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NMPC of a Continuous Fermenter Using Wiener-Hammerstein Model Developed from Irregularly Sampled Multi-rate Data

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Abstract: Control of a bio-reactor is a complex task due to inherent non-linearities and unavailability of measurements of the quality variables at regular sampling intervals. In this work, it proposed to identify Wiener-Hammerstein type fast-rate time series models for the quality variables directly from the irregularly sampled multi-rate input-output data. The identified models are further used to develop a multi-rate nonlinear predictive controller. The efficacy of the proposed modelling and control scheme is demonstrated by conducting simulation studies on a continuous fermenter system that exhibits input multiplicity and gain reversal in the desired operating region.

Keywords: Irregular sampling, Multi-rate systems, Hammerstein-Wiener model, Inferential Control, Non-linear Model Predictive Control

1. INTRODUCTION

Control of a biological processes is a complex task due to inherent non-linearities in the dynamic behaviour, uncertainties arising due to changes in the behaviour of the biomass and changes in the conditions. These difficulties are compounded due to the fact that many variables of interest, such as bio-mass concentration or certain product concentrations, are not measurable on-line. Even if these measurements are made available for feedback control through lab assays, these measurements are likely to be available at irregular sampling intervals. From the view point of control, such systems in which either measurements and / or manipulated inputs are available at regular / irregular but different sampling intervals are called multi-rate sampled data systems. With advances in digital control hardware, it is now possible to develop special approaches to deal with these multi-rate sampled data systems (Gudi et al. [1997]).

From the view point of control of bio-reactors, a subclass of multi-rate systems, in which measurements are available at irregular sampling rate but the manipulated input moves are made at a regular and fast sampling rate, are of particular interest. To achieve tight control of such processes, it is important generate reliable estimates of the monitored output at a fast rate and use these estimates in developing a control law. If a reasonably accurate unstructured mechanistic model is available for the bioprocess under consideration, then this can be achieved by constructing inter-sample estimates of the unmeasured variables using any standard state estimation technique such as extended Kalman filtering (EKF) or moving horizon estimation (MHE). However, developing a reliable dynamic model from the first principles can prove to be a difficult and time consuming task. Recently, Srinivasarao et al. [2007] have proposed to use nonlinear fast-rate time series models, which are identified from multi-rate input-output data, for improving control of multi-rate sampled data systems. This appears to be an attractive option as the time required for model development and, in turn, the cost of the model development exercise, is significantly less when compared with a mechanistic model.

If it is desired to develop a nonlinear time series model for a process, then the first step towards model building is selection of appropriate model structure. Block oriented models, such as Wiener or Hammerstein model, are probably the simplest and most popular form of nonlinear time series models used in the process control literature(Pearson and Ogunnaike [1997]). These models consist of a linear dynamic component and a static nonlinear component, which appears either at input of the linear dynamic component (Hammerstein structure) or at the output of the linear dynamic component (Wiener structure). Srinivasarao et al. [2007] have explored the use of only the Wiener type models in the multi-rate scenario. The linear dynamic component of these models is parametrized using orthonormal bass filters (OBF). In the present work, it is proposed to extend this approach to identify Hammerstein and Wiener-Hammerstein models, in which the static nonlinear blocks appear at the input as well as the output ends of the linear dynamic component, from the multi-rate input-output data. When estimates of the irregularly sampled variable become available at the fast rate, the problem that needs to be addressed next is the choice of appropriate control strategy. Over the last two decades, nonlinear model predictive control (NMPC) has emerged as the prime tool to handle the control problems associated with industrial systems exhibiting highly nonlinear dynamics(Qin and Badgwell [2003]). Thus, we proceed to use the identified Hammerstein / Wiener-Hammerstein

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models to develop a nonlinear predictive controller that can provide offset free closed loop behaviour.

This paper is organized in four sections. In the nest section the proposed nonlinear block oriented model structure and model parameter identification from an irregularly sampled multi-rate data are presented. In Section 3, an NMPC scheme is developed based on the identified model. Simulation studies on a continuous fermenter system are presented next.

