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Offset-free Nonlinear Model Predictive Control Based on Moving Horizon Estimation for an Air Separation Unit ⋆

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Abstract: Air separation units (ASU) pose a classic problem for nonlinear system control. This paper proposes a framework that integrates nonlinear model predictive control (NMPC) and moving horizon estimation (MHE). We prove that the proposed method achieves offset free regulatory behavior, even in the presence of plant-model mismatches. If the plant uncertainty structure is known, the proposed framework can be modified to estimate the uncertainty parameters. Thus, the model used in the NMPC and MHE can be adaptively modified online. Finally, the proposed method is applied on a large scale air separation unit, and the steady state offset free behavior is observed.

1. INTRODUCTION

Electricity is not readily stored and must be used or wasted after production. Therefore, power plants must be able to ramp up and down frequently to meet the fluctuating power demand. Huang et al. [2009b] reported a successful application of nonlinear model predictive control (NMPC) based on a rigorous dynamic model for an air separation unit (ASU) in an Integrated Gasification Combined Cycle (IGCC) power plant. It is shown by Huang et al. [2009b] that the NMPC strategy is able to steer the ASU during the ramping processes and demonstrates superior performance against linear MPC. This study is based on the assumption that all the states of the ASU process are measured. In practical applications, however, state information is usually not completely available for measurement. Thus, a state estimator is normally applied to reconstruct the state information from the plant output.

In practice, disturbances and modeling errors are usually present and often not predictable. Furthermore, as time progress or operating conditions change drastically, they develop into non-vanishing plant-model mismatches. Thus, it is essential to ensure controller's performance in the presence of the plant-model mismatches. Offset free regulatory behavior for plant outputs is an important requirement in control applications. In order to achieve offset free behavior, several strategies based on state augmentation for linear systems have been proposed by Pannocchia and Kerrigan [2005] and Pannocchia and Bemporad [2007]. Moreover, a target state resetting strategy is proposed by Rajamani et al. [2009]. For nonlinear systems, Meadows and Rawlings [1997] describes a conventional offset free (N)MPC method which adds an output disturbance into the objective function for the entire predictive horizon. The output disturbance is generated by comparing the measured plant output to the model prediction at the current time step. Huang et al. [2009a] proposed an offset free NMPC formulation that integrates both the state and output disturbances from nonlinear extended observers, such as extended Kalman filter and extended Luenberger observer. In addition, the proposed method yields better disturbance-rejecting performance compared to the conventional method and it can be applied to open-loop unstable systems.

In this work, we propose to extend the NMPC formulation to integrate Moving Horizon Estimation (MHE) as the state estimator. It can be shown that the proposed method will yield offset free steady state control behavior even when there are non-vanishing plant-model mismatches. Then the proposed method with rigorous dynamic model is applied to the large scale air separation unit in Huang et al. [2009b]. We see that the NMPC and MHE successfully regulate the outputs of the unit at the desired set points without any offset.

2. MOVING HORIZON ESTIMATION AND NONLINEAR MODEL PREDICTIVE CONTROL

This work considers a general nonlinear dynamic system with possible plant-model mismatches,

$$x_{k+1} = f(x_k, u_k, \theta_k) \quad (1a)$$
$$y_k = h(x_k), \quad k \geq 0 \quad (1b)$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, $y_k \in \mathbb{R}^{n_y}$ and $\theta_k \in \Omega_\theta \subset \mathbb{R}^{n_\theta}$ are the plant states, controls, outputs and uncertainty parameters, respectively, defined at time steps $k \geq 0$, and Ω_θ is a compact set. Here if $\theta_k = 0$, equation (1) represents the nominal plant. Without losing generality, we assume that the given plant (1) has an equilibrium point at the origin, that is $f(0,0,0) = 0$. Moreover, no trajectory of this system exhibits finite escape time.

In this section, two formulations that integrate the NMPC and MHE are proposed in the sequel.

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2.1 *Formulation with State and Output Disturbances*

In this scenario, we assume that the uncertainty structure is unknown. Hence, a nominal value 0 for the uncertainty parameter is used in the predictive model for both MHE (2a) and NMPC (3a). To compensate for the plant-model mismatch, state and output disturbances are used.

