider the simplified model used in (Antonelli et al., 2003) and more recently by Grognard and Bernard (2006), where saturated controls were considered for the stability analysis of the AM1 model.

The idea of dissipative control for wastewater treatment plant was considered in (Ito, 2004). In recent years, general "energy-based" approaches for nonlinear control design were developed and successfully used in process control engineering. One interesting approach is to represent the nonlinear process as a Hamiltonian system with dissipation. One example in chemical engineering was given recently by Otero-Muras et al. (2008) who studied the stability of a reaction network using its dissipative Hamiltonian representation. However, one limitation associated with the study of non-mechanical nonlinear systems using dissipative Hamiltonian is to derive a suitable Hamiltonian function for the problem. As discussed in (Johnsen and Allgöwer, 2007), applications of Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) techniques is difficult since the concept of "energy" is usually ill-defined for process control applications, for example when mass balances are considered. In (Cheng et al., 2005), a matching approach to transform control affine systems into port-controlled Hamiltonian systems was proposed. In particular, for a fixed desired closed loop structure and a free Hamiltonian function $H(x)$, the problem leads to a set of PDEs parameterized by the feedback controller to be implemented. Relaxing the need for exact matching, a non-exact matching IDA-PBC approach was recently developed and applied successfully to chemical reactor process stabilization (Ramírez et al., 2009). Finally, it was shown in (Wang et al., 2007) how k-th degree approximate dissipative Hamiltonian systems can be used to solve the realization problem and how associated k-th degree approximate Lyapunov functions can be used to study the stability of such systems. Another aspect of the construction in (Wang et al., 2007) is the fact that the drift dynamics can be decomposed in a gradient direction and a direction tangential to the constant level sets of a known Hamiltonian functions (see also Wang et al. (2003)). In the present paper, we seek to construct a damping controller based on a geometric decomposition of the drift dynamics. A similar decomposition was used in the context of passivity-based control design by Sira-Ramírez and Angulo-Núñez (1997) where the drift dynamics was decomposed into a dissipative, a non-dissipative and an invariant components with respect to a known storage function. In the proposed approach, the storage function is assumed to be unknown *a priori* and constructed using a geometric decomposition of the drift system.

More precisely, we propose to use the tools of exterior calculus to construct a dissipative potential for the system and design a stabilizing controller following the Jurdjevic–Quinn approach (see for example Malisoff and Mazenc (2009, Chapter 4) for a review). In Hudon et al. (2008), a local decomposition method based on homotopy is proposed. In particular, it is shown how a dissipative potential associated to the exact part of a one-form obtained by taking the interior product of a non-vanishing two-form with respect to the drift vector field can be computed.

In Hudon and Guay (2009), the approach is used for stabilization purposes. The problem of Lyapunov functions construction for the stabilization of time-independent nonlinear control affine systems satisfying Jurdjevic–Quinn conditions is considered. It is shown that a Lyapunov function can be computed for the closed-loop vector field subject to damping feedback control using the dissipative potential obtained from the aforementioned decomposition. Along this line, the objective of this paper is to study the potential of damping controllers for wastewater treatment stabilization. With that respect, as a first step, the present paper assumes full-state feedback and known model and parameters. For this paper, constraints are not considered explicitly in the design. Using simulations, the application of the proposed controller for stabilization in a neighborhood of the desired equilibrium is presented. Then, it is shown that the controller keeps the pollutant level at that equilibrium level when the inflow concentrations is increased.

The paper is organized as follows. Section 2 reviews the AM1 model system. In Section 3, the decomposition and damping feedback controller constructions from (Hudon et al., 2008; Hudon and Guay, 2009) is presented. A feedback controller for the AM1 model is constructed following this approach in Section 4. Numerical applications are given in Section 5 using the parameters from (Bernard et al., 2001). Conclusions and future areas of investigation are outlined in Section 6.