2. DEVELOPMENT OF FAST-RATE BLOCK ORIENTED MODEL

Consider a continuously operated bio-process governed by a generalised unstructured model of the form

$$\frac{d\mathbf{z}}{dt} = \mathbf{F}\left[\mathbf{z}, \mathbf{u}_T(t), \mathbf{d}(t), \theta(t)\right] \tag{1}$$

$$\mathbf{y}(t) = \mathbf{G}\left[\mathbf{z}, \theta(t)\right] + \upsilon_y(t) \tag{2}$$

where $\mathbf{z} \in R^s$ represents state vector, $\mathbf{u}_T \in R^m$ represents the true value of manipulated inputs, $\mathbf{d} \in \mathbf{R}^d$ represents unmeasured disturbances, $\mathbf{y} \in R^r$ represents the vector of measured outputs corrupted with measurement noise $\upsilon_y(t)$ and $\theta \in R^\nu$ represents parameter vector. It is assumed that

- $\mathbf{u} \in R^m$ represents known (or computed) value of manipulated input which are related to the true values as follows
$$\mathbf{u}_T(t) = \mathbf{u}(t) + \upsilon_u(t) \tag{3}$$
where $\upsilon_u \in R^m$ denotes an unknown input disturbance, which is assumed to be a zero mean stationary signal.
- Variations of signals $\mathbf{d}(t)$ and process parameters $\theta(t)$ around their mean values, denoted as $\upsilon_d(t)$ and $\upsilon_p(t)$, respectively, can be represented as zero mean stationary stochastic processes and
$$\mathbf{d}(t) = \overline{\mathbf{d}} + \upsilon_d(t) \tag{4}$$
$$\theta(t) = \overline{\theta} + \upsilon_d(t) \tag{5}$$
where $\overline{\mathbf{d}}$ and $\overline{\theta}$ represent mean values, which remain unaltered through out the identification process, of these signals. But these can can change abruptly during operation of the plant but with a low frequency.
- the process has a fading memory or in other words does not contain any integrating/ unstable modes.
- the measured outputs are available irregularly such that the sampling intervals are integer multiple of the *shortest sampling time,* T.

It is further assumed that the plant is perturbed deliberately by injecting multi-level perturbations in the manipulated inputs and the input-output data is collected. The actuators are manipulated at discrete time instants $\{t_k = kT : k = 0, 1, 2, ...\}$ where sampling instants $\{t_k\}$ are called as *minor sampling instants*. The measurements of i^{th} output are assumed to be available at a slower rate only at sampling instants given by the sub-sequence $\{k_{i0}, k_{i1}, k_{i2}, ...\}$, called as *major sampling instants*, such that the difference $k_{il} - k_{il-1} = q_{il}$ (≥ 1) where q_{il} is an integer. For regularly sampled multi-rate system q_{il} is constant and independent of l, else, the system is an irregularly sampled system. Thus, the information available from the plant is the sampled sequence of input at fast rate, i.e. set $\mathcal{U}_N = \{\mathbf{u}(k) : k = 0, 1, 2, ...N\}$ and and the corresponding irregularly sampled multi-rate output data sets

$$\mathcal{Y}_{i,N} = \{y_i(k_l) : k_{i,l} = k_{i,1}, k_{i,2}, ..., k_{i,q}; k_{i,q} \leq N\}$$

for $i = 1, 2, ..., r$ collected from the plant where k_{il} represents major sampling instants. It may be noted that, while generating data set for model identification, measurement noises and plant disturbance are introduced through the vectors $\mathbf{d}$ and θ drift as given by equations (4) and (5), respectively.

2.1 Development of Fast Rate Block Oriented Models

It is desired to develop multiple (r) input single output (MISO) model for all the outputs of interest. Thus, to simplify the notation, sub-script i denoting the i'th output is dropped in this section. Given input-output data set $\Sigma_N = (\mathcal{Y}_N, \mathcal{U}_N)$, the identification problem can be formulated as identification of a non-linear operator $\Xi[\cdot]$

$$y(k_l) = \Xi\left[\varphi(k_l), \theta\right] + e(k_l) \tag{6}$$

in such a way that, a suitable norm of model residuals $\{e(k_l) : \forall k_l; k_l = k_0, k_1, k_2...k_n\}$ is minimized with respect to parameter vector θ. The regressor vector $\varphi[\cdot]$ can either be chosen as function of past known inputs alone (i.e. nonlinear output error or NOE model structure) or as a function of past inputs and the past output measurements (i.e. nonlinear ARX or NARX model structure) (Sjoberg et al. [1995]). The models with NOE structure are known to have excellent long range prediction ability and are ideal candidates for developing nonlinear predictive control schemes. Thus, we restrict ourselves to the development of NOE models from Σ_N.

