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Adaptive Quasi-Infinite Horizon NMPC of a Continuous Fermenter ⋆

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Abstract: Uncertainties inherent in biological systems make control of continuous fermenters a challenging task. This work proposes to integrate the quasi-infinite horizon nonlinear model predictive control (NMPC) algorithm, with an adaptive state and parameter estimator. As a result, the quasi-infinite horizon NMPC can be applied to systems with only part of the states measured. Moreover, the adaptive parameter estimation can update the uncertainty parameter online in the presence of plant-model mismatch. Two variations of target setting optimization problem are proposed to adjust the equilibrium points when they drift away due to the plant-model mismatch. The proposed method is applied to a fermentor with partial state feedback.

1. INTRODUCTION

Biochemical processes exhibit highly nonlinear and complex dynamics. Uncertainties inherent in biological systems render control problems associated with fermenters and bioreactor difficult. Over the last decade, nonlinear model predictive control has emerged as a promising approach for achieving tight control of such uncertain nonlinear systems. Many researchers have contributed important issues to guarantee the nominal stability of NMPC. Keerthi and Gilbert [1988] and Mayne and Mischalska [1990] proposed a *terminal equality constraint* scheme, requiring the states to be zero at the end of each finite prediction horizon. However, it is hard to get solution since it is difficult to determine the prediction horizon length to satisfy the terminal equality constraint. In order to avoid this, Michalska and Mayne [1993] proposed a *terminal inequality constraint* approach and suggested a dual-mode receding horizon control. NMPC with this scheme is implemented by switching between the two controllers, depending on the states being inside or outside the *terminal region*. Chen and Allgöwer [1998] proposed a *quasi-infinite horizon* NMPC scheme that optimizes an objective functional consisting of a finite horizon cost and a *terminal cost* subject to system dynamics, input constraints and an additional *terminal region constraint*. The terminal states are penalized such that the terminal cost bounds the infinite horizon cost of the nonlinear system controlled by a local linear state feedback controller. It has the advantage that the local linear state feedback controller is never implemented; it is only used to determine the terminal region and the terminal penalty matrix offline. In addition, it has computational advantages in NMPC applications as well. The prediction horizon can be selected shorter than in common practice, which chooses long prediction horizons to achieve nominal stability. As a result, the quasi-infinite horizon scheme has become the standard formulation in the NMPC literature with stability analyses, such as in Mayne et al. [2000], Magni and Scattolini [2007], Limon et al. [2009] and etc.

The above mentioned studies are based on the assumption that all the states in nonlinear systems are measurable. However, in practice, the outputs of systems are part of the states, or are nonlinear functions of the states. Consequently, a state estimator is usually employed to reconstruct the states from the outputs. In addition, disturbances and modeling errors are usually present due to parameter drift and changes of operating conditions in industrial NMPC applications, causing the presence of plant-model mismatch. The plant-model mismatch may lead to biased state estimates or cause the NMPC controller to be unstable. Patwardhan et al. [2009] proposed a moving horizon framework to adaptively estimate states and uncertainty parameters for nonlinear discrete-time systems simultaneously. This proposed scheme has the advantage that constraints can be easily introduced on the uncertainty parameters.

The aim of this work is to incorporate the quasi-infinite NMPC scheme with an adaptive state estimator, in order to deal with systems with partial state feedback and the presence of plant-model mismatch. In the next section, we briefly recall the procedure to calculate the terminal penalty matrix and terminal region constraints in the quasi-infinite horizon NMPC given by Chen and Allgöwer [1998]. Then an adaptive extended Kalman filter (EKF) for continuous-time systems, similar to that in Patwardhan et al. [2009], is proposed. In addition, we propose two target setting schemes which adjust the equilibrium points of nonlinear systems in the presence of plant-model mismatch. Finally, the adaptive quasi-infinite horizon NMPC is proposed based on the adaptive EKF and the target setting optimization problems. The proposed method is illustrated by simulation studies of a fermentation process with partial state feedback.

