Proceedings of the 9th International Symposium on
Dynamics and Control of Process Systems (DYCOPS 2010),
Leuven, Belgium, July 5-7, 2010
Mayuresh Kothare, Moses Tade, Alain Vande Wouwer, Ilse Smets (Eds.)

Sequential and Iterative Distributed Model Predictive Control of Nonlinear Process Systems Subject to Asynchronous Measurements

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Abstract: In this work, we focus on sequential and iterative distributed model predictive control (DMPC) of large scale nonlinear process systems subject to asynchronous measurements. Assuming that there is an upper bound on the maximum interval between two consecutive asynchronous measurements, we design DMPC schemes that take into account asynchronous feedback explicitly via Lyapunov techniques. Sufficient conditions under which the proposed distributed control designs guarantee that the states of the closed-loop system are ultimately bounded in regions that contain the origin are provided. The theoretical results are illustrated through a catalytic alkylation of benzene process example.

Keywords: Distributed model predictive control; Nonlinear systems; Asynchronous measurements.

1. INTRODUCTION

Model predictive control (MPC) is a popular control strategy based on using a model of the system to predict its future evolution from the current state along a given prediction horizon. Using these predictions, the manipulated input trajectory is optimized by minimizing a given performance index. To obtain finite dimensional optimization problems, MPC optimizes over a family of piecewise constant trajectories with a fixed sampling time and a finite prediction horizon. Once the optimization problem is solved, only the first manipulated input value is implemented, discarding the rest of the trajectory and repeating the optimization in the next sampling step. Typically, MPC is studied from a centralized control point of view in which all the manipulated inputs of a control system are optimized with respect to an objective function in a single optimization problem. When the number of the state variable and manipulated inputs of the process, however, becomes large, the computational burden of the centralized optimization problem may increase significantly and may impede the applicability of a centralized MPC system, especially in the case where nonlinear process models are used in the MPC. One feasible alternative to overcome this problem is to utilize a distributed MPC (DMPC) architecture in which the manipulated inputs are computed by more than one optimization problems in a coordinated fashion.

In our previous work (Liu et al. (2009) and Liu et al. (in press)), we proposed two different DMPC architectures for nonlinear systems. One DMPC architecture is the sequential DMPC in which distributed controllers use a one-directional communication strategy, are evaluated in sequence and each controller is evaluated only once when a new measurement is available. The other DMPC architecture is the iterative DMPC in which the distributed controllers utilize a bi-directional communication strategy, are evaluated in parallel and iterate to improve closed-loop performance. The results obtained in Liu et al. (2009) and Liu et al. (in press) are based on the assumption that continuous state feedback is available. In the present work, we consider the design of DMPC schemes in a more common setting for chemical processes; that is, measurements of the states are not available continuously but asynchronously. With respect to other available results on DMPC design, several DMPC methods have been proposed in the literature that deal with the coordination of separate MPC controllers (Camponogara et al. (2002); Rawlings and Stewart (2008); Dunbar (2007); Richards and How (2007); Magni and Scattolini (2006)). All of the above results are based on the assumption of continuous sampling and perfect communication between the sensor and the controller. Previous work on MPC design for systems subject to asynchronous measurements has primarily focused on centralized MPC design (Muñoz de la Peña and Christofides (2008)) and little attention has been given to the design of DMPC for systems subject to asynchronous measurements except our recent work (Liu et al. (2010)) on the

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design of DMPC schemes for nonlinear systems subject to asynchronous and delayed measurements. However, in Liu et al. (2010), we only considered the design of sequential DMPC involving two distributed controllers for nonlinear systems subject to asynchronous measurements, and no attention was given to the design of iterative DMPC for nonlinear systems subject to asynchronous measurements.

In this work, we focus on the design of sequential and iterative DMPC schemes for large scale nonlinear process systems subject to asynchronous measurements. We first extend the results obtained in Liu et al. (2010) for sequential DMPC to include multiple distributed controllers, and then modify the iterative DMPC scheme presented in Liu et al. (in press) to take explicitly into account asynchronous measurements in the design. The distributed controllers are designed via Lyapunov-based MPC technique Mhaskar et al. (2005). Sufficient conditions under which the proposed distributed control designs guarantee that the states of the closed-loop system are ultimately bounded in regions that contain the origin are provided. The theoretical results are illustrated through a catalytic alkylation of benzene process example.