At time step k, the MHE problem based on the nominal uncertainty value is formulated as

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{N_e} (\zeta_{k-N_e+j}^T \Pi_y \zeta_{k-N_e+j} + \xi_{k-N_e+j}^T \Pi_x \xi_{k-N_e+j}) \\
& + (\hat{x}_{k-N_e} - \bar{x}_{k-N_e})^T \Pi_0 (\hat{x}_{k-N_e} - \bar{x}_{k-N_e}) \\
\text{s.t.} \quad & \hat{x}_{k-N_e+j+1} = f(\hat{x}_{k-N_e+j}, u_{k-N_e+j}, 0) + \xi_{k-N_e+j} \quad (2a) \\
& \hat{y}_{k-N_e+j} = h(\hat{x}_{k-N_e+j}) \quad (2b) \\
& \zeta_{k-N_e+j} = y_{k-N_e+j} - \hat{y}_{k-N_e+j} \quad (2c) \\
& \hat{x}_{k-N_e+j} \in \mathbb{X}, \zeta_{k-N_e+j} \in \Omega_\zeta, \xi_{k-N_e+j} \in \Omega_\xi \quad (2d) \\
& j = 0, \ldots, N_e - 1, \quad (2e)
\end{aligned}
$$

where N_e is the estimation horizon length, Π_y, Π_x, Π_0 are symmetric positive definite tuning matrices, $\hat{x}_k$, $\hat{y}_k$ are the estimated state and output values, and ξ_k, ζ_k are the state and output disturbances which are assumed to be bounded in compact sets Ω_ζ and Ω_ξ, respectively. In addition, $\bar{x}_{k-N_e}$ is the most likely *prior* value of x_{k-N_e}. After the MHE problem is solved, we choose $\hat{x}_{k-Ne+1}$ as the prior value $\bar{x}_{k-N_e+1}$ for the arrival cost at time step $k+1$.

Note that though we consider a noise free plant (1) for notational simplicity, the proposed MHE and NMPC can easily incorporate the state and output noises. In addition, state noise can be considered as a special form of the uncertainty parameter θ.

With the estimated states ($\hat{x}_k$) and the state, output disturbances (ξ_k, ζ_k) at the current time step k, the NMPC with state and output disturbances is formulated as:

$$
\begin{aligned}
\min \quad & \sum_{j=0}^{N_p} (l_{k+j} - y_r)^T \Gamma_y (l_{k+j} - y_r) + \sum_{i=0}^{N_c-1} \triangle v_{k+i}^T \Gamma_u \triangle v_{k+i} \\
\text{s.t} \quad & z_{k+j+1} = f(z_{k+j}, v_{k+j}, 0) + \xi_k, \;\; j = 0, \ldots, N_p - 1 \quad (3a) \\
& z_k = \hat{x}_k, \; z_{k+j} \in \mathbb{X} \quad (3b) \\
& l_{k+j} = h(z_{k+j}) + \zeta_k, \; \triangle v_{k+i} = v_{k+i+1} - v_{k+i} \quad (3c) \\
& v_{k+j} = v_{k+t_i} \text{ for } t_i \le j < t_{i+1}, \; v_{k+i} \in \mathbb{U}, \quad (3d) \\
& t_0 = 0 \le t_1 \le t_2 \le, \ldots, \le N_p - 1 \quad (3e)
\end{aligned}
$$

where N_p, N_c are the prediction and control horizon length respectively, z_k, l_k, v_k are the predicted state, output and control movement, respectively. The estimated state is used as the initial condition in the NMPC problem. In typical NMPC applications, fewer degrees of freedom are available for the control movement. The last two constraints indicate that the control action is the input blocking form, ensuring that the available degrees of freedom spread over the entire prediction horizon. After the NMPC problem is solved, the first manipulated variable v_k is injected into the plant, i.e. $u_k = v_k$.

Note that unlike conventional NMPC formulation, the predictive model is perturbed by the state and output disturbances (ξ_k and ζ_k) to compensate for the unknown uncertainty parameter θ_k. It is worth mentioning that ζ_k and ξ_k, which are the calculated value at the end of the estimation horizon in MHE, are parameters in the NMPC problem.

Now, we pursue the analysis to show that the proposed method yields zero steady state offset. The analysis is similar to that in Meadows and Rawlings [1997], but does not depend on the target setting optimization problem.

Theorem 1. If

(1) the set point y_r is feasible for the perturbed predictive model $z_{k+j+1} = f(z_{k+j}, v_{k+j}, 0) + \xi_k$ and $l_{k+j} = h(z_{k+j}) + \zeta_k$,
(2) the NMPC controller (3) is asymptotically stabilizing for the perturbed predictive model,
(3) the closed-loop system goes to a steady state,
(4) the perturbed predictive model is observable at the steady state,

then the system controlled by the MHE (2) and the NMPC (3) has zero steady state offset.