Given Σ_N Srinivasarao et al. [2007] have proposed to develop a Wiener type fast-rate NOE model of the form

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) \tag{7}$$

$$y(k_l) = \Omega\left[\mathbf{x}(k_l)\right] + v(k_l) \tag{8}$$

Here, $\mathbf{x}(k) \in R^n$ represents the state vector updated at the fast rate and $\Omega[\cdot]$ represents a static map relating the states with the output. It may be noted that the measurement model (8) holds only at the major sampling instants. The matrices $(\mathbf{\Phi}, \mathbf{\Gamma})$ appearing in the linear dynamic component are parametrized using orthonormal basis filters (OBF), which represent an orthonormal basis for the set of strictly proper stable transfer functions (denoted as $\mathcal{H}_2$). Ninness and Gustafson [1997] have shown that a complete orthogonal set in $\mathcal{H}_2$ can be constructed as follows

$$F_k(z, \xi) = \frac{\sqrt{(1 - |\xi_k|^2)}}{(z - \xi_k)} \prod_{j=1}^{k-1} \frac{(1 - \xi_k^* z)}{(z - \xi_j)} \tag{9}$$

where $\{\xi_k : k = 1, 2, ...\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs. The nonlinear state output map $\Omega[\cdot] : R^n \rightarrow R$ was chosen to be simple multi-dimensional quadratic polynomial functions of the form(Srinivasarao et al. [2007])

$$\Omega\left[\cdot\right] = \mathbf{C}\,\mathbf{x}(k_l) + \mathbf{x}(k_l)^T\,\mathbf{D}\,\mathbf{x}(k_l) \tag{10}$$

This model is referred to as Wiener-OBF model in the rest of the text.

In the present work, we propose two modifications in the above structure

- **OBF-Hammerstein model**: One possibility is to introduce static non-linearity at input side in equation (7) as follows

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{\Lambda}\left[\mathbf{u}(k)\right] \tag{11}$$

$$\mathbf{\Lambda}(k) = \left[\, f^{(1)}\left[\mathbf{u}(k)\right]\; f^{(2)}\left[\mathbf{u}(k)\right]\; ...\; f^{(M)}\left[\mathbf{u}(k)\right]\right]^T \tag{12}$$

Here, $f^{(j)}\left[\mathbf{u}(k)\right] : R^m \rightarrow R \quad \text{for } j = 1, 2, ..., M$ represent some chosen continuous nonlinear functions. For example, a simplest choice can be a polynomial functions of the form

$$f^{(i)}\left[\mathbf{u}(k)\right] = \mathbf{u}_i(k) \tag{13}$$

$$f^{(m+l)}\left[\mathbf{u}(k)\right] = \mathbf{u}_i(k)\mathbf{u}_j(k) \tag{14}$$

$$\textit{for } i = 1, 2, ...m, \quad j = i, 2, ...m$$

$$l = 1, 2, ..., m(m+1)/2$$

In fact, $f^{(i)}\left[\mathbf{u}(k)\right]$ can be viewed as pseudo-linear input entering the state dynamics. The state to output map in this case is chosen to be linear as follows

$$\Omega\left[\,\mathbf{x}(k_l)\right] = \mathbf{C}\,\mathbf{x}(k_l) \tag{15}$$

In the literature on the development of Hammerstein models, the input non-linearity is often modelled as

$$\mathbf{\Lambda}\left[\mathbf{u}(k)\right] = \sum_{i=1}^{M} \alpha_i f^{(i)}\left[\mathbf{u}(k)\right]$$

where the parameters $\{\alpha_i\}$ are either estimated apriori based on some other considerations (Ljung [1999]) or as a as part of the model parameter estimation exercise (Gomez and Baeyens [2004]). The parameter estimation problem can become complex in the later case since the model residuals are complex functions of these parameters. On the other hand, the major advantage of formulating the Hammerstein model in this manner is that the parameter identification problem is reduced to estimating matrices $(\mathbf{\Phi}, \mathbf{\Gamma})$ and estimation of elements of vector $\mathbf{C}$, which is a liner parameter estimation problem.

- **Wiener-OBF-Hammerstein model:** This model is obtained by combining the OBF-Hammerstein model with Wiener-OBF model as follows

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{\Lambda}\left[\mathbf{u}(k)\right]$$

$$y(k_l) = \Omega\left[\,\mathbf{x}(k_l)\right] + v(k_l) \tag{16}$$

The state to output map can be chosen as a polynomial function, such as equation (10). With this choice, the state to output map at any major sampling instant can be expressed as follows

$$y(k_l) = \mathbf{\Theta}^T\,\chi(k_l) + v(k_l) \tag{17}$$

where

$$\chi(k_l) = \left[\,\mathbf{x}(k_l)^T\;\; F(k_l)^T\,\right]^T \tag{18}$$

$$F(k_l) = \left[\,(\mathbf{x}_1(k_l))^2\;\; 2\mathbf{x}_1(k_l)\mathbf{x}_2(k_l)\;\; ....\,\right]^T \tag{19}$$

and

$$\mathbf{\Theta} = \left[\,\mathbf{C}\;\; \mathbf{D}_{1,1}\;\; \mathbf{D}_{1,2}\;\; ....\;\; \mathbf{D}_{n,n}\,\right]^T \tag{20}$$