2. PRELIMINARIES

2.1 Problem formulation

We consider nonlinear process systems described by the following state-space model:

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))u_i(t) + k(x(t))w(t) \qquad (1)$$

where $x(t) \in R^{n_x}$ denotes the vector of process state variables, $u_i(t) \in R^{m_{u_i}}$, $i = 1, \dots, m$, are m sets of control (manipulated) inputs and $w(t) \in R^{n_w}$ denotes the vector of disturbance variables. The m sets of inputs are restricted to be in m nonempty convex sets $U_i \subseteq R^{m_{u_i}}$, $i = 1, \dots, m$, which are defined as follows: $U_i := \{u_i \in R^{m_{u_i}} : |u_i| \le u_i^{\max}\}$ [2], $i = 1, \dots, m$, where $u_i^{\max}$, $i = 1, \dots, m$, are the magnitudes of the input constraints. The disturbance vector is bounded, i.e., $w(t) \in W$, where $W := \{w \in R^{n_w} : |w| \le \theta, \theta > 0\}$.

We assume that $f(x)$, $g_i(x)$, $i = 1, \dots, m$, and $k(x)$ are locally Lipschitz vector functions and that the origin is an equilibrium of the unforced nominal system (i.e., system of Eq. 1 with $u_i(t) = 0$, $i = 1, \dots, m$, $w(t) = 0$ for all t) which implies that $f(0) = 0$.

2.2 Modeling of asynchronous measurements

We assume that the state of the system of Eq. (1), $x(t)$, is available asynchronously at time instants t_k where $\{t_{k\ge 0}\}$ is a random increasing sequence of times. The distribution of $\{t_{k\ge 0}\}$ characterizes the time needed to obtain a new measurement. In general, if there exists the possibility of arbitrarily large periods of time in which a new measurement is not available, then it is not possible to provide guaranteed stability properties. This is because there exists a non-zero probability that the system may operate in open-loop for a period of time large enough for the state to leave the stability region. In order to study the stability properties in a deterministic framework, in the present work, we assume that there exists an upper bound T_m on the interval between two successive measurements, i.e., $\max_k \{t_{k+1} - t_k\} \le T_m$. This assumption is reasonable from a process control perspective.

2.3 Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $h(x) = [h_1(x) \ \dots \ h_m(x)]^T$ with $u_i = h_i(x)$, $i = 1, \dots, m$, which renders the origin of the nominal closed-loop system asymptotically stable while satisfying the input constraints for all the states x inside a certain stability region. We note that this assumption is essentially equivalent to the assumption that the process is stabilizable or that the pair (A, B) in the case of linear systems is stabilizable. Using converse Lyapunov theorems (e.g., Lin et al. (1996)), this assumption implies that there exist functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ of class $\mathcal{K}$ [3] and a continuously differentiable Lyapunov function $V(x)$ for the nominal closed-loop system which is continuous and bounded in R^{n_x}, that satisfy the following inequalities:

$$\begin{aligned} &\alpha_1(|x|) \le V(x) \le \alpha_2(|x|) \\ &\frac{\partial V(x)}{\partial x}(f(x) + \sum_{i=1}^{m} g_i(x)h_i(x)) \le -\alpha_3(|x|) \\ &|\frac{\partial V(x)}{\partial x}| \le \alpha_4(|x|),\ h_i(x) \in U_i,\ i = 1, \dots, m \end{aligned} \qquad (2)$$

for all $x \in D \subseteq R^{n_x}$ where D is an open neighborhood of the origin. We denote the region $\Omega_\rho \subseteq D$ [4] as the stability region of the closed-loop system under the Lyapunov-based controller $h(x)$. The construction of $V(x)$ can be carried out in a number of ways using systematic techniques like, for example, sum-of-squares methods.

By continuity, the local Lipschitz property assumed for the vector fields $f(x)$, $g_i(x)$, $i = 1, \dots, m$, and $k(x)$ and taking into account that the manipulated inputs u_i, $i = 1, \dots, m$, and the disturbance w are bounded in convex sets, there exists a positive constant M such that

$$|f(x) + \sum_{i=1}^{m} g_i(x)u_i + k(x)w| \le M \qquad (3)$$

for all $x \in \Omega_\rho$, $u_i \in U_i$, $i = 1, \dots, m$, and $w \in W$. In addition, by the continuous differentiable property of the Lyapunov function $V(x)$ and the Lipschitz property assumed for the vector field $f(x)$, there exist positive constants L_x, L_{u_i}, $i = 1, \dots, m$, and L_w such that

$$\begin{aligned} &|\frac{\partial V}{\partial x}f(x) - \frac{\partial V}{\partial x}f(x')| \le L_x|x - x'|,\ |\frac{\partial V}{\partial x}k(x)| \le L_w \\ &|\frac{\partial V}{\partial x}g_i(x) - \frac{\partial V}{\partial x}g_i(x')| \le L_{u_i}|x - x'|,\ i = 1, \dots, m \end{aligned} \qquad (4)$$

for all $x, x' \in \Omega_\rho$, $u_i \in U_i$, $i = 1, \dots, m$, and $w \in W$.