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2. QUASI-INFINITE HORIZON NONLINEAR MODEL PREDICTIVE CONTROL

In this section, we briefly summarize the quasi-infinite horizon NMPC method for continuous systems given by Chen and Allgöwer [1998]. The class of systems to be controlled is described by the following general nonlinear ordinary differential equations (ODEs):

$$\frac{dx(t)}{dt} = f(x(t), u(t), \theta(t)) \tag{1a}$$

$$y(t) = h(x(t)) \tag{1b}$$

where $x(t) \in \mathbb{R}^{n_x}$, $y(t) \in \mathbb{R}^{n_y}$ and $u(t) \in \mathbb{R}^{n_u}$ represent the state vector, output vector and input vector, respectively. $\theta(t) \subset \Omega_\theta \in \mathbb{R}^{n_\theta}$ is the uncertainty parameter vector that is in a compact set Ω_θ, with $\mathbf{0}$ as its nominal value. The system is subject to input constraint

$$u(t) \in \mathbb{U}, \forall t \geq 0. \tag{2}$$

Moreover, this system is assumed to have equilibrium point at the origin, i.e.

$$0 = f(0,0,0),\ 0 = h(0). \tag{3}$$

In practice, if the equilibrium point is not 0, one can shift the origin to the equilibrium point so that this assumption holds true.

If all the states are measurable, i.e. $y(t) = x(t)$, the quasi-infinite horizon NMPC is formulated as follows:

$$\begin{aligned}
\min \quad & \int_k^{k+N_p} \left(z(\tau)^T W_x z(\tau) + v(\tau)^T W_u v(\tau) \right) d\tau \\
& + z(k+N_p)^T W_\infty z(k+N_p) \\
\text{s.t} \quad & \frac{dz(t)}{dt} = f(z(t), v(t), 0) && (4a) \\
& z(k) = x(k) && (4b) \\
& v(t) \in \mathbb{U},\ z(k+N_p) \in \Omega_x && (4c) \\
& t \in [k,\ k+N_p], && (4d)
\end{aligned}$$

where N_p is the prediction horizon, $z(t)$ and $v(t)$ are the future predicted states and control movements. W_x and W_u are tuning matrices, denoting positive-definite weights for the states and inputs, respectively. W_∞ and Ω_x represent the *terminal penalty matrix* and *terminal region constraint*, which are computed using local linearization of the nominal model (1a) at the terminal state. Note that in the controller, the uncertainty parameter is set to its nominal value.

The procedure to calculate the terminal penalty W_∞ and terminal region Ω_x given by Chen and Allgöwer [1998] is as follows:

(1) Consider the Jacobian linearization of the system (1) at the origin

$$\frac{dx}{dt} = Ax + Bu \tag{5}$$

where $A := \frac{\partial f}{\partial x}|_{(0,0,0)}$ and $B := \frac{\partial f}{\partial u}|_{(0,0,0)}$. If equation (5) is stabilizable, then solve a linear state feedback gain K, i.e. $u = -Kx$, by linear-quadratic regulator (LQR) algorithm, such that $A_k := A + BK$ is asymptotically stable.

(2) Choose $\kappa \in [0, \infty)$ that satisfies

$$\kappa < -\lambda_{\max}(A_k), \tag{6}$$

and obtain the positive-definite symmetric W_∞ by solving

$$(A_k + \kappa I)^T W_\infty + W_\infty (A_k + \kappa I) = -W^*, \tag{7}$$

where $W^* = W_x + K^T W_u K$ is a positive-definite symmetric matrix.

(3) Find the largest possible α_1 such that $Kx \in \mathbb{U}$, $\forall x \in \Omega_{\alpha_1}$, where $\Omega_{\alpha_1} := \{x \in \mathbb{R}^{n_x} | x^T W_\infty x \leq \alpha_1\}$.