2. PROBLEM FORMULATION

We use the AM1 model of anaerobic digestion presented in (Bernard et al., 2001) and analyzed for control design purposes in (Antonelli et al., 2003) and (Grognard and Bernard, 2006). The model is given by

$$\dot{X}_1 = (\mu_1(S_1) - \alpha D)X_1 \tag{1}$$

$$\dot{X}_2 = (\mu_2(S_2) - \alpha D)X_2 \tag{2}$$

$$\dot{S}_1 = -k_1\mu_1(S_1)X_1 + D(S_{1,in} - S_1) \tag{3}$$

$$\dot{S}_2 = k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 + D(S_{2,in} - S_2), \tag{4}$$

where the state variables X_1, X_2, S_1, and S_2 are positive and represent the biomass concentrations and the pollutant concentrations in tank 1 and tank 2. The manipulate variable is the input flow rate D and $\alpha \in [0,1]$ represents the proportion of bacteria that are not fixed in the bed, *e.g.*, an ideal fixed bed would be modeled using $\alpha = 0$ while an ideal mixed stirred tank reactor would be modeled with $\alpha = 1$. The yield coefficients k_1, k_2, and k_3 are assumed to be known.

Following (Bernard et al., 2001), the microbial kinetics $\mu_1(S_1)$ and $\mu_2(S_2)$ are assumed to be of Monod and Haldane type, respectively:

$$\mu_1(S_1) = \frac{\mu_{1,max}S_1}{K_{S1} + S_1} \tag{5}$$

$$\mu_2(S_2) = \frac{\mu_{2,max}S_2}{K_{S2} + S_2 + \frac{S_2^2}{K_{I2}}}. \tag{6}$$

The above controlled dynamics is control affine, and can be written as

$$\dot{x} = f(x) + g(x)u \tag{7}$$

with $x = [X_1,\ X_2,\ S_1,\ S_2]^T$ where

$$f(x) = \begin{bmatrix} \mu_1(x_3)x_1 \\ \mu_2(x_4)x_2 \\ -k_1\mu_1(x_3)x_1 \\ k_2\mu_1(x_3)x_1 - k_3\mu_2(x_4)x_2 \end{bmatrix},\ g(x) = \begin{bmatrix} -\alpha x_1 \\ -\alpha x_2 \\ (x_{3,in} - x_3) \\ (x_{4,in} - x_4) \end{bmatrix}. \tag{8}$$

In the next section, we will construct a dissipative potential $\psi(x)$ using a local decomposition of the drift dynamics $f(x)$ centered at a desired equilibrium $x^* = [x_1^*,\ x_2^*,\ x_3^*,\ x_4^*]^T$. A damping feedback controller will then be design as

$$u(x) = -\kappa\nabla^T\psi(x) \cdot g(x). \tag{9}$$

3. DAMPING FEEDBACK STABILIZATION CONSTRUCTION

The construction of a local dissipative potential based on a homotopy decomposition, outlined in Section 3.1, is presented in Section 3.2. Due to space limitations, we omit reviews of exterior calculus on $\mathbb{R}^n$, presented for example in Edelen (2005) and summarized in Hudon et al. (2008) and Hudon and Guay (2009). We denote a smooth vector field as

$$X(x) = \sum_{i=1}^{n} v^i(x)\partial_{x_i} \tag{10}$$

and a smooth differential one-form as

$$\omega(x) = \sum_{i=1}^{n} \omega_i(x)dx_i, \tag{11}$$

where $v^i(x)$ and $\omega_i(x)$ are smooth functions on $\mathbb{R}^n$.

Consider a control affine system

$$\dot{x} = f(x) + \sum_{k=1}^{m} u_k g_k(x), \quad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^m, \tag{12}$$

for some $f, g_1 \cdots, g_m \in \mathcal{C}^\infty(\mathbb{R}^n;\mathbb{R}^n)$ and assume that $f(0) = 0$. Moreover, assume that for every $x \in \mathbb{R}^n \setminus \{0\}$,

$$\text{span}\{f(x), \text{ad}_f^k g(x),\ k \in \mathbb{N}\} = \mathbb{R}^n. \tag{13}$$

Consider the feedback law $u = (u_1, \dots, u_m)^T$ defined by

$$u_k = -\nabla^T\psi(x) \cdot g(x), \quad \forall\ k \in 1, \dots, m, \tag{14}$$

with $\psi(x)$ a weak Jurdjevic–Quinn function (Malisoff and Mazenc, 2009, Chapter 4), *i.e.*, such that $\psi(x) > 0$ and $(\nabla^T\psi \cdot g)(x) < 0$ for all x in a neighborhood $\mathcal{O} \subset \mathbb{R}^n \setminus \{0\}$, $\psi(0) = 0$ and $(\nabla^T\psi \cdot g)(0) = 0$.