Proof: In the following analysis, the super script *ss* denotes the steady state value. Since y_r is reachable for the perturbed predictive model and the NMPC control law is asymptotically stable, the stage cost in NMPC (3) is zero at steady state, i.e.

$$l^{ss} = y_r. \quad (4)$$

Moreover, at the steady state, the predictive state remains constant, $z^{ss} = \hat{x}^{ss}$. The control action is also a constant, i.e. $u^{ss} = v^{ss}$. Thus the following equations hold true,

$$
\begin{aligned}
\hat{x}^{ss} &= f(\hat{x}^{ss}, u^{ss}, 0) + \xi^{ss}, \quad (5a) \\
l^{ss} &= h(\hat{x}^{ss}) + \zeta^{ss}. \quad (5b)
\end{aligned}
$$

In addition, at the steady state the MHE evolves according to

$$
\begin{aligned}
\hat{x}^{ss} &= f(\hat{x}^{ss}, u^{ss}, 0) + \xi^{ss}, \quad (6a) \\
\hat{y}^{ss} &= h(\hat{x}^{ss}), \quad (6b) \\
\zeta^{ss} &= y^{ss} - \hat{y}^{ss}. \quad (6c)
\end{aligned}
$$

Since the predictive model in the MHE (6a) is exactly the same as that in the NMPC (5a), by combining equations (5b), (6b) and (6c), we see

$$l^{ss} = y^{ss}. \quad (7)$$

Then by virtue of equations (4) and (7), the following equation can be derived

$$y^{ss} = y_r. \quad (8)$$

It indicates that the plant output equals to the set point at the steady state. ■

Remark 1. If the set point y_r is not feasible, then the proposed approach minimizes the steady state output difference, i.e. $(y^{ss} - y_r)^T \Gamma_y (y^{ss} - y_r)$.

Remark 2. Note that in this analysis, the observer is in a general formulation (6). Thus, the Theorem also applies to the NMPC incorporated with the nonlinear recursive observers.

2.2 *Formulation with State and Parameter Estimation*

In this scenario, we assume that the uncertainty parameter structure is known. Then instead of compensating for the uncertainty by state and output disturbances, MHE can estimate both the state and uncertainty together. Thus the model for the MHE and the NMPC is modified adaptively. At time step k, the MHE is formulated as:

$$\min \sum_{j=1}^{N_e} (\zeta^T_{k-N_e+j}\Pi_y\zeta_{k-N_e+j}) + \hat{\theta}^T_k\Pi_\theta\hat{\theta}_k$$

$$+ (\hat{x}_{k-N_e} - \bar{x}_{k-N_e})^T\Pi_0(\hat{x}_{k-N_e} - \bar{x}_{k-N_e})$$

$$\text{s.t.}\quad \hat{x}_{k-N_e+j+1} = f(\hat{x}_{k-N_e+j}, u_{k-N_e+j}, \hat{\theta}_k) \tag{9a}$$

$$\hat{y}_{k-N_e+j} = h(\hat{x}_{k-N_e+j}) \tag{9b}$$

$$\zeta_{k-N_e+j} = y_{k-N_e+j} - \hat{y}_{k-N_e+j} \tag{9c}$$

$$\hat{x}_{k-N_e+j} \in \mathbb{X}, \zeta_{k-N_e+j} \in \Omega_\zeta, \hat{\theta}_k \in \Omega_\theta \tag{9d}$$

$$j = 0, \ldots, N_e - 1, \tag{9e}$$

where $\hat{\theta}_k$ is the estimated uncertainty parameter which is bounded in a compact set Ω_θ. Since this MHE formulation smoothes the uncertainty parameter over the entire estimation horizon, the estimated state is also smoothed over the horizon.

Consequently, the NMPC is able to adaptively update the predictive model using the estimated parameter.

$$\min \sum_{j=0}^{N_p} (l_{k+j} - y_r)^T\Gamma_y(l_{k+j} - y_r) + \sum_{j=0}^{N_c-1} \triangle v^T_{k+j}\Gamma_u\triangle v_{k+j}$$

$$\text{s.t}\quad z_{k+j+1} = f(z_{k+j}, v_{k+j}, \hat{\theta}_k),\ \ j = 0, \ldots, N_p - 1 \tag{10a}$$

$$z_k = \hat{x}_k,\ z_{k+j} \in \mathbb{X} \tag{10b}$$

$$l_{k+j} = h(z_{k+j}) + \zeta_k,\ \triangle v_{k+j} = v_{k+j+1} - v_{k+j} \tag{10c}$$

$$v_{k+j} = v_{k+t_i} \text{ for } t_i \le j < t_{i+1},\ v_{k+j} \in \mathbb{U}, \tag{10d}$$

$$t_0 = 0 \le t_1 \le t_2 \le, \ldots, \le N_p - 1. \tag{10e}$$

To show that the formulation with state and parameter estimation (MHE (9) and NMPC (10)) provides zero steady state offset, an analysis similar to Theorem 1 can be performed. It is omitted here for the sake of brevity.

Similar to the analysis in Theorem 1, we can show that the NMPC and MHE with parameter estimation (MHE (9) and NMPC (10)) is able to provide offset-free control behavior. The proof follows in the same way as that in Theorem 1, and is omitted here.