(4) Find $\Omega_x := \{x \in \mathbb{R}^{n_x} | x^T W_\infty x \leq \alpha\}$ by making iterations of an optimization problem

$$\max_x \{x^T W_\infty \phi(x) - \kappa x^T W_\infty x | x^T W_\infty x \leq \alpha\} \tag{8}$$

for the chosen κ by reducing α from α_1 until the optimum value given by (8) is nonpositive. Here $\alpha \in (0, \alpha_1]$ and

$$\phi(x) = f(x, Kx) - A_k x. \tag{9}$$

Chen and Allgöwer [1998] have shown that this quasi-infinite horizon formulation can guarantee asymptotic closed-loop stability under nominal conditions if all the states are measured. In practice, however, the state information are usually not fully measurable. Hence, a state estimator such as EKF is usually implemented in parallel to reconstruct the initial state in order to solve the NMPC problem. In addition, plant-model mismatch is usually present. As a result, the state estimation may be biased and the equilibrium point may drift away from the origin.

3. ADAPTIVE EXTENDED KALMAN FILTER

In this section, we present an adaptive EKF scheme based on a moving horizon framework that estimates both the state and the uncertainty parameter in the system (1).

At time step k, a *nominal* extended Kalman filter (EKF) is carried out as follows:

$$x^-(k+1) = \hat{x}(k) + \int_k^{k+1} f(x(\tau), u(\tau), 0) d\tau \tag{10a}$$

$$\hat{x}(k) = x^-(k) + L(k)(y(k) - h(x^-(k))), \tag{10b}$$

where x^- and $\hat{x}$ are *a priori* and *a posteriori* estimates respectively. $L(k)$ represents the time-varying Kalman gain matrix, which is calculated by

$$L(k) = P_{XY}(k) P_{YY}(k)^{-1}. \tag{11}$$

where $P_{XY}(k)$ represents the *cross-covariance matrix* relating $x(k)$ and $y(k)$, while $P_{YY}(k)$ represents the *auto-covariance matrix* of signal $y(k)$. These matrices are approximated using the following set of equations

$$P_{XY}(k) = P(k)^- C(k)^T \tag{12a}$$

$$P_{YY}(k) = C(k) P(k)^- C(k)^T + R \tag{12b}$$

$$P(k) = (I - L(k)C(k)P(k)^- \tag{12c}$$

$$P(k+1)^- = \Phi(k) P(k) \Phi(k)^T + Q \tag{12d}$$

where $P(k)^-$ and $P(k)$ are called *a priori* and *a posteriori* covariance matrices, and $\Phi(k) := \exp(A(k)T)$, $A(k) := \frac{\partial f}{\partial x}|_{(x^-(k), u(k), 0)}$, $C(k) := \frac{\partial h}{\partial x}|_{x^-(k)}$, T is the sampling time, R and Q are tuning matrices. The EKF is called nominal because the prediction is based on the nominal plant model.

In the presence of plant-model mismatch, the *a priori* and *a posteriori* error sequences will not converge to zero. In order to get an accurate estimate of the state and to improve the model for the further NMPC application, an adaptive method to update the model parameter online is proposed by Patwardhan et al. [2009], based on observer error information for nonlinear discrete-time systems. The idea is that at each time step k, we solve an optimization problem over a time horizon N to get an optimal value of the uncertain parameter θ to minimize the output error. This can be modified for continuous-time systems as follows:

$$
\begin{aligned}
\hat{\theta}(k) = &\arg\min_{\theta} \sum_{j=1}^{N} |y(k-N+j) - h(z^{-}(k-N+j))|^2_{W(k-N+j)} \\
&\text{s.t. } z(k-N+j+1)^{-} = \hat{z}(k-N+j) \\
&\qquad + \int_{k-N+j}^{k-N+j+1} f(z(\tau), u(k-N+j), \theta) d\tau \\
&\quad \hat{z}(k-N+j) = z^{-}(k-N+j) \\
&\qquad + L(k-N+j)(y(k-N+j) - h(z(k-N+j)^{-})) \\
&\quad \hat{z}(k-N) = \hat{x}(k-N), \\
&\quad j = 1 \ldots N,\ \theta \in \Omega_\theta \subset \mathbb{R}^{n_\theta}
\end{aligned} \tag{13}
$$