3. DMPC WITH ASYNCHRONOUS MEASUREMENTS

In this section, we design sequential and iterative DMPC schemes, taking into account asynchronous measurements

[2] $|\cdot|$ denotes Euclidean norm of a vector.

[3] A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

[4] We use Ω_ρ to denote the set $\Omega_\rho := \{x \in R^{n_x} | V(x) \le \rho\}$.

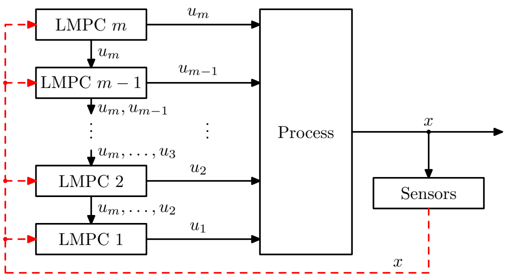

Fig. 1. Sequential DMPC for nonlinear systems subject to asynchronous measurements.

explicitly in their designs, that provide deterministic closed-loop stability properties. In each proposed architecture, we will design m Lyapunov-based MPC (LMPC) controllers to compute u_i, $i = 1, \ldots, m$, and refer to the LMPC computing the input trajectories of u_i as LMPC i.

3.1 Sequential DMPC formulation

A schematic diagram of the proposed sequential DMPC design for systems subject to asynchronous measurements is shown in Fig. 1. We propose to take advantage of the MPC scheme when feedback is lost to update the control inputs based on a state prediction obtained by the model and to have the control actuators store and implement the last computed optimal input trajectories (Muñoz de la Peña and Christofides (2008); Liu et al. (2010)). Specifically, the proposed implementation strategy is as follows:

1. When a new measurement is available at t_k, all the LMPCs receive the state measurement $x(t_k)$ from the sensors.
2. For $j = m$ to 1
   - 2.1. LMPC j receives the entire future input trajectories of u_i, $i = m, \ldots, j+1$, from LMPC $j+1$ and evaluates the future input trajectory of u_j based on $x(t_k)$ and the received future input trajectories.
   - 2.2. LMPC j sends the entire input trajectories of u_j to its actuators and the entire input trajectories of u_i, $i = m, \ldots, j$, to LMPC $j-1$.

Note that in the above implementation strategy, each LMPC sends its own computed input trajectories and the other input trajectories it received to the next LMPC controller (i.e., LMPC j sends input trajectories to LMPC $j-1$). This implies that LMPC j, $j = m, \ldots, 2$, does not have any information about the values of u_i, $i = j-1, \ldots, 1$, that will take when the optimization problems of the LMPC controllers are evaluated. In order to make a decision, LMPC j, $j = m, \ldots, 2$ must assume trajectories for u_i, $i = j-1, \ldots, 1$, along the prediction horizon. To this end, the Lyapunov-based controller $h(x)$ is used. In order to inherit the stability properties of the controller $h(x)$, each control input u_i, $i = 1, \ldots, m$ must satisfy a set of constraints that guarantee a given minimum contribution to the decrease rate of the Lyapunov function $V(x)$ in the case of asynchronous measurements.

In order to proceed, we define $\hat{x}(\tau|t_k)$ for $\tau \in [0, N\Delta]$ as the nominal sampled trajectory of the system of Eq. 1 associated with the feedback control law $h(x)$ and sampling time Δ starting from $x(t_k)$. This nominal sampled trajectory is obtained by integrating the following differential equation recursively:

$$\dot{\hat{x}}(\tau|t_k) = f(\hat{x}(\tau|t_k)) + \sum_{i=1}^{m} g_i(\hat{x}(\tau|t_k))h_i(\hat{x}(l\Delta|t_k)), \quad \forall \tau \in [(l\Delta, (l+1)\Delta)) \tag{5}$$

where $l = 0, \ldots, N-1$. Based on $\hat{x}(\tau|t_k)$, we can define the following input trajectories:

$$u_{n,j}(\tau|t_k) = h_j(\hat{x}(l\Delta|t_k)),\ j = 1, \ldots, m, \quad \forall \tau \in [l\Delta, (l+1)\Delta),\ l = 0, \ldots, N-1 \tag{6}$$

which will be used in the design of the LMPCs. Specifically, the design of LMPC j, $j = 1, \ldots, m$, is based on the following optimization problem:

$$\min_{u_{s,j} \in S(\Delta)} \int_0^{N\Delta} \left[\tilde{x}^j(\tau)^T Q_c \tilde{x}^j(\tau) + \textstyle\sum_{i=1}^m u_{s,i}(\tau)^T R_{ci} u_{s,i}(\tau)\right] d\tau \tag{7a}$$

$$\text{s.t. } \dot{\tilde{x}}^j(\tau) = f(\tilde{x}^j(\tau)) + \textstyle\sum_{i=1}^m g_i(\tilde{x}^j(\tau))u_{s,i} \tag{7b}$$

$$\dot{\hat{x}}^j(\tau) = f(\hat{x}^j(\tau)) + \textstyle\sum_{i=1}^j g_i(\hat{x}^j(\tau))u_{n,i}(\tau|t_k) + \textstyle\sum_{i=j+1}^m g_i(\hat{x}^j(\tau))u_{s,i} \tag{7c}$$

$$u_{s,i}(\tau) = u_{n,i}(\tau|t_k), i = 1, \ldots, j-1 \tag{7d}$$

$$u_{s,i}(\tau) = u^*_{s,i}(\tau|t_k), i = j+1, \ldots, m \tag{7e}$$

$$u_{s,j}(\tau) \in U_j \tag{7f}$$

$$\tilde{x}^j(0) = \hat{x}^j(0) = x(t_k) \tag{7g}$$

$$V(\tilde{x}^j(\tau)) \le V(\hat{x}^j(\tau)),\ \forall \tau \in [0, N_R\Delta] \tag{7h}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling time Δ, N is the prediction horizon, Q_c and R_{ci}, $i = 1, \ldots, m$, are positive definite weighting matrices, and N_R is the smallest integer satisfying $T_m \le N_R\Delta$. The vector $\tilde{x}^j$ is the predicted trajectory of the nominal system with u_j computed by the above optimization problem (i.e., LMPC j) and the other control inputs defined by Eqs. 7d-7e. The vector $\hat{x}^j$ is the predicted trajectory of the nominal system with $u_j = u_{n,j}(\tau|t_k)$ and the other control inputs defined by Eqs. 7d-7e. In order to fully take advantage of the prediction, we choose $N \ge N_R$. The optimal solution to this optimization problem is denoted $u^*_{s,j}(\tau|t_k)$ and is defined for $\tau \in [0, N\Delta)$.

The constraint of Eq. 7b is the nominal model of the system, which is used to generate the trajectory $\tilde{x}^j$; the constraint of Eq. 7c defines a reference trajectory of the nominal system (i.e., $\hat{x}^j$) when the input u_j is defined by $u_{n,j}(\tau|t_k)$; the constraint of Eq. 7d defines the value of the inputs evaluated after u_j (i.e., u_i with $i = 1, \ldots, j-1$); the constraint of Eq. 7e defines the value of the inputs evaluated before u_j (i.e., u_i with $i = j+1, \ldots, m$); the constraint of Eq. 7f is the constraint on the manipulated input u_j; the constraint of Eq. 7g sets the initial state for the optimization problem; and the constraint of Eq. 7h guarantees that the contribution of input u_j to the decrease rate of the time derivative of the Lyapunov function from t_k to $t_k + N_R\Delta$, if $u_j = u^*_{s,j}(\tau|t_k)$, $\tau \in [0, N_R\Delta)$ is applied, is bigger or equal to the value obtained when $u_j = u_{n,j}(t - t_k|t_k)$, $t \in [t_k, t_k + N_R\Delta)$ is applied. This constraint guarantees that the proposed sequential DMPC design maintains the stability of the Lyapunov-based controller $h(x)$ implemented in a sample-and-hold fashion and with open-loop state estimation in the presence of asynchronous measurements.

The manipulated inputs of the closed-loop system under the above sequential DMPC are defined as follows:

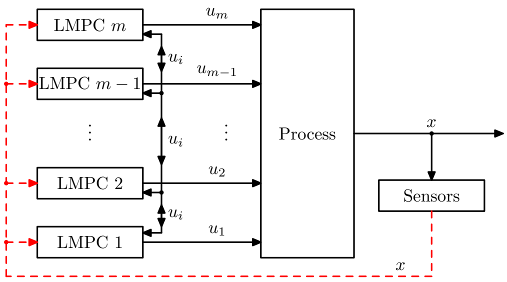

Fig. 2. Iterative DMPC for nonlinear systems subject to asynchronous measurements.

$$u_i(t) = u^*_{s,i}(t - t_k|t_k),\ i = 1, \ldots, m, \forall t \in [t_k, t_{k+1}). \quad (8)$$

The stability of the proposed sequential DMPC with asynchronous measurements is summarized in Theorem 1.