2.3 *Illustrative CSTR Example*

We consider a simulated NMPC scenario with a nonlinear continuous stirred tank reactor (CSTR) developed by Hicks and Ray [1971]. The CSTR is represented by the following differential equations:

$$\frac{dz_c}{dt} = (z_c - 1)/u_2 + k_0 z_c \exp(-E_a/z_T) \tag{11a}$$

$$\frac{dz_T}{dt} = (z_T - z^f_T)/u_2 + k_0 z_c \exp(-E_a/z_T) + \nu u_1(z_T - z^{cw}_T). \tag{11b}$$

This system involves two states $z = [z_c, z_T]$ corresponding to dimensionless concentration and temperature, and two manipulated inputs, corresponding to the cooling water flow rate u_1 and the inverse of dilution rate u_2. The model parameters are $z^{cw}_T = 0.38$, $z^f_T = 0.395$, $E_a = 5$, $\nu = 1.95 \times 10^4$ and k_0 is an uncertainty parameter in the plant with nominal value $k_0 = 300$ in the model. The system is operated at steady state $z_{ss} = [0.12466, 0.74068]$ corresponding to $u_{ss} = [378, 20]$. In this simulation, it is assumed that both the states are measured, i.e. the output mapping function in (1b) is chosen to be $h(\cdot) = \text{diag}[1,1] \times [z_c, z_T]^T$.

This model is regulated by the framework with both proposed methods. The horizon of the MHE, N_e is chosen as 6 with

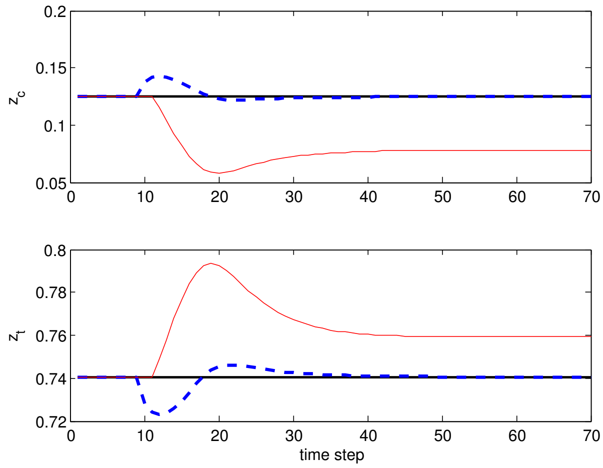

Fig. 1. State profile in scenario 1, the blue dashed line is the plant, the red line is the estimated value, the black line is the set point

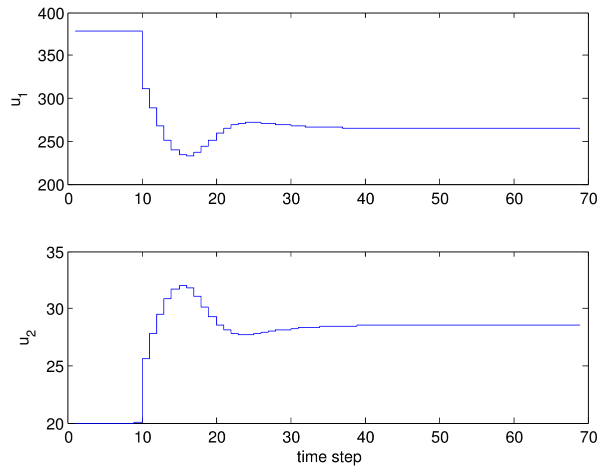

Fig. 2. Control profile in scenario 1

sampling time 1. Let $Q = \text{diag}[1,1]$ and $R = \text{diag}[1,1]$, and define $A_{ss} = \frac{\partial f}{\partial z}|_{z_{ss},u_{ss}}$, $B_{ss} = \frac{\partial f}{\partial u}|_{z_{ss},u_{ss}}$, $C_{ss} = \frac{\partial h}{\partial z}|_{z_{ss}}$ and $V_{ss} = R + C_{ss}QC^T_{ss}$. The weighting matrices are chosen to be inverse of the covariance information which is calculated similar to the extended Kalman filter, i.e., $\Pi_0^{-1} = A_{ss}QA^T_{ss} + B_{ss}QB^T_{ss} - A_{ss}QC^T_{ss}V^{-1}_{ss}C_{ss}QA^T_{ss}$, $\Pi_x^{-1} = Q$, $\Pi_y^{-1} = R$ and $\Pi_\theta = 0$. The NMPC is tuned with prediction horizon $N_p = 10$, control horizon $N_c = 5$, and the tuning matrices $\Gamma_y = \text{diag}[10^6,\ 10^6]$, $\Gamma_u = 0$.

In the first simulation, the plant is controlled by the formulation with state and output disturbances (MHE (2) and NMPC (3)). The plant starts from the nominal steady state value. At time step 10, the uncertainty parameter k_0 is reduced to 70% of its nominal value as shown at the bottom of Figure 3. The resulting closed-loop responses are shown in Figures 1 and 2. We see that the state estimates are biased after the plant-model mismatch is introduced. However, the proposed method is able to regulate the plant outputs at the desired set points.