where $W(k-N+j)$ is the weighting matrix of each time step, $z^{-}(k-N+j)$ and $\hat{z}(k-N+j)$ are the *a priori* and *a posteriori* state estimates in the optimization problem, its initial condition is chosen to be the estimated value from the observer at time step $k-N$. Moreover, the Kalman gain matrix $L(k-N+j)$ is calculated in the same way as in equation (11). Finally, the estimated states are smoothed with the updated uncertainty parameter, i.e. $\hat{x}(k-N+j) = \hat{z}(k-N+j, \hat{\theta}(k-N+j)),\ j = 0, \ldots, N$. It means that the adaptive EKF automatically updates the estimated state over the horizon N according to the estimated uncertainty parameter. Patwardhan et al. [2009] pointed out that unlike state and parameter estimation by augmenting systems, this formulation allows us to add constraints for the uncertain parameter θ. Moreover, if the uncertainty parameter is observable, the adaptive EKF (13) can guarantee the estimated uncertainty parameter converges to the true plant value, even for multiple uncertainty parameters.

4. TARGET SETTING

Due to the presence of plant-model mismatch, the equilibrium point of the plant may drift away from the origin. In this section, we propose to solve a target setting optimization problem that adjusts the equilibrium point. The new target state will be used as the state setpoint in the NMPC formulation. Moreover, the terminal penalty matrix and terminal region constraint will be calculated based on the adjusted equilibrium point. Depending on different objectives, we propose two formulations of the target setting optimization problem. The first one considers providing offset free behavior for the plant output. The objective function is to minimize the difference between the setpoint of plant output and the calculated target output. The second formulation aims to gain the best profit of the plant. Hence the objective function is to maximize an economic criteria, such as production rate, economic profit value, etc.

The following formulation is to calculate the target values of state $x(k)^{ts}$ and input $u(k)^{ts}$ that minimize the difference between the target output $y(k)^{ts}$ and the setpoint y_r.

$$
\begin{aligned}
\min\ & |y^{ts}(k) - y_r|^2_{W_y} \\
\text{s.t. } & 0 = f(x(k)^{ts}, u(k)^{ts}, \hat{\theta}(k)) \\
& y(k)^{ts} = h(x(k)^{ts}) \\
& u(k)^{ts} \in \mathbb{U},
\end{aligned} \tag{14}
$$

where W_y is the corresponding weighting matrix, $\hat{\theta}(k)$ is the calculated uncertainty parameter from the adaptive EKF (13). Unlike the target setting optimization problem based on state and output errors proposed by Huang et al. [2009], this formulation is based on the updated uncertainty parameter $\hat{\theta}(k)$ from the adaptive EKF.

Alternatively, one can solve an equivalent infinite horizon dynamic optimization problem to obtain the target steady-state.

$$
\begin{aligned}
\min & \int_{k}^{k+\infty} \left((y(\tau) - y_r)^T W_y (y(\tau) - y_r)\right) d\tau \\
\text{s.t.} & \frac{dz(t)}{dt} = f(z(t), u^{ts}(k), \hat{\theta}(k)) \\
& y(t) = h(z(t)) \\
& z(k) = \hat{x}(k), \quad u^{ts}(k) \in \mathbb{U} \\
& t \in [k,\ k+\infty].
\end{aligned} \tag{15}
$$

Note that $u^{ts}(k)$ is the calculated target input for the entire horizon, and the initial condition is the state estimation from the adaptive EKF (13). The target state is then chosen to be $x^{ts}(k) = z^{*}(k+\infty)$, which is the solution at the end of the horizon. In practice, we approximate the steady state solution by replacing ∞ with a finite number.