Theorem 1. Consider the system of Eq. 1 in closed-loop with the DMPC design of Eqs. 7-8 based on the controller $h(x)$ that satisfies the conditions of Eq. 2 with class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$. Let $\Delta, \epsilon_s > 0$, $\rho > \rho_{\min} > 0$, $\rho > \rho_s > 0$ and $N \geq N_R \geq 1$ satisfy the following inequalities:

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + L^* M \leq -\epsilon_s/\Delta \quad (9)$$

and

$$-N_R\epsilon_s + f_V(f_W(N_R\Delta)) < 0 \quad (10)$$

with $L^* = L_x + \sum_{i=1}^m L_{u_i} u_i^{\max}$, N_R being the smallest integer satisfying $N_R\Delta \geq T_m$, $f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + M_v s^2$ and $f_W(\tau) = R_w\theta(e^{R_x\tau} - 1)/R_x$ (M_v, R_w, R_x are positive numbers depending on system characteristics). If the initial state of the closed-loop system $x(t_0) \in \Omega_\rho$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_a} \subseteq \Omega_\rho$ where

$$\rho_a = \rho_{\min} + f_V(f_W(N_R\Delta))$$

with $\rho_{\min} = \max\{V(\hat{x}(t+\Delta)) : V(\hat{x}(t)) \leq \rho_s\}$ and $\hat{x}$ being the nominal trajectory of the system of Eq. 1 under the control law $h(x)$ applied in sample-and-hold fashion.

The proof of Theorem 1 can be found in Liu et al. (submitted) and is omitted here due to space limitations.

3.2 Iterative DMPC formulation

In contrast to the one-directional communication of the sequential DMPC architecture, the iterative DMPC architecture utilizes a bi-directional communication strategy in which all the distributed controllers are able to share their future input trajectories information after each iteration. In the presence of asynchronous measurements, the iterative DMPC in Liu et al. (in press) cannot guarantee closed-loop stability. In this subsection, we modify the implementation strategy and the formulation of the distributed controllers to take into account asynchronous measurements (see Fig. 2). Specifically, the proposed implementation strategy is as follows:

1. When a new measurement is available at t_k, all the LMPCs receive the state measurement $x(t_k)$ from the sensors.
2. At iteration c ($c \geq 1$):
   2.1. All the distributed LMPCs exchange their latest future input trajectories.
   2.2. Each LMPC evaluates its own future input trajectory based on $x(t_k)$ and the latest received input trajectories of all the other LMPCs.
3. If a termination condition is satisfied, each LMPC sends its entire future input trajectory to its actuators; if the termination condition is not satisfied, go to step 2 ($c \leftarrow c + 1$).

For the iterations in this DMPC design, there are different choices of the termination condition. For example, the number of iterations c may be restricted to be smaller than a maximum iteration number $c_{\max}$ (i.e., $c \leq c_{\max}$) or the iterations may be terminated when the difference of the performance between two consecutive iterations is smaller than a threshold value or when a maximum computational time is reached.

The proposed design of the LMPC j, $j = 1, \ldots, m$, at iteration c is based on the following optimization problem:

$$\min_{u_{p,j} \in S(\Delta)} \int_0^{N\Delta} \left[\tilde{x}^j(\tau)^T Q_c \tilde{x}^j(\tau) + \textstyle\sum_{i=1}^m u_{p,i}(\tau)^T R_{ci} u_{p,i}(\tau)\right] d\tau \quad (11a)$$

$$\text{s.t. } \dot{\tilde{x}}^j(\tau) = f(\tilde{x}^j(\tau)) + \textstyle\sum_{i=1}^m g_i(\tilde{x}^j(\tau)) u_{p,i} \quad (11b)$$

$$u_{p,i}(\tau) = u_{p,i}^{*,c-1}(\tau|t_k),\ \forall i \neq j \quad (11c)$$

$$\left|u_{p,j}(\tau) - u_{p,j}^{*,c-1}(\tau|t_k)\right| \leq \Delta u_j,\ \forall \tau \in [0, N_R\Delta] \quad (11d)$$

$$u_{p,j}(\tau) \in U_j \quad (11e)$$

$$\tilde{x}^j(0) = x(t_k) \quad (11f)$$

$$\begin{aligned} &\frac{\partial V(\tilde{x}^j)}{\partial \tilde{x}^j}\left(\frac{1}{m} f(\tilde{x}^j(\tau)) + g_j(\tilde{x}^j(\tau)) u_{p,j}(\tau)\right) \\ &\quad \leq \frac{\partial V(\hat{x})}{\partial \hat{x}}\left(\frac{1}{m} f(\hat{x}(\tau|t_k)) + g_j(\hat{x}(\tau|t_k)) u_{n,j}(\tau|t_k)\right), \\ &\quad \forall \tau \in [0, N_R\Delta] \end{aligned} \quad (11g)$$