With this feedback, one has for all $x \in \mathbb{R}^n \setminus \{0\}$

$$\frac{d\psi}{dt}(x) = f(x) \cdot \nabla\psi(x) - \sum_{k=1}^{m}(g_k(x) \cdot \nabla\psi(x))^2 < 0. \tag{15}$$

Therefore, the equilibrium $0 \in \mathbb{R}^n$ is asymptotically stable in closed-loop. The function $\psi(x)$ is not a control Lyapunov function (CLF) in general. In (Hudon and

Guay, 2009), a deformation approach of the function $\psi(x)$ was presented. We refer to (Malisoff and Mazenc, 2009, Definition 2.2) for CLF construction methods based on the prior knowledge of a function $\psi(x)$ satisfying the weak Jurdjevic–Quinn conditions. In the next section, we seek to use the drift vector field $f(x)$ structure to design the dissipative potential $\psi(x)$.

3.1 Homotopy Operator

We first show how to construct a radial homotopy operator $\mathbb{H}$, *i.e.*, a linear operator on elements of one forms on $\mathbb{R}^n$ that satisfies the identity

$$\omega = d(\mathbb{H}\omega) + \mathbb{H}d\omega, \tag{16}$$

for a given differential form ω.

The first step in the construction of a homotopy operator is to define a star-shaped domain on $\mathbb{R}^n$. An open subset S of $\mathbb{R}^n$ is said to be star-shaped with respect to a point $p^0 = (x_1^0, \ldots, x_n^0) \in S$ if the following conditions hold:

- S is contained in a coordinate neighborhood U of p^0.
- The coordinate functions of U assign coordinates $(x_1^0, \ldots, x_n^0)$ to p^0.
- If p is any point in S with coordinates $(x_1, \ldots, x_n)$ assigned by functions of U, then the set of points $(x + \lambda(x - x^0))$ belongs to S, $\forall \lambda \in [0, 1]$.

A star-shaped region S has a natural associated vector field $\mathfrak{X}$, defined in local coordinates by

$$\mathfrak{X}(x) = (x_i - x_i^0)\partial_{x_i}, \quad \forall x \in S. \tag{17}$$

For a differential form ω of degree k on a star-shaped region S centered at an equilibrium x^0, the homotopy operator is defined, in coordinates, as

$$(\mathbb{H}\omega)(x) = \int_0^1 \mathfrak{X}(\lambda(x_i - x_i^0)) \lrcorner \omega(x_i^0 + \lambda(x_i - x_i^0))\lambda^{k-1} d\lambda, \tag{18}$$

where $\omega(x_i^0 + \lambda(x_i - x_i^0))$ denotes the differential form evaluated on the star-shaped domain in the local coordinates defined above.

The important properties of the homotopy operator that are used in the sequel are the following:

(i) $\mathbb{H}$ maps $\Lambda^k(S)$ into $\Lambda^{k-1}(S)$ for $k \geq 1$ and maps $\Lambda^0(S)$ identically to zero.
(ii) $d\mathbb{H} + \mathbb{H}d =$ identity for $k \geq 1$ and $(\mathbb{H}df)(x) = f(x) - f(x_0)$ for $k = 0$.
(iii) $(\mathbb{H}\mathbb{H}\omega)(x_i) = 0, \quad (\mathbb{H}\omega)(x_i^0) = 0$.
(iv) $\mathfrak{X}\lrcorner\mathbb{H} = 0, \quad \mathbb{H}\mathfrak{X}\lrcorner = 0$.

The first part of the right hand side of (16), $d(\mathbb{H}\omega)$, is obviously a closed form, since $d \circ d(\mathbb{H}\omega) = 0$. Since by property (i) of the homotopy operator, for $\omega \in \Lambda^k(S)$, we have $(\mathbb{H}\omega) \in \Lambda^{k-1}(S)$, $d(\mathbb{H}\omega)$ is also exact on S. We denote the exact part of ω by $\omega_e = d(\mathbb{H}\omega)$ and the anti-exact part by $\omega_a = \mathbb{H}d\omega$. It is possible to show that ω vanishes on $\mathbb{R}^n$ if and only if ω_e and ω_a vanish together Edelen (2005).