To improve the process profit in the presence of plant-model mismatch, the second variation of the target setting optimization problem can be formulated as follows:

$$
\begin{aligned}
\max\ & P(x(k)^{ts}, u(k)^{ts}) \\
\text{s.t. } & 0 = f(x(k)^{ts}, u(k)^{ts}, \hat{\theta}(k)) \\
& y(k)^{ts} = h(x(k)^{ts}) \\
& u(k)^{ts} \in \mathbb{U},
\end{aligned} \tag{16}
$$

where $P(\cdot,\cdot)$ is an economic profit value which is a function of the target state $x(k)^{ts}$ and target input $u(k)^{ts}$. The alternative equivalent dynamic optimization problem is formulated as follows:

$$
\begin{aligned}
\max\ & P\left(z(k+\infty), u^{ts}(k)\right) \\
\text{s.t.} & \frac{dz(t)}{dt} = f(z(t), u^{ts}(k), \hat{\theta}(k)) \\
& y(t) = h(z(t)) \\
& z(k) = \hat{x}(k), \quad u^{ts}(k) \in \mathbb{U} \\
& t \in [k,\ k+\infty].
\end{aligned} \tag{17}
$$

The dynamic formulations (15) and (17) are essentially approximations of the steady state formulations (14) and (16). In our experiences, the dynamic formulations (15) and (17) make the problem easier to converge even though they result in larger dynamic optimization problems.

5. ADAPTIVE QUASI-INFINITE HORIZON NMPC

Once the state estimation, the optimal uncertainty parameter and the updated target steady state are available, the adaptive quasi-infinite horizon NMPC can be formulated as:

$$
\begin{aligned}
\min & \int_{k}^{k+N_p} \left((z(\tau) - x(k)^{ts})^T W_x (z(\tau) - x(k)^{ts}) \right. \\
& \quad \left. + (v(\tau) - u^{ts}(k))^T W_u (v(\tau) - u^{ts}(k))\right) d\tau \\
& \quad + (z(k+N_p) - x(k)^{ts})^T W_\infty(k) (z(k+N_p) - x(k)^{ts})
\end{aligned}
$$

$$
\text{s.t} \quad \frac{dz(t)}{dt} = f(z(t), v(t), \hat{\theta}(k)) \tag{18a}
$$

$$
z(k) = \hat{x}(k) \tag{18b}
$$

$$
z(k+N_p) \in \Omega_x(k),\ \left(v(t) - u^{ts}(k)\right) \in \mathbb{U} \tag{18c}
$$

$$
t \in [k,\ k+N_p]. \tag{18d}
$$

Unlike the quasi-infinite horizon NMPC (4), this adaptive formulation is initialized with the estimated state and the predictive model is adaptively updated with the estimated uncertainty parameter. Moreover, the target state, the terminal penalty matrix and the terminal inequality region are calculated online at each time step.

At the time step k, the center of the terminal region is the updated target state $x^{ts}(k)$. The terminal penalty matrix $W_\infty(k)$ and the constant $\alpha(k)$, which determine the shape and size of the terminal region, are calculated by the following procedure:

(1) Consider the Jacobian linearization of the system (1) at the target:
$$\frac{dx}{dt} = A(k)x + B(k)u \tag{19}$$
where $A(k) := \frac{\partial f}{\partial x}|_{(x^{ts}(k),u^{ts}(k),\hat{\theta}(k))}$ and $B(k) := \frac{\partial f}{\partial u}|_{(x^{ts}(k),u^{ts}(k),\hat{\theta}(k))}$. If equation (19) is stabilizable, then solve a linear state feedback gain $K(k)$, such that $A_k(k) := A(k) + B(k)K(k)$ is asymptotically stable.

(2) Choose $\kappa \in [0,\infty)$ that satisfies
$$\kappa < -\lambda_{\max}\left(A_k(k)\right), \tag{20}$$
and obtain the positive-definite symmetric $W_\infty(k)$ by solving
$$(A_k(k) + \kappa I)^T W_\infty(k) + W_\infty(k)(A_k(k) + \kappa I) = -W^*(k), \tag{21}$$
where $W^*(k) = W_x + K(k)^T W_u K(k)$ is a positive-definite symmetric matrix.

(3) Let $\triangle x := (x - x^{ts}(k))$. Find the largest possible $\alpha_1(k)$ such that $K(k)\triangle x \in \mathbb{U}$, $\forall \triangle x$ that satisfy $\triangle x^T W_\infty(k) \triangle x \leq \alpha_1(k)$.