The second simulation scenario is the same as the first one, except that the plant is regulated by the formulation with state and parameter estimation (MHE (9) and NMPC (10)). As shown at the bottom of Figure 6, the estimated uncertainty

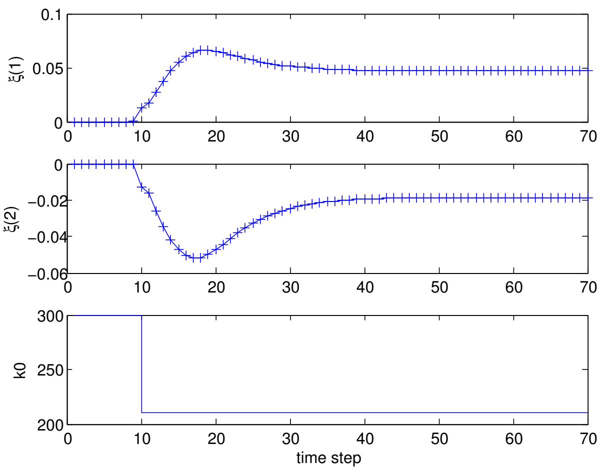

Fig. 3. Error profile in scenario 1

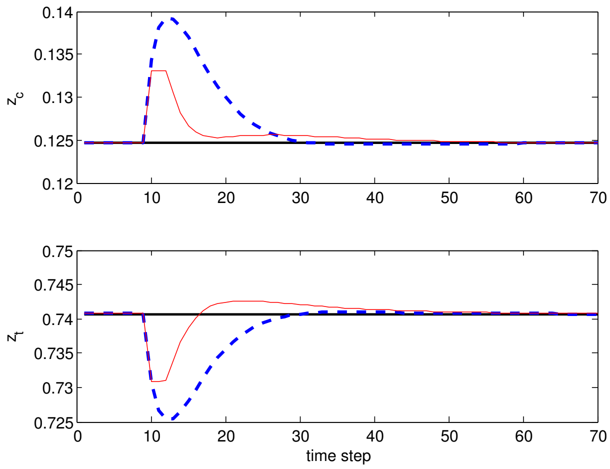

Fig. 4. State profile in scenario 2, the blue dashed line is the plant, the red line is the estimated value, the black line is the set point.

gradually converges to the plant value after the plant-model mismatch is introduced at time step 10. Thus, after time step 20, the uncertainty parameter in the model equals to that in the plant, removing the plant-model mismatch. Figure 4 shows that the proposed method quickly rejects the disturbance and yields the offset free control behavior. In addition, the estimated states converge to the measured plant states after the plant-model mismatch is eliminated.

It is interesting to compare Figure 1 and 4 to see that the formulation with parameter estimation (equations (9) and (10)) rejects the plant-model mismatch faster than the formulation with state and output disturbances (equations (2) and (3)). Therefore, it is recommend to estimate the state and parameter together if the uncertainty structure information is available.

3. ASU SIMULATION

In this study, we consider an industrial size air separation unit in IGCC power plants, reported in Huang et al. [2009b]. The unit contains two integrated cryogenic distillation columns as shown in Figure 7. The high pressure column (bottom) has 40 trays and operates at 5-6 bars, while the low pressure column (top) operates at 1-1.5 bars and also has 40 trays. An air feed flow is split into two substreams. The high pressure air (MA)

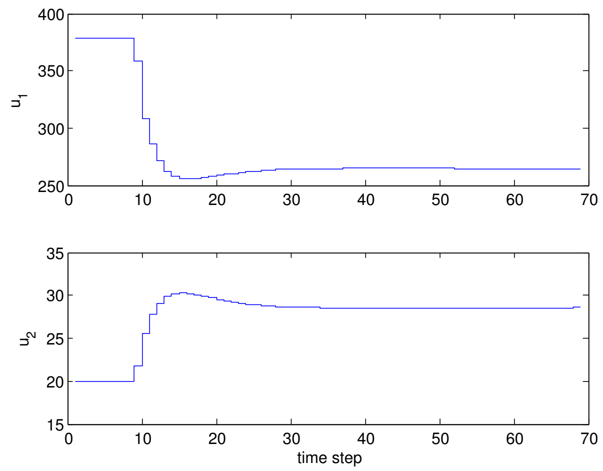

Fig. 5. Control profile in scenario 2

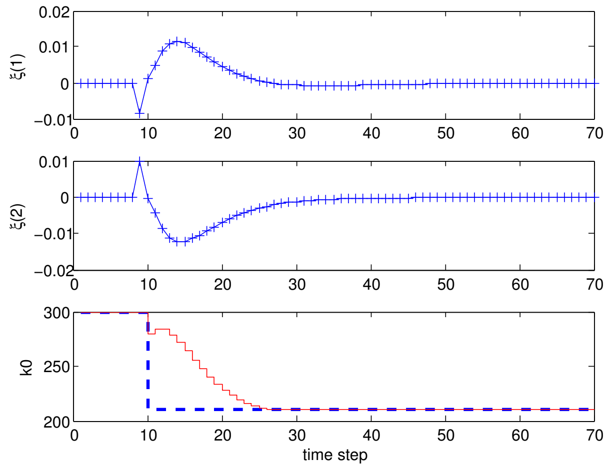

Fig. 6. Error profile in scenario 2, the blue dashed line is the plant, the red line is the estimated value.