where $\tilde{x}^j$ is the predicted trajectory of the nominal system with u_j computed by the LMPC of Eq. 11 and all the other inputs are the optimal input trajectories at iteration $c-1$ of the rest of the distributed controllers. The optimal solution to this optimization problem is denoted $u_{p,j}^{*,c}(\tau|t_k)$ which is defined for $\tau \in [0, N\Delta)$. Accordingly, we define the final optimal input trajectory of LMPC j (that is, the optimal trajectories computed at the last iteration) as $u_{p,j}^*(\tau|t_k)$ which is also defined for $\tau \in [0, N\Delta)$.

Note that for the first iteration of each distributed LMPC, the input trajectories defined in Eq. 6 based on the trajectory generated in Eq. 5 are used as the initial guesses of the input trajectories; that is, $u_{p,i}^{*,0} = u_{n,i}$ with $i = 1, \ldots, m$.

The constraint of Eq. 11d puts a limit on the input change in two consecutive iterations. This constraint allows LMPC j to take advantage of the input trajectories received at the last iteration (i.e., $u_{p,i}^{*,c-1}$, $\forall i \neq j$) to more accurately predict the future evolution of the system state. For LMPC j (i.e., u_j), the magnitude of input change in two consecutive iterations is restricted to be smaller than a positive constant Δu_j. The constraint of Eq. 11g is used to guarantee the closed-loop stability.

The manipulated inputs of the closed-loop system under the above iterative DMPC are defined as follows:

$$u_i(t) = u^*_{p,i}(t - t_k|t_k),\ i = 1, \ldots, m, \forall t \in [t_k, t_{k+1}). \quad (12)$$

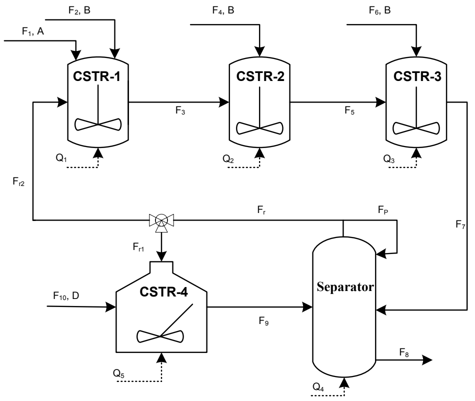

Fig. 3. Process flow diagram of alkylation of benzene.

The stability of the proposed iterative DMPC with asynchronous measurements is summarized in Theorem 2.

Theorem 2. Consider the system of Eq. 1 in closed-loop with the DMPC design of Eqs. 11-12 based on the controller $h(x)$ that satisfies the conditions of Eq. 2 with class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1,2,3,4$. Let $\Delta, \epsilon_s > 0$, $\rho > \rho_{\min} > 0$, $\rho > \rho_s > 0$ and $N \geq N_R \geq 1$ satisfy the condition of Eq. 9 and the following inequality:

$$-N_R\epsilon_s + f_X(N_R\Delta) + f_V(f_W(N_R\Delta)) < 0 \tag{13}$$

with N_R, $f_V(\cdot)$, $f_W(\cdot)$ are defined as in Theorem 1, $f_X(\tau) = \sum_{i=1}^{m}(\frac{1}{m}L_x + L_{u_i}u_i^{\max})(\frac{1}{C_{1,i}}f_{X,i}(\tau) - \frac{C_{2,i}}{C_{1,i}}\tau)$ and $f_{X,i}(\tau) = \frac{C_{2,i}}{C_{1,i}}(e^{C_{1,i}\tau} - 1)$ ($C_{1,i}$ and $C_{2,i}$, $i = 1,\ldots,m$, are positive numbers depending on system characteristics). If the initial state of the closed-loop system $x(t_0) \in \Omega_\rho$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_b} \subseteq \Omega_\rho$ where

$$\rho_b = \rho_{\min} + f_X(N_R\Delta) + f_V(f_W(N_R\Delta))$$

with $\rho_{\min}$ defined as in Theorem 1.

The proof of Theorem 2 can be found in Liu et al. (submitted) and is also omitted here due to space limitations.

4. APPLICATION TO AN ALKYLATION OF BENZENE PROCESS

The process of alkylation of benzene with ethylene to produce ethylbenzene consists of four continuously stirred tank reactors (CSTRs) and a flash tank separator, as shown in Fig. 3. Please refer to Liu et al. (in press) for the detailed description and modeling of the process.