(4) Find the largest constant $\alpha(k) \in (0, \alpha_1(k)]$ by making iterations of an optimization problem
$$\max_{\triangle x}\{\triangle x^T W_\infty(k)\phi(\triangle x) - \kappa \triangle x^T W_\infty(k)\triangle x | \\ \triangle x^T W_\infty(k)\triangle x \leq \alpha(k)\} \tag{22}$$
for the chosen κ by reducing $\alpha(k)$ from $\alpha_1(k)$ until the optimum value given by (22) is nonpositive. Here
$$\phi(\triangle x) = f(\triangle x, K(k)\triangle x) - A_k(k)\triangle x. \tag{23}$$

As a result, the terminal region $\Omega_x(k)$ centered around the target state $x^{ts}(k)$ is defined by
$$\Omega_x(k) = \{x \in \mathbb{R}^{n_x} | \left(x - x^{ts}(k)\right)^T W_\infty(k)\left(x - x^{ts}(k)\right) \leq \alpha(k)\}. \tag{24}$$

6. SIMULATION EXAMPLES

In this section, the proposed adaptive quasi-infinite horizon NMPC is applied to a fermentation process reported by Henson and Seborg [1990], Patwardhan and Madhavan [1993]. The dynamic model equations for this process are given below:
$$\begin{aligned}
\frac{dX}{dt} &= -DX + \mu X \\
\frac{dS}{dt} &= D(S_f - S) - \frac{1}{Y_{x/s}}\mu X \\
\frac{dP}{dt} &= -DP + (\alpha\mu + \beta)X \\
Y_1 &= X, \text{ and } Y_2 = S,
\end{aligned} \tag{25}$$
where the X represents the biomass concentration, S represents the substrate concentration, and P denotes the product concentration. Assume X and S are measured outputs, while the dilution rate D and the feed substrate concentration S_f are the input variables. Model parameter $Y_{x/s}$ represent the cell-mass yield, and α and β are kinetic parameters. μ is the specific growth rate, which is calculated as
$$\mu = \frac{\mu_m(1 - \frac{P}{P_m})}{K_m + S + \frac{S^2}{K_i}}. \tag{26}$$
Equation (26) contains four model parameters: the maximum specific growth rate μ_m, the product saturation constant P_m, the substrate saturation constant K_m and the substrate inhabitation constant K_i. The nominal values of the model parameters are listed in Table 1.

Table 1. Nominal Model Parameters

parameter	nominal value	parameter	nominal value
$Y_{x/s}$	0.4 g/g	P_m	50 g/L
α	2.2 g/g	K_m	1.2 g/L
β	0.2 h^{-1}	K_i	22 g/L
μ_m	0.48 h^{-1}		

At the nominal condition, the optimal steady-state values are listed in Table 2.

Table 2. Optimal Steady State

variable	value	variable	value	variable	value
X^*	7.293 g/L	S^*	5.169 g/L	P^*	24.9 g/L
D^*	0.164 h^{-1}	S_f^*	23.4 g/L		

6.1 *Target Setting with Offset Free*

In the first scenario, the control objective is to maintain the plant output at the nominal setpoint without offset, even in the presence of plant-model mismatch. Consequently, target setting optimization problem (15) is used. The cell-mass yield $Y_{x/s}$ is considered as an uncertainty parameter.