From the decomposition outlined above, we have

$$\omega - \omega_a = \omega_e. \tag{19}$$

Taking the exterior derivative on both sides and using the fact that ω_e is closed, we have

$$d(\omega - \omega_a) = d(\omega_e) = 0. \tag{20}$$

In the sequel, we apply the homotopy operator on one-forms.

3.2 Construction of a Dissipative Potential

First, we define a non-vanishing closed two-form $\Omega(x)$ on $\mathbb{R}^n$ as

$$\Omega = \sum_{1 \leq i < j \leq n} dx_i \wedge dx_j. \tag{21}$$

In the present paper, the non-vanishing two-form Ω is not necessarily defined in a canonical way, since the objective is ultimately to compute an admissible dissipative potential (and not a minimal one). For example, if $n = 3$, we would have,

$$\Omega = dx_1 \wedge dx_2 + dx_1 \wedge dx_3 + dx_2 \wedge dx_3. \tag{22}$$

The orientation of the two-form will be fixed, if necessary, by checking the sign of the obtained dissipative function, $\psi(x)$.

We obtain a first one-form associated to the system by contracting this two-form with respect to the drift vector field,

$$\omega_0(x) = (f \lrcorner \Omega)(x). \tag{23}$$

From the above discussion, we know that we can locally construct a homotopy operator on $\mathbb{R}^n$ such that $\omega_0 = \omega_{0,e} + \omega_{0,a}$. Since $\omega_{0,e}$ is exact, it is given as the exterior derivative of a potential function and we rewrite

$$\omega_0 = d\psi + \omega_{0,a}. \tag{24}$$

We assume that $\psi(x)$, obtained after application of the homotopy operator (*i.e.*, $\psi(x) = (\mathbb{H}\omega_{0,e})$)(x), is such that $\nabla^T\psi(x) \cdot f < 0$ for $x \in \mathbb{R}^n \setminus \{0\}$. In practice, one may use an integrating factor $\gamma(x)$ to guarantee that

$$\psi(x) = (\mathbb{H}(\gamma\omega_0))(x) \tag{25}$$

has the desired properties. In the present paper, the anti-exact part that does not contribute locally to the dissipative dynamics is not taken into account for the design. In practice, a feedback gain κ is used to dominate the tangential dynamics, *i.e.*, we construct the damping feedback controller

$$u_k(x) = -\kappa(\nabla^T\psi \cdot g)(x). \tag{26}$$

However, if one was considering the problem of deforming $\psi(x)$ to assign a Lyapunov function for the closed-loop dynamics, it was shown in (Hudon and Guay, 2009) that $\omega_{0,a} \equiv 0$ has to hold locally by building an integrating factor. Essentially, this last condition is equivalent to the equality of mixed partial derivatives for the construction of storage function for dissipative systems.

In the next section, we apply this construction to the AM1 model.

4. APPLICATION TO AM1 MODEL

We denote the drift vector field $f(x)$ from Section 2 as $f(x) = \sum_{i=1}^{4}(f_i\partial_{x_i})(x)$. We define the two-form Ω as

$$\Omega = \sum_{1\le i\le j\le 4} dx_i \wedge dx_j. \tag{27}$$

Contracting this differential form with respect to $f(x)$, we obtain

$$\omega_0(x) = \sum_{i=1}^{4} \omega_i(x)dx_i, \tag{28}$$

where the functions ω_i, $i = 1, \ldots, 4$, are given as

$$\omega_1(x) = -(f_2 + f_3 + f_4)(x) \tag{29}$$
$$\omega_2(x) = (f_1 - f_3 - f_4)(x) \tag{30}$$
$$\omega_3(x) = (f_1 + f_2 - f_4)(x) \tag{31}$$
$$\omega_4(x) = (f_1 + f_2 + f_3)(x). \tag{32}$$

We re-organize $\omega_0(x)$ in terms of the kinetics expression, to obtain

$$\omega_0(x) = \phi_1(x_3)x_1\xi_1 + \phi_2(x_4)x_2\xi_2 \tag{33}$$

with

$$\begin{aligned}\xi_1 &= (k_1 - k_2)dx_1 + (1 + k_1 - k_2)dx_2 \\ &\quad + (1 - k_2)dx_3 + (1 + k_1)dx_4 \end{aligned}\tag{34}$$
$$\begin{aligned}\xi_2 &= (1 + k_3)dx_1 + (k_3)dx_2 \\ &\quad + (1 + k_3)dx_3 + dx_4. \end{aligned}\tag{35}$$