Here, $\mathbf{\Theta}$ is a $L \times 1$ vector with $L = n \times (n+3)/2$, $\mathbf{x}_i(k)$ represents i'th element of vector $\mathbf{x}(k_l)$ and $\mathbf{D}_{j,l}$ represents (j,l)'th element of matrix $\mathbf{D}$.

2.2 Estimation of Model Parameters

The estimation of OBF poles and the parameters of state-output map of the proposed OBF-Wiener model as well as the Wiener-OBF-Hammerstein model can be carried out using a nested optimization approach as proposed by Srinivasarao et al. [2007]. Thus, given an input-output data set Σ_N, the least squares estimate of the parameters can be obtained by solving the following minimization problem(Srinivasarao et al. [2007]).

$$(\widehat{\mathbf{\Theta}}, \widehat{\xi}) = \begin{array}{c}\arg\min\\ \xi\end{array} \frac{1}{q}\sum_{k=k_0}^{k_q}\left[\widehat{\mathbf{v}}(k_l, \widehat{\mathbf{\Theta}}(\xi)\right]^2 \tag{21}$$

subject to

$$|\xi_j| < 0 \;\; \textit{for}\, j = 1, 2....n \tag{22}$$

$$\widehat{\mathbf{\Theta}}(\xi) = \left[\overline{E}\left(\varphi(k_l)\varphi(k_l)^T\right)\right]^{-1}\overline{E}\left(\varphi(k_l)y(k_l)\right) \tag{23}$$

where $\overline{E}(\cdot)$ represents the expected value operator. Here, $\varphi(k_l) \equiv \mathbf{x}(k_l)$ for OBF-Hammerstein model and $\varphi(k_l) \equiv \chi(k_l)$ for Wiener-OBF-Hammerstein model.

3. MULTI-RATE NMPC FORMULATION

The model identification exercise yields r MISO models of the form

$$\mathbf{x}^{(i)}(k+1) = \mathbf{\Phi}^{(i)}\mathbf{x}^{(i)}(k) + \mathbf{\Gamma}^{(i)}\mathbf{\Lambda}^{(i)}\left[\mathbf{u}(k)\right]$$

$$\mathbf{y}_i(k_{i,l}) = \Omega^{(i)}\left[\mathbf{x}^{(i)}(k)\right] + \mathbf{v}_i(k_{i,l}) \tag{24}$$

where $i = 1, 2, ...r$, which are used for current state estimation and future set-point trajectory predictions. In this section, the predictive control formulation is developed using the Wiener-OBF-Hammerstein models as Wiener-OBF model and OBF-Hammerstein model form special cases of this generic model.

3.1 Current state and Inter-sample Output Estimation

Given initial state estimates $\{\widehat{\mathbf{x}}^{(i)}(0|-1) : i = 1, 2, .., r\}$ and input sequence $\{\mathbf{u}(k) : k = 0, 1, 2, ...\}$, on-line *open loop* state estimators can be constructed as follows

$$\widehat{\mathbf{x}}^{(i)}(k+1|k) = \mathbf{\Phi}^{(i)}\widehat{\mathbf{x}}^{(i)}(k|k-1) + \mathbf{\Gamma}^{(i)}\mathbf{\Lambda}^{(i)}\left[\mathbf{u}(k)\right]$$

Since the process is assumed to be open loop stable and poles of matrices $\left\{\mathbf{\Phi}^{(i)} : i = 1, 2, .., r\right\}$ are inside the unit circle by construction, $\widehat{\mathbf{x}}^{(i)}(0|-1) = \overline{0}$ can also serve as a reasonable initial guess. The MISO models can be used for inter-sample estimation of outputs at any minor sampling instant as follows

$$\widehat{\mathbf{y}}_i(k) = \Omega^{(i)}\left[\mathbf{x}^{(i)}(k)\right] \quad \textit{for} \quad k_{i,l-1} \leq k < k_{i,l} \tag{25}$$