The manipulated inputs to the process are the heat injected to or removed from the five vessels, Q_1, Q_2, Q_3, Q_4 and Q_5, and the feed stream flow rates to CSTR-2 and CSTR-3, F_4 and F_6. The states of the process consist of the concentrations of benzene (A), ethylene (B), ethylbenzene (C), and 1,3-diethylbenzene (D) in each of the five vessels and the temperatures of the vessels. We consider a steady state (operating point), x_s, of the process which is defined by the steady-state inputs Q_{1s}, Q_{2s}, Q_{3s}, Q_{4s}, Q_{5s}, F_{4s} and F_{6s} (see Liu et al. (in press)). The steady-state temperatures in the five vessels are the following:

$$T_{1s} = 477.24\ K,\ T_{2s} = 476.97\ K,\ T_{3s} = 473.47\ K,$$
$$T_{4s} = 470.60\ K,\ T_{5s} = 478.28\ K.$$

The control objective is to regulate the system from an initial state to the steady state. The initial temperatures of the five vessels are the following:

$$T_{1o} = 443.02\ K,\ T_{2o} = 437.12\ K,\ T_{3o} = 428.37\ K,$$
$$T_{4o} = 433.15\ K,\ T_{5o} = 457.55\ K.$$

The first distributed controller (LMPC 1) will be designed to compute the values of Q_1, Q_2 and Q_3, the second distributed controller (LMPC 2) will be designed to compute the values of Q_4 and Q_5, and the third distributed controller (LMPC 3) will be designed to compute the values of F_4 and F_6. Taking this into account, the process model belongs to the class of nonlinear systems: $\dot{x}(t) = f(x) + g_1(x)u_1(x) + g_2(x)u_2(x) + g_3(x)u_3(x)$ where the state x is the deviation of the state of the process from the steady state, $u_1^T = [u_{11}\ u_{12}\ u_{13}] = [Q_1 - Q_{1s}\ Q_2 - Q_{2s}\ Q_3 - Q_{3s}]$, $u_2^T = [u_{21}\ u_{22}] = [Q_4 - Q_{4s}\ Q_5 - Q_{5s}]$ and $u_3^T = [u_{31}\ u_{32}] = [F_4 - F_{4s}\ F_6 - F_{6s}]$ are the manipulated inputs which are subject to input constraints. We use the same design of $h(x)$ as in Liu et al. (in press) based on a quadratic Lyapunov function $V(x) = x^T P x$ with P being the following weight matrix: $P = diag^{5}([1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1])$. Based on $h(x)$, we design the sequential DMPC and the iterative DMPC with the weighting matrices being $Q_c = diag([1\ 1\ 1\ 1\ 10^3\ 1\ 1\ 1\ 1\ 10^3\ 10\ 10\ 10\ 10\ 3000\ 1\ 1\ 1\ 1\ 10^3\ 1\ 1\ 1\ 1\ 10^3])$, $R_{c1} = diag([1\times10^{-8}\ 1\times10^{-8}\ 1\times10^{-8}])$, $R_{c2} = diag([1\times10^{-8}\ 1\times10^{-8}])$ and $R_{c3} = diag([10\ 10])$. The sampling time of the LMPCs is chosen to be $\Delta = 30$ *sec*. For the iterative DMPC designs, Δu_i is chosen to be $0.25u_i^{\max}$ for all the distributed LMPCs and maximum iteration numbers (i.e., $c \leq c_{\max}$) are applied as the termination conditions. In all the simulations, bounded process noise is added to the process model to simulate disturbances/model uncertainty.

We consider that the state of the process is sampled asynchronously and that the maximum interval between two consecutive measurements is $T_m = 75$ *sec*. The asynchronous nature of the measurements is introduced by the measurement difficulties of the full state given the presence of several species concentration measurements. We will compare the proposed sequential and iterative DMPC for systems subject to asynchronous measurements with a centralized LMPC which takes into account asynchronous measurements explicitly (Muñoz de la Peña and Christofides (2008)). The centralized LMPC uses the same weighting matrices, sampling time and prediction horizon as used in the DMPCs. To model the time sequence $\{t_{k\geq0}\}$, we apply an upper bounded random Poisson process. The Poisson process is defined by the number of events per unit time W. The interval between two successive state sampling times is given by $\Delta_a = \min\{-ln\chi/W,\ T_m\}$, where χ is a random variable with uniform probability distribution between 0 and 1. This generation ensures that $\max_k\{t_{k+1} - t_k\} \leq T_m$. In the simulations, W is chosen to be 30 and the time sequence generated by this bounded Poisson process is shown in Fig. 4. For this set of simulations, we choose the prediction horizon of all the LMPCs to be $N = 3$ and choose $N_R = N$ so that $N_R\Delta \geq T_m$.