Before projecting the dynamics on a star-shaped domain centered at the origin, we re-labeled the state variables as $y_i = x_i - x_i^*$. Evaluating the one-form locally, we have

$$\begin{aligned}\tilde{\omega}_0 &= \lambda\left(\phi_1(x_3^* + \lambda y_3)x_1^* + \phi_2(x_4^* + \lambda y_4)x_2^*\right) \\ &\quad + \lambda^2\left(\phi_1(x_3^* + \lambda y_3)\tilde{\xi}_1 + \phi_2(x_4^* + \lambda y_4)\tilde{\xi}_2\right)d\lambda, \end{aligned}\tag{36}$$

with

$$\begin{aligned}\tilde{\xi}_1 &= (k_1 - k_2)y_1^2 + (1 + k_1 - k_2)y_1y_2 \\ &\quad + (1 - k_2)y_1y_3 + (1 + k_1)y_1y_4 \end{aligned}\tag{37}$$
$$\begin{aligned}\tilde{\xi}_2 &= (1 + k_3)y_1y_2 + k_3y_2^2 \\ &\quad + (1 + k_3)y_3y_3 + y_3y_4. \end{aligned}\tag{38}$$

The dissipative potential is thus computed as the homotopy of the sum of four terms that can be integrated symbolically:

$$\psi(y) = \mathbb{H}\omega_0 \tag{39}$$
$$= x_1^*\Psi_1 + x_2^*\Psi_2 + \Psi_3\tilde{\xi}_1 + \Psi_4\tilde{\xi}_2, \tag{40}$$

with

$$\Psi_1 = \int_0^1 \lambda\phi_1(x_3^* + \lambda y_3)d\lambda \tag{41}$$
$$\Psi_2 = \int_0^1 \lambda\phi_2(x_4^* + \lambda y_4)d\lambda \tag{42}$$
$$\Psi_3 = \int_0^1 \lambda^2\phi_1(x_3^* + \lambda y_3)d\lambda \tag{43}$$
$$\Psi_4 = \int_0^1 \lambda^2\phi_2(x_4^* + \lambda y_4)d\lambda. \tag{44}$$

Due to space constraints, the expressions for Ψ_i, $i = 1, \ldots, 4$ are not reported here.

The damping controller is given by

$$u(x) = -\kappa(\nabla^T\psi \cdot g)(y) \tag{45}$$
$$\begin{aligned}&= -\kappa_1\frac{\partial\psi}{\partial y_1}\cdot\alpha(y_1 + x_1^*) - \kappa_2\frac{\partial\psi}{\partial y_2}\cdot\alpha(y_2 + x_2^*) \\ &\quad - \kappa_3\frac{\partial\psi}{\partial y_3}\cdot\alpha(y_3 + x_3^* - x_{3,in}) \\ &\quad - \kappa_4\frac{\partial\psi}{\partial y_4}\cdot\alpha(y_4 + x_4^* - x_{4,in}). \end{aligned}\tag{46}$$

In practice the gains κ may be adjusted to adjust the convergence of the system to the desired equilibrium. In the next section, we present a numerical application of this controller.

5. NUMERICAL APPLICATION

We now present the numerical applications of the feedback controllers derived in the previous section. Simulation parameters are taken directly from (Bernard et al., 2001). Figure 1 shows the time trajectory of the closed-loop system initialized at $x_0 = [0.1,\ 0.1,\ 15\ 15]^T$. with $S_{1,in} = S_{2,in} = 15$. It shows stabilization of the system to a neighborhood of the equilibrium parameterized by $D^* = 0.27$ and $\alpha = 0.5$, given by $x^* = [0.67,\ 0.39,\ 0.92, 2.12]^T$. As mentioned earlier, no saturation and constraints were considered in this study. Figure 2 shows the manipulated variable trajectory.