$\widehat{\mathbf{y}}_i(k)$ represents an estimate of $\mathbf{y}_i(k)$ generated at the fast rate. Whenever a measurement becomes available for the i'th output (i.e. at a major sampling instant k_l), the the model residual can be estimated as follows

$$\mathbf{v}_i(k_{i,l}) = \mathbf{y}_i(k_{i,l}) - \widehat{\mathbf{y}}_i(k_{i,l}) = \mathbf{y}_i(k_{i,l}) - \Omega^{(i)}\left[\mathbf{x}^{(i)}(k_{i,l})\right] \tag{26}$$

3.2 Future Trajectory Predictions

In the multi-rate system under consideration, the control action is taken at every minor sampling instant k. When the above r MISO models are used to develop a predictive control scheme, it is unlikely that the operating conditions remain identical to those during the model identification exercise. The parameters of the bio-process under consideration may change with with time, which can result in bias between the estimated model outputs and measured process outputs. Thus, if it is desired to achieve offset-free control of the bio-reactor, it becomes necessary to take additional measures to account for such model-plant mismatch while developing a predictive control scheme. In this work, it is proposed to use a modified version of the dead-beat type disturbance observer to correct the future trajectory predictions. By this approach, given a set of future manipulated input sequence at the k'th instant

$$\mathcal{U}_{F,k} = \{\mathbf{u}(k|k), \mathbf{u}(k+1|k), ...\mathbf{u}(k+p-1|k)\} \tag{27}$$

the fast-rate future state predictions for the i'th model over a prediction horizon of p can be generated as follows:

$$\mathbf{x}^{(i)}(k+j+1|k) = \Phi^{(i)}\mathbf{x}^{(i)}(k+j|k) + \Gamma^{(i)}\mathbf{\Lambda}^{(i)}\left[\mathbf{u}(k+j|k)\right]$$

for $j = 1, 2, ...p$. The output predictions are generated by combining the dead-beat disturbance observer of the form

$$\widehat{\mathbf{y}}(k+j|k) = \widehat{\mathbf{y}}_i(k) = \Omega^{(i)}\left[\mathbf{x}^{(i)}(k+j|k)\right] + \mathbf{d}(k+j|k) \tag{28}$$

$$\mathbf{d}(k+j+1|k) = \mathbf{d}(k+j|k) \tag{29}$$

$$\mathbf{d}(k|k) = \mathbf{v}_f(k) \tag{30}$$

Here, k_l represents the last major sampling instant at which the measurement was available for the concern output and $\mathbf{v}_{f,i}(k_l)$ represent filtered model residual obtained by passing $\mathbf{v}_i(k_{i,l})$ through a unity gain filter

$$\mathbf{v}_{i,f}(k+1) = \lambda_i\mathbf{v}_{i,f}(k) + [1-\lambda_i]\,\overline{\mathbf{v}}_i(k)$$
$$\overline{\mathbf{v}}_i(k) = \mathbf{v}_{i,f}(k_{i,l}) \quad for \quad k_{i,l} \le k < k_{i,l+1}$$
$$\lambda_i = \exp\left[\frac{-T}{\tau_{i,f}}\right]$$

Here, $\tau_{i,f} > 0$ represent filter tuning parameter for the i'th output. It may be noted that the proposed scheme for model-plant mismatch compensation introduces integral action in the controller formulation.

In practice, the degrees of freedom available in choosing future manipulated input move for future trajectory manipulation (denoted as q and called as control horizon) are fewer than the prediction horizon (p). Moreover, these can be spread across the prediction horizon through input blocking as follows

$$\mathbf{u}(k+j|k) = \mathbf{u}(k|k) \;:\forall j; j = 1, ...m_1 - 1 \tag{31}$$

$$\mathbf{u}(k+j|k) = \mathbf{u}(k+m_1|k) \;:\forall j; j = m_1 + 1(1)m_2 - 1 \tag{32}$$

....

$$\mathbf{u}(k+j|k) = \mathbf{u}(k+m_{q-1}|k) \;:\forall j; j = m_{q-1} + 1(1)p - 1 \tag{33}$$

where m_j are selected such that

$$0 < m_1 < m_2 < .... < m_{q-1} \tag{34}$$

3.3 NMPC Formulation

Given a future set-point trajectory $\{\mathbf{r}(k+j|k) : j = 1, 2, ...p\}$, the future prediction error vector $\mathbf{e}_f(k+j|k)$ is defined as follows

$$\mathbf{e}_f(k+j|k) = \mathbf{r}(k+j|k) - \widehat{\mathbf{y}}(k+j|k) \tag{35}$$