In this simulation, the sampling time T is chosen as 0.5 hr. The adaptive EKF (13) is tuned with $Q = \left[\frac{\partial f}{\partial u}|_{x^*,u^*}\right]\bar{Q}\left[\frac{\partial f}{\partial u}|_{x^*,u^*}\right]^T$, $\bar{Q} = \text{diag}[0.01, 0.09]$, $R = \text{diag}[0.01, 0.01]$. The horizon of the adaptive EKF is chosen to be 5 time units. The prediction horizon of the target setting optimization problem (15) is chosen to be 40 time units, and the output weighting matrix is tuned as $W_y = \text{diag}[1,1]$. The prediction horizon of the adaptive NMPC (18) is chosen to be 20 time units. The calculated control action is chosen to be the input blocking form with 2 steps as the control horizon. It indicates that there are 2 degrees of freedom, and they are spread over the entire prediction horizon, satisfying $v(1) = v(2) =, \cdots, v(10)$ and $v(11) = v(12) =, \cdots, v(20)$. The weighting matrix are $W_x = \text{diag}[1,1,1]$ and $W_u = 0.5 \times \text{diag}[1,1]$. The simulation is performed in Matlab, the plant is propagated using ODE45, the optimization problems (13), (15), (17), (18) and (22) are solved using fmincon, which is based on Sequential Quadratic Programming (SQP) algorithm.

The simulation starts from the nominal point. Then at 12.5 hrs, the uncertainty parameter $Y_{x/s}$ is perturbed by 100%, to 0.8 g/g. Figures 1 and 2 show the closed-loop plant responses. It is observed that the adaptive EKF can track the changes of the uncertainty parameters in few time steps, removing the plant-model mismatch. The proposed approach can quickly reject the disturbance and regulate the plant at the target. The plant output X and S are controlled at their setpoints without any offset. Moreover, after the estimated uncertainty parameter converges

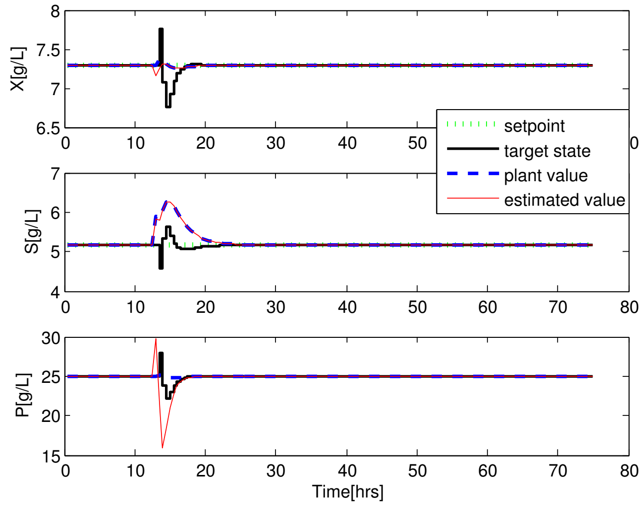

Fig. 1. State profile of scenario 1.

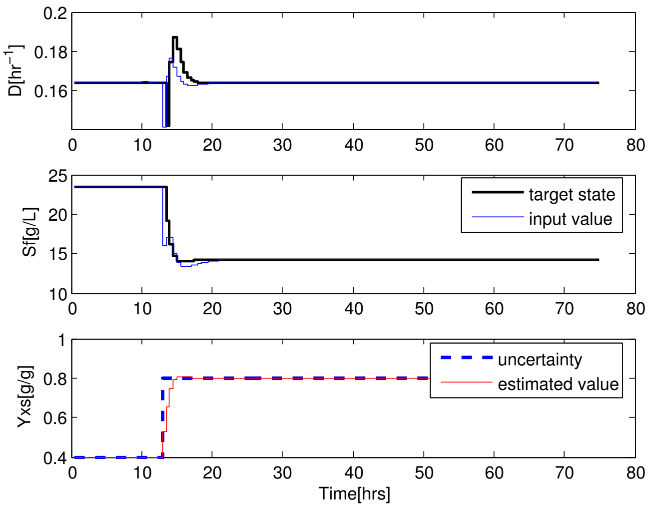

Fig. 2. Input and uncertainty profiles of scenario 1.

to the plant value and the plant-model mismatch is removed, the EKF yields unbiased state estimation as shown in Figure 1.