We first compare the proposed DMPC designs for systems subject to asynchronous measurements with the centralized LMPC from a stability point of view. Figure 5 shows the trajectory of the Lyapunov function $V(x)$ under these

[5] $diag(v)$ denotes a matrix with its diagonal elements being the elements of vector v and all the other elements being zeros.

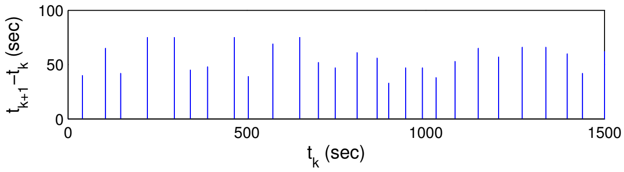

Fig. 4. Asynchronous measurement sampling times $\{t_{k\geq 0}\}$ with $T_m = 75$ *sec*: the $x-$axis indicates $\{t_{k\geq 0}\}$ and the $y-$axis indicates the size of the interval between t_k and t_{k-1}.

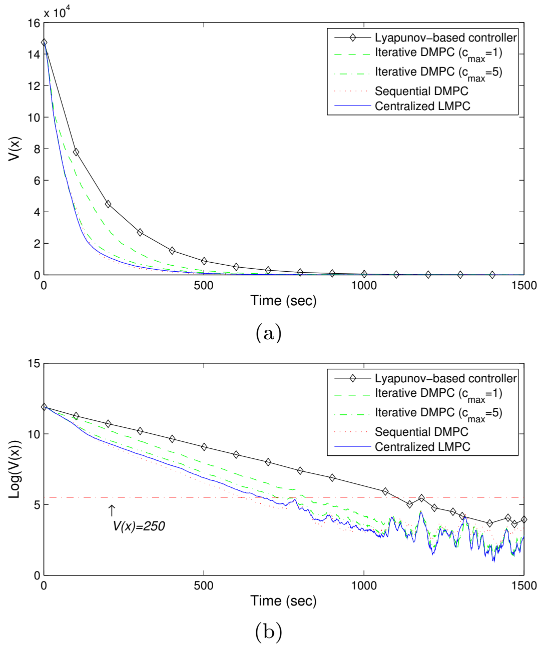

Fig. 5. Trajectories of the Lyapunov function under the Lyapunov-based controller $h(x)$ implemented in a sample-and-hold fashion and with open-loop state estimation, the iterative DMPC with $c_{\max} = 1$ and $c_{\max} = 5$, the sequential DMPC and the centralized LMPC: (a) $V(x)$; (b) $Log(V(x))$.

control designs. From Fig. 5, we see that the proposed DMPC designs as well as the centralized LMPC design are able to drive the system state to a region very close to the desired steady state. From Fig. 5, we can also see that the sequential DMPC, the centralized LMPC and the iterative DMPC with $c_{\max} = 5$ give very similar trajectories of $V(x)$. Another important aspect we can see from Fig. 5(b) is that at the early stage of the closed-loop system simulation, because of the strong driving force related to the difference between the steady-state and the initial condition, the process noise/disturbance has small influence on the process dynamics, even though the controller(s) has/have to operate in the presence of asynchronous measurements. When the states get close to the steady-state, the Lyapunov function starts to fluctuate due to the domination of noise/disturbance over the vanishing driving force. However, the proposed DMPC designs are able to maintain practical stability of the closed-loop system and keep the trajectory of the Lyapunov function in a bounded region ($V(x) \leq 250$) very close to the steady state.

Next, we compare the evaluation times of the LMPCs in these control designs. The simulations are carried out by Java programming language in a *Pentium 3.20 GHz* computer. The optimization problems are solved by the open source interior point optimizer Ipopt. We evaluate the LMPC optimization problems for 100 runs. The mean evaluation time of the centralized LMPC is about 23.7 *sec*. The mean evaluation time for the sequential DMPC scheme, which is the sum of the evaluation times (1.9 *sec*, 3.6 *sec* and 3.2 *sec*) of the three LMPCs, is about 8.7 *sec*. The mean evaluation time of the iterative DMPC scheme with one iteration is 6.3 *sec* which is the largest evaluation time among the evaluation times (1.6 *sec*, 6.3 *sec* and 4.3 *sec*) of the three LMPCs. The mean evaluation time of the iterative DMPC architecture with four iterations is 18.7 *sec* with the evaluation times of the three LMPCs being 6.9 *sec*, 18.7 *sec* and 14.0 *sec*. From this set of simulations, we see that the proposed DMPC designs lead to a significant reduction in the controller evaluation time compared with a centralized LMPC design though they provide a very similar performance.

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