In it's simplest form, the NMPC at the sampling instant k is formulated as a constrained optimization problem whereby the future manipulated input moves $\mathbf{u}(k|k), \mathbf{u}(k+1|k)......\mathbf{u}(k+m_{q-1}-1|k)$ are determined by minimizing an objective function

$$\min_{\mathbf{u}(k|k),...\mathbf{u}(k+m_{q-1}|k)} \sum_{i=1}^{p} \mathbf{e}_f(k+i|k)^T\mathbf{W}_E\mathbf{e}_f(k+i|k)$$
$$+\sum_{i=1}^{q-1}\left\{\Delta\mathbf{u}(k+i|k)^T\,\mathbf{W}_U\,\Delta\mathbf{u}(k+i|k)\right\}$$

subject to the following constraints

$$\mathbf{u}^L \le \mathbf{u}(k+i|k) \le \mathbf{u}^H \quad for \;\; i = 0, 1, .., q-1$$
$$\Delta\mathbf{u}^L \le \Delta\mathbf{u}(k+i|k) \le \Delta\mathbf{u}^H \quad for \;\; i = 0, 1, .., q-1$$

where

$$\Delta\mathbf{u}(k+i|k) = \mathbf{u}(k+i|k) - \mathbf{u}(k+i-1|k)$$

$\mathbf{W}_U$ (Input weighting matrix) and $\mathbf{W}_E$ (Output weighting matrix) are the tuning parameters, chosen by keeping process economics, importance of a particular input or output in mind. The resulting constrained optimization problem can be solved using any nonlinear programming technique. The controller is implemented in a moving horizon framework. Thus, after solving the optimization problem, only the first move $\mathbf{u}_{opt}(k|k)$ is implemented on the plant, i.e.

$$\mathbf{u}(k) = \mathbf{u}_{opt}(k|k)$$

and the optimization problem is reformulated at the next sampling instant based on the updated information from the plant.

4. SIMULATION STUDIES

4.1 Model Identification

To investigate the efficacy of the proposed non-linear identification schemes, simulation study was conducted on a continuously operated fermenter system. Production of ethanol by fermentation of glucose using saccharomyces cerevisiae yeast is a widely used fermentation process. A simplified version for such a continuous anaerobic fermenter system can be represented by the generalised model as described below (Henson and Seborg [1992])

$$\frac{dX}{dt} = -DX + \mu(P,S)X \tag{36}$$

$$\frac{dS}{dt} = D(S_F - S) - \frac{1}{Y_{X|S}}\mu(P,S)X \tag{37}$$

$$\frac{dP}{dt} = -DP + (\alpha\mu(P,S) + \beta)X \tag{38}$$

Here, X ($\equiv \mathbf{y}_1$) represents biomass or effluent cell mass concentration, S represents substrate concentration, P ($\equiv$

$\mathbf{y}_2$) represents the product concentration, D ($\equiv \mathbf{u}_1$) represents dilution rate and S_F ($\equiv \mathbf{u}_2$) represents the feed substrate concentration. The specific growth rate $\mu(P,S)$ exhibits both substrate and product inhibition and is given as

$$\mu(P,S) = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S^2}{K_i}} \tag{39}$$

The nominal model parameters used for simulation study are taken from Henson and Seborg [1992].

The maximum specific growth rate (μ_m) is sensitive to changes in operating conditions. In the present work, it is assumed that the plant parameter μ_m and the actuators fluctuate stochastically as reposted in Srinivasarao et al. [2007]. It may be noted that the continuous fermenter has very slow dynamics and open loop settling time is of the order of 40 hours. Thus, the *shortest sampling interval* (T) for model identification is chosen as 1 hour. The measurements of $\mathbf{X}$ and $\mathbf{P}$, which are sampled at uniformly distributed random intervals with minimum sampling interval of 1 hour and maximum sampling interval of 3 hours. It is further assumed that the measurements are corrupted with zero mean Gaussian white noise with standard deviations 0.075 and 0.225, respectively.

The optimum point, at which the biomass and product concentrations attain their maximum, is selected as the operating point of the fermenter. To generate the identification and validation data sets, both the inputs were simultaneously perturbed using multilevel random signal with standard deviations of $\sigma_{u_1} = 0.04$ and $\sigma_{u_2} = 6$, respectively. These signals are obtained by modifying the PRBS signals generated with switching time of 4 hours using the '*idinput*' function in the System Identification Toolbox of MATLAB. Model identification is carried out using data collected for 4000 hours of data. Additional 990 hours of data is used for dynamic model validation.

Model performance is compared on the basis of *Percentage prediction error(PPE)*, *Percentage estimation error(PEE)* (Srinivasarao et al. [2007]), *Akaike information criterion(AIC)* values (Pearson and Ogunnaike [1997]) and the steady-state behaviour.