enters the bottom of the high pressure column and the expanded air (EA) enters the 20^{th} tray of the low pressure column . Crude nitrogen gas (GN) from the main heat exchanger is also added to the 25^{th} tray of the high pressure column. The reboiler of the low pressure column is integrated with the condenser of the high pressure column. The main products of the high pressure column are pure nitrogen (PNI) ($> 99.99\%$) and crude liquid oxygen ($\sim 50\%$). The crude oxygen stream is fed into the 19^{th} tray of the low pressure column. In addition, an intermediate side stream from the 15^{th} tray of the high pressure column (LN) is fed into the top of the low pressure column. A high purity separation is achieved in the low pressure column, leading to nitrogen gas with $\sim 99\%$ purity and oxygen (POX) with $\sim 97\%$ purity as products.

The ASU model is represented by tray-by-tray equations consisting of mass balances (overall and component balances of nitrogen, oxygen and argon), energy balances, phase equilibrium, hydraulic and summation equations.

Overall Mass Balance

$$\frac{dM_i}{dt} = L_{i-1} + V_{i+1} - L_i - V_i + F_i \tag{12}$$

where i is the index of each tray, starting from the top of the column. M_i is the liquid mole holdup ($[mol]$) on tray i, L_i and V_i are liquid and vapor molar flow rates, respectively and F_i is the

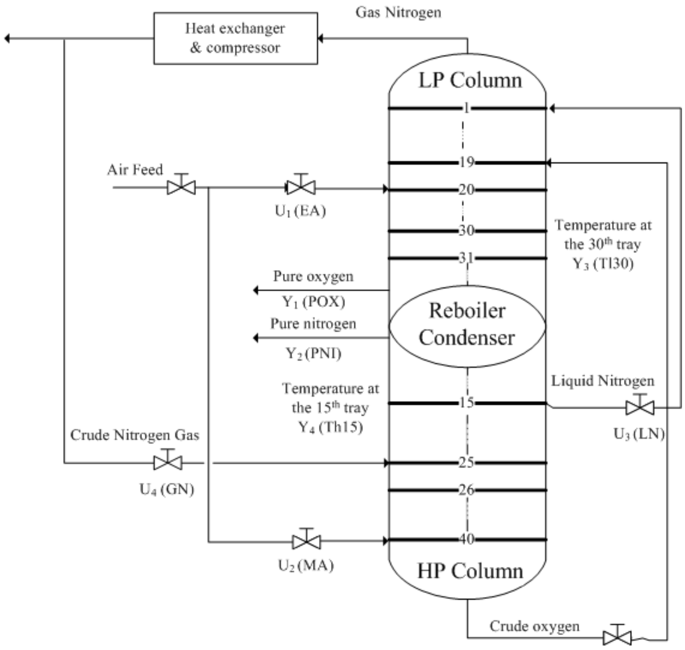

Fig. 7. ASU flow sheet.

molar feed ($[\frac{mol}{min}]$). If there is no feed to tray i, then $F_i = 0$. In this case, the only nonzero values of F_i are those corresponding to expanded air (EA, U_1), pure air (MA, U_2), liquid nitrogen (LN, U_3), crude gas nitrogen (GN, U_4) and crude oxygen stream as shown in Figure 7.

Component Balances

$$M_i \frac{dc_{i,j}}{dt} = L_{i-1}(c_{i-1,j} - c_{i,j}) + V_{i+1}(d_{i+1,j} - c_{i,j}) \\ -V_i(d_{i,j} - c_{i,j}) + F_i(c^f_{i,j} - c_{i,j}) \quad (13)$$

where $j \in$ COMP is the index of each component, $c_{i,j}$ and $d_{i,j}$ are component mole fractions in the liquid and vapor phases, $c^f_{i,j}$ are the mole fractions of the feed.