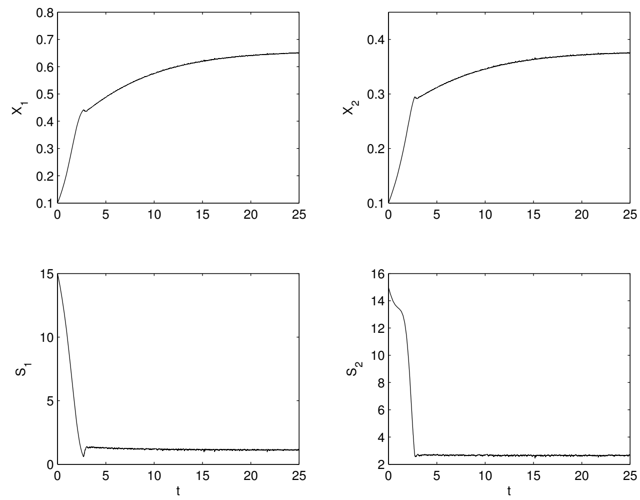

Fig. 1. Stabilization to a desired equilibrium

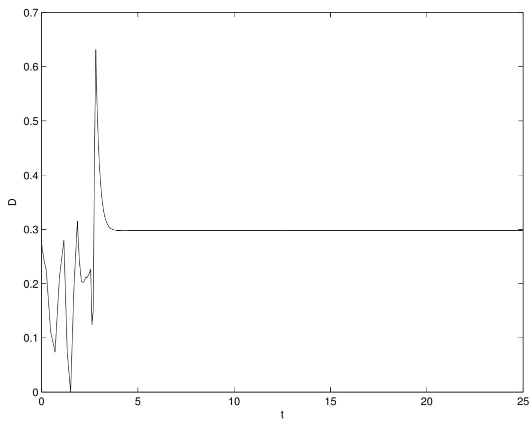

Fig. 2. Manipulated variable trajectory

An interesting aspect of the proposed dissipative feedback design methodology is depicted in Figure 3. In this case the value of both inlet flows concentration is increased from $S_{i,in} = 15$ to $S_{i,in} = 25$ at time $t = 25$. To achieve disturbance rejection, the gains κ_3 and κ_4 in Equation (46) have to be increased over κ_1 and κ_2. As a net result result, the biomass concentration increases to their new equilibrium value, while the pollutant concentrations S_1 and S_2 stay in a neighborhood of the desired equilibrium values.

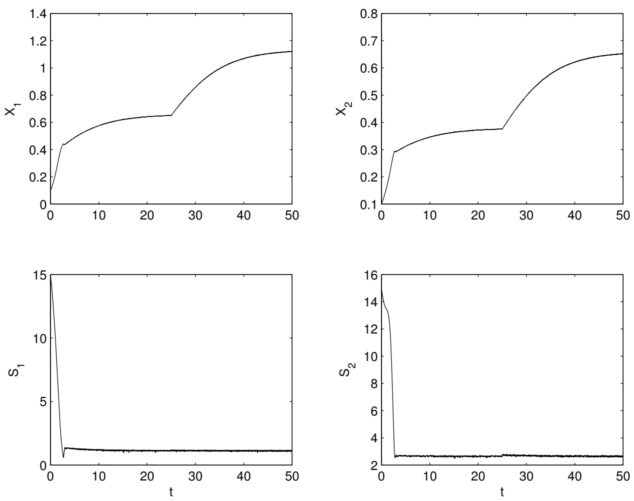

Fig. 3. State trajectories with inlet flows concentration disturbance at $t = 25$

6. CONCLUSION

In this paper, a procedure to construct stabilizing controllers using local homotopy-based decomposition of the drift vector field for watewater plant treatment was considered. Taking the interior product of a non vanishing two-form with respect to the drift vector field, we first obtained a non-closed characteristic one-form for the system. Constructing a locally defined homotopy operator on a star-shaped domain centered at the desired equilibrium point, we presented how to decompose locally the obtained form into an exact and an anti-exact one-forms. From (Hudon et al., 2008), we know that the exact part is associated to a dissipative (stable) potential. The obtained anti-exact form is associated to a non dissipative potential which generated tangential dynamics that do not contribute to the value of the dissipative potential locally on the star-shaped domain. A stabilizing controller was designed using the Jurdjevic–Quinn approach, following the construction developed in (Hudon and Guay, 2009). Application of the technique for stabilization of the desired equilibrium was presented. It was shown, through numerical simulations that input flows concentration disturbance could be handled by proposed controller. Future research will focus on incorporation of input constraints in the feedback scheme and robustness considerations with respect to parameter uncertainties.

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