Optimal pole location of the identified block oriented nonlinear models are reported in Table 2. The results(PPE and PEE values) of multi-rate model validation are summarized in Table 1. Fig. 3 compares the results of predictions obtained using proposed Hammerstein and Hammerstein-Wiener models with Wiener model developed by Srinivasarao et al. [2007] from irregularly sampled input-output data. For multi-rate model prediction efficiency, i.e. PPE values, of Hammerstein and Wiener models are comparable. However, there is significant improvement in prediction capability when process is modelled using Hammerstein-Wiener structure. Fig. 1 and Fig. 2 present the comparison of the steady state behaviour of the plant with the identified models for biomass concentration (X) and product concentration (P) with respect to dilution rate (D) and substrate concentration (S_F). Performance (Fig. 1) of the Hammerstein model was found to be poorest among the three. Between the remaining two identified models, Hammerstein-Wiener captures the steady state behaviour of the system over a wide operating range in a better way when compared with the Wiener model.

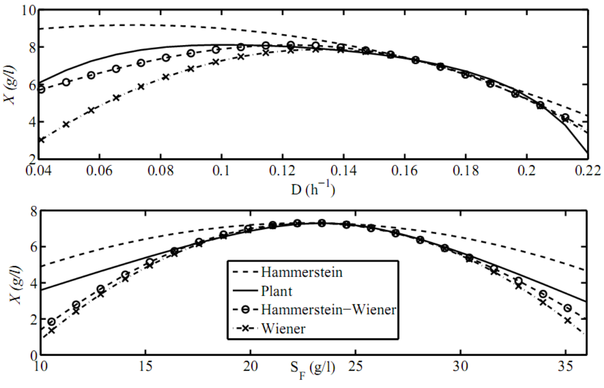

Fig. 1. Steady-state map of cell mass concentration(X)

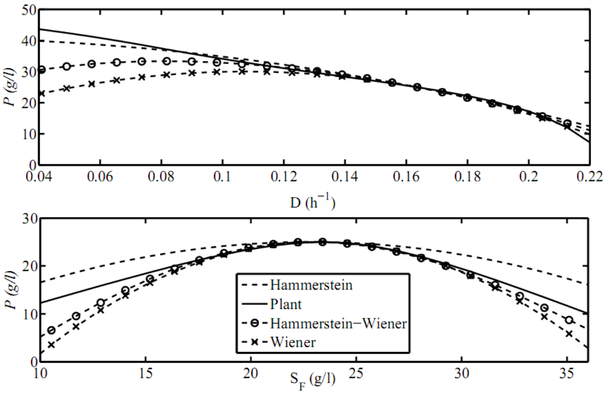

Fig. 2. Steady-state map of product concentration(P)

From Table 1, it can be observed that PEE values obtained using Hammerstein-Wiener model are significantly smaller than the PEE values obtained using Hammerstein or Wiener model for the multi-rate case. Despite of the fact that number of model parameters is high in case of Wiener-Hammerstein model, AIC value (Table 1) of the same is the lowest among all three. In addition to this, the identified Hammerstein-Wiener model generates excellent inter-sample predictions(Fig. 3).

Table 1. Dynamic model validation statistics

Output	Model	N_m	PPE	PEE	AIC
X	Wiener	18	14.5	14.037	−2866
X	Hammerstein	20	13.2	17.77	−2373
X	Hammerstein-Wiener	42	8.25	8.6481	−3747
P	Wiener	18	10.1	9.60	1898
P	Hammerstein	20	7.11	10.94	2173
P	Hammerstein-Wiener	42	5.32	5.632	930

4.2 Multi-rate Predictive Control

The identified Wiener Hammerstein model was further used to develop a multi-rate NMPC scheme. The performance of the NMPC scheme is evaluated for the following servo and regulatory changes:

- Positive step change(25%) in the plant parameter(μ_m)
- Shifting operation from a sub-optimal point to an optimal operating point.