Energy Balance

$$M_i\left(\frac{\partial h^L_i}{\partial T_i}\bar{T}_i + \sum_{j\in \text{COMP}} \frac{\partial h^L_i}{\partial c_{i,j}}\bar{c}_{i,j}\right) = L_{i-1}(h^L_{i-1} - h^L_i) \\ + V_{i+1}(h^V_{i+1} - h^L_i) - V_i(h^V_i - h^L_i) + F_i(h^f_i - h^L_i) \quad (14)$$

where $h^L_i = f^{hl}(T_i, P_i)$ and $h^V_i = f^{hv}(T_i, P_i)$ are liquid and vapor enthalpies in $[\frac{kJ}{mol}]$, p_i is the total pressure on tray i, and h^f_i is the feed enthalpy, T_i is the tray temeperature. Expressions and data to compute h^V_i and h^L_i can be found in a number of standard references. Moreover

$$\bar{c}_{i,j} := \frac{dc_{i,j}}{dt} = \big(L_{i-1}(c_{i-1,j} - c_{i,j}) + V_{i+1}(d_{i+1,j} - d_{i,j}) \\ -V_i(d_{i,j} - c_{i,j}) + F_i(c^f_{i,j} - c_{i,j})\big)/M_i, \quad (15)$$

and

$$\bar{T}_i := \frac{dT_i}{dt} = -\frac{\sum_{j\in\text{COMP}}\left[c_{i,j}\sum_{k\in\text{COMP}}\left(\frac{\partial K_{i,j}}{\partial c_{i,k}}\bar{c}_{i,k}\right) + K_{i,j}\bar{c}_{i,k}\right]}{\sum_{j\in\text{COMP}} c_{i,j}\partial K_{i,j}/\partial T_i} \quad (16)$$

Here we define $K_{i,j} = \gamma_{i,j} p^{sat}_{i,j}/p_i$, where $p^{sat}_{i,j} = f^p(T_i)$ is the saturation pressure of pure component j on tray i, and $\gamma_{i,j}$ denotes the liquid activity coefficient describing the non-ideal vapor-liquid equilibrium calculated as in Huang et al. [2009b].

Summation Equation

$$1 = \sum_{j\in\text{COMP}} y_{i,j} \quad (17)$$

Hydraulic Equation

$$L_i = k_d M_i \quad (18)$$

where k_d is considered as an uncertainty parameter with $k_d = 0.5\ min^{-1}$ as the nominal value.

Vapor-Liquid Equilibrium

$$y_{i,j} p_i = \gamma_{i,j} x_{i,j} p^{sat}_{i,j}. \quad (19)$$

After modeling each tray in the distillation columns by equations (12) - (19), the ASU model is composed of 320 differential equations and 1,200 algebraic equations.

The control structure is set the same as that reported in Huang et al. [2009b]. We choose the molar flow rate of pure oxygen (POX-Y_1), pure nitrogen (PNI-Y_2), the temperature at 30^{th} tray in the low pressure column (Tl30-Y_3), and temperature at the 15^{th} tray in the high pressure column (Th15-Y_4) as output variables. Four stream flow rates are considered as manipulated variables, including the expanded air feed (EA-U_1), main air feed (MA-U_2), reflux liquid nitrogen (LN-U_3) and crude gas nitrogen (GN-U_4). The objective is to regulate the outputs at their set-points in the presence of the plant-model mismatch.

Since there are many uncertainties in the ASU process, e.g. thermodynamic properties, tray efficiencies, etc., it is not trivial to determine the uncertainty structure of the ASU model in practice. Therefore, we choose to use the formulation with state and output disturbances. The prediction (N_p) horizon and control (N_c) horizon in the NMPC formulation are chosen to be 20, with 5 minutes sampling time. Γ_y is chosen to be a diagonal matrix with 3×10^{-2} corresponding to the Tl30, Th15, and 1×10^{-4} corresponding to the POX, PNI, while Γ_u is set to be a null matrix. For the MHE formulation, the sampling time is still 5 minutes, but the estimation horizon (N_e) is chosen to be 5. The MHE is tuned with Π_y as a diagonal matrix with 1×10^{-2} corresponding to Tl30, Th15, and 1×10^{-4} corresponding to POX, PNI; Π_x is chosen as a diagonal matrix with 1 corresponding to the $c_{i,j}, \forall i, j$ and 1×10^{-5} corresponding to $M_i, \forall i$; and Π_0 is a diagonal matrix with 5 corresponding to the $c_{i,j}, \forall i, j$ and 5×10^{-5} corresponding to $M_i, \forall i$.

The simulation starts from the nominal steady state with $k_d = 0.5$. At 25 minutes, the k_d in the plant is increased to 0.6, while $k_d = 0.5$ in the model, introducing a plant-model mismatch. The closed-loop plant output is presented in Figure 8, and the control action is shown in Figure 9. It is observed that the proposed MHE and NMPC strategy rejects the disturbance and regulates the plant outputs at their set points without any steady-state offset.