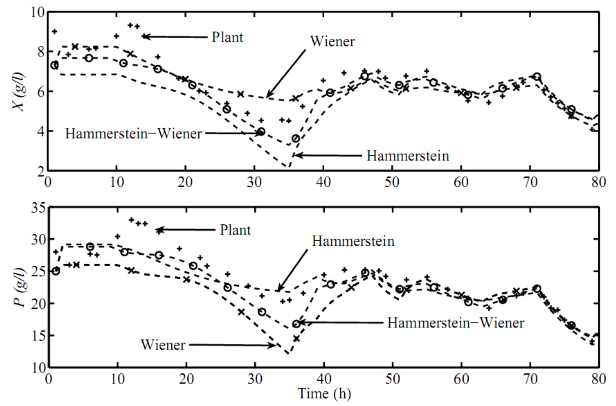

Fig. 3. Dynamic model validation for multi-rate model

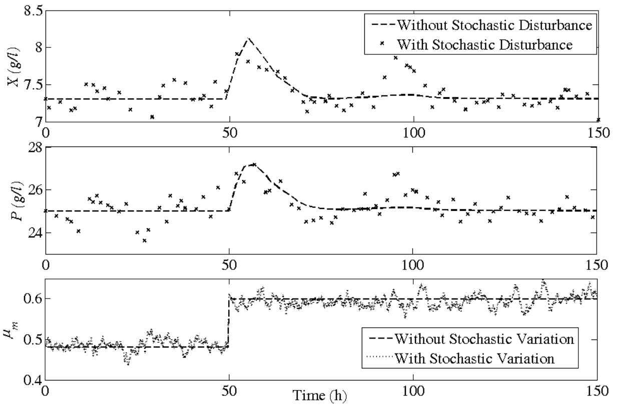

Fig. 4. Multi-rate regulatory control: Variation of controlled output and plant parameter (μ_m)

Dilution rate (D) and substrate concentration (S_F) are manipulated using the above mentioned NMPC scheme to control biomass (X) and product (P) concentrations. The tuning parameters used for controller tuning are as follows: $\mathbf{W}_E = diag[1\ 1]$ $\mathbf{W}_U = diag[0\ 0]$ $p = 20$ $q = 5$. NMPC scheme with equal input blocking($= [4\ 4\ 4\ 4\ 4]$) is implemented using Wiener-Hammerstein model along with the scenario that measurements are available at an irregular rate as discussed earlier. The multi-rate NMPC is found to be robust to step change in plant parameter μ_m (Fig. 4). The NMPC successfully moves the system from a sub-optimal operating point to the optimum operating point, which is singular point where gain changes sign, and maintain operation at the peak (Fig. 5).

5. CONCLUSION

In this work, it proposed to use Wiener-Hammerstein type fast-rate time series models for achieving a tight

Table 2. Optimum GOBF poles[2] of Hammerstein(I) and Wiener-Hammerstein(II) models

Input	Model I(X)	Model II(X)	Model I(P)	Model II(P)
$\mathbf{u}_1$	[4.2 10.0]	[5.3 14.5]	[5.81 10.1]	[7.7 14.5]
$\mathbf{u}_2$	[2.1 0.1]	[7.19 7.2]	[13.9 13.9]	[8.1 8.1]
$\mathbf{u}_1^2$	[9.6 9.6]	[16.8]	[10.9 10.9]	[19.8]
$\mathbf{u}_2^2$	[6.3 6.3]	[6.3]	[13.4 3.1]	[10.8]
$\mathbf{u}_1 u_2$	[11.7 1.6]	[18.1]	[7.3 7.3]	[20.2]

[2] *Discrete-time poles(ζ) are reported in terms of equivalent continuous-time time constants(τ), where $\zeta = \exp(-T/\tau)$.*

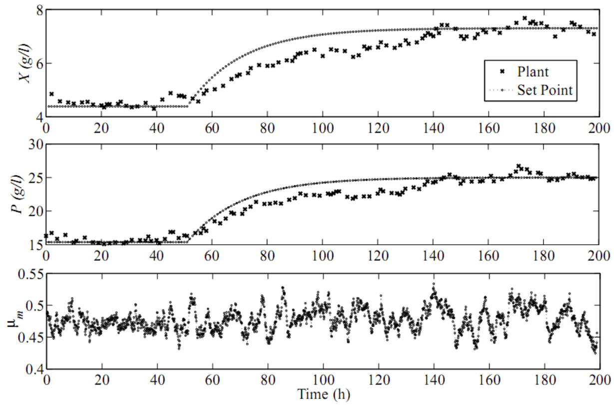

Fig. 5. Multi-rate servo control: Variation of controlled output and plant disturbance

control of a continuous fermenter. The efficacy of the proposed modelling and control scheme is demonstrated by conducting simulation studies on a continuous fermenter system that exhibits input multiplicity and gain reversal in the desired operating region. The modelling studies reveal that the proposed Wiener-Hammerstein model has better dynamic and steady state prediction capability when compared with Wiener or Hammerstein model. The multi-rate NMPC scheme developed using the Wiener-Hammerstein model produces satisfactory servo and regulatory responses.

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