It is known that the ternary distillation problem is not observable if only tray temperatures and flow rates are measured. However in this control structure setting, there are only 4 inputs, which means that the offset free control behavior can be achieved for up to 4 outputs. We choose to use the same 4 measurements as reported in Huang et al. [2009b]. The purities in the product steams POX and PNI are not directly mea-

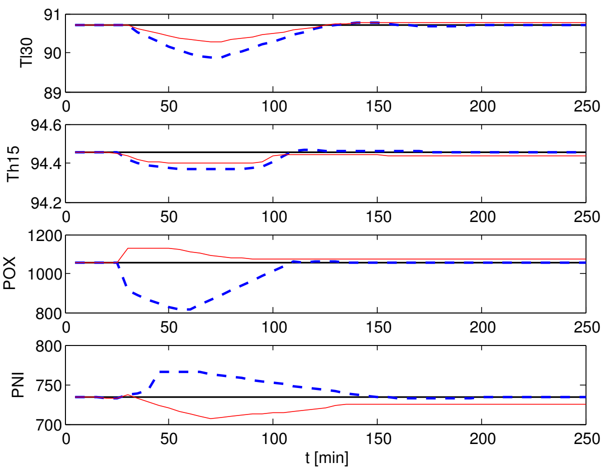

Fig. 8. ASU output. The blue dashed line is the plant, the red line is the estimated value and the black line the set point.

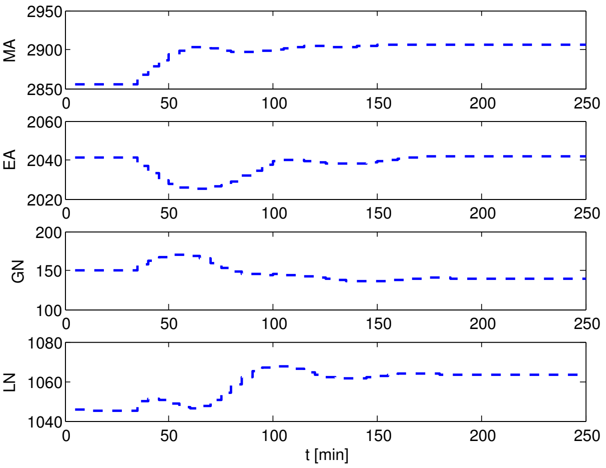

Fig. 9. Control profile for the ASU.

sured. Nevertheless, it is interesting to note in Figure 10 that the oxygen and nitrogen purity in the product streams satisfy the requirement. Moreover, the control action which is solved based on the estimated states, yields offset free behavior for the 4 measured outputs, even though the system is not observable. We believe it is due to the fact that the ASU is open-loop stable and the MHE and NMPC problems at each time step are well initialized from their previous solutions, respectively.

The NMPC and MHE problems for the ASU model are solved using simultaneous collocation-based approach, as described by Biegler et al. [2002]. After discretization, the resulting Nonlinear Programming (NLP) problem corresponding to the NMPC problem (20 finite elements and 3 collocations) has 116,900 constraints and 117,140 variables; while the NLP corresponding to the MHE problem (5 finite elements and 3 collocations) contains 29,285 constraints and 30,885 variables. Both the NLPs are solved using AMPL and IPOPT on a Intel DuoCore 2.4 GHz personal computer. The NMPC problems take up to 6 IPOPT iterations and 200 CPU seconds to solve, while the MHE problems take up to 15 iterations and 90 CPU seconds to solve.

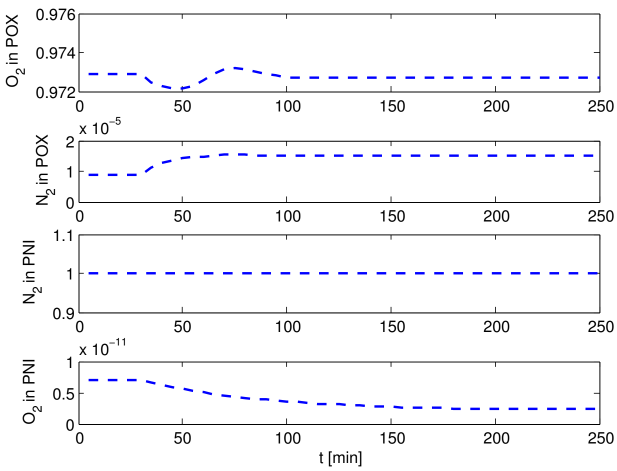

Fig. 10. Product purity profile.

4. SUMMARY

We extend the previous proposed offset free NMPC framework to incorporate MHE as the state estimator. It can be shown that the proposed method achieves offset free behavior, if the set point is feasible for the perturbed predictive model and the control law asymptotically converge to the steady state. In addition, an integrated state and parameter estimation strategy is proposed using MHE and incorporated into the NMPC framework. Thus, the NMPC and MHE can adaptively accommodate plant-model mismatches. A better control performance is observed using the framework with state and parameter estimation, provided that the uncertainty structure is known. The proposed method is successfully implemented on a large scale air separation unit.

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