Figure 3 shows the profile of α and the eigenvalues of W_∞, which determine the size and shape of the terminal region as in equation (24). It is worth emphasizing that the center of the terminal region is the calculated target state which is shown as the thick solid lines in Figure 1. Before 12.5 hrs, the terminal penalty matrix is

$$W_\infty(k) = \begin{bmatrix} 58.5791 & -3.0981 & -15.8994 \\ -3.0981 & 5.3694 & 4.0738 \\ -15.8994 & 4.0738 & 6.3555 \end{bmatrix}$$

and $\Omega_x(k) = \{x \in \mathbb{R}^{n_x} | (x - x^{ts}(k))^T W_\infty(k) (x - x^{ts}(k)) \leq 1.8792\}$. After the disturbance at 12.5 hrs, the calculated terminal penalty matrix is

$$W_\infty(k) = \begin{bmatrix} 59.1187 & -1.4220 & -14.6016 \\ -1.4220 & 4.9367 & 1.8887 \\ -14.6016 & 1.8887 & 4.1183 \end{bmatrix}$$

and $\Omega_x(k) = \{x \in \mathbb{R}^{n_x} | (x - x^{ts}(k))^T W_\infty(k) (x - x^{ts}(k)) \leq 1.9895\}$.

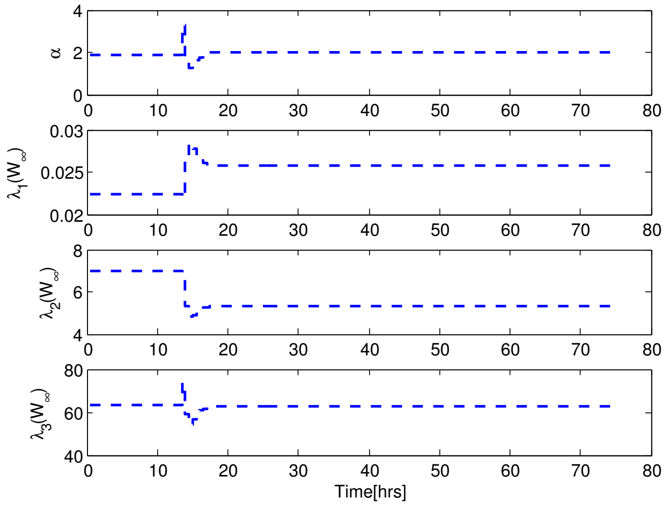

Fig. 3. Terminal region profile of scenario 1.

6.2 *Target Setting with Maximizing Economic Profit*

In the second scenario, the control objective is to maximize the product concentration P in the presence of plant-model mismatch. Consequently, target setting optimization problem (17) is used. The objective function of the target setting problem is to maximize P at steady states. The maximum specific growth rate μ_m is considered as an uncertainty parameter. The tuning parameters are chosen to be exactly the same as those in the previous simulation.

Similarly, the simulation starts from the nominal point. Then at 35 hrs, the uncertainty parameter μ_m is perturbed by 50%, to 0.72 h^{-1}. Figure 4 and 5 show the closed-loop plant responses. Before the perturbation, the proposed approach adjusts the plant setpoint to increase the product concentration P from 24.9 g/L to the optimal value 36.13 g/L. Correspondingly, the setpoints of biomass concentration X and substrate concentration S are changed to 8.21 g/L and 5.137 g/L, respectively. The plant is gradually moved to the new target state. After the introduction of the perturbation, the adaptive EKF can track the change of the uncertainty parameter. The setpoint of the biomass concentration X is increased to 9.85 g/L to maintain the optimal product concentration. Moreover, the plant states are controlled at the desired targets both before and after the disturbance.

Figure 6 shows the profile of α and the eigenvalues of W_∞ in the second simulation scenario. It may be noted that there is significant change in size of the terminal region in this case. When the system reaches the optimal target state before the perturbation at 35 hrs, the calculated terminal penalty matrix is

$$W_\infty(k) = \begin{bmatrix} 119.76 & -9.9521 & -11.572 \\ -9.9521 & 13.183 & 4.3029 \\ -11.572 & 4.3029 & 2.0405 \end{bmatrix}$$

and $\Omega_x(k) = \{x \in \mathbb{R}^{n_x} | (x - x^{ts}(k))^T W_\infty(k) (x - x^{ts}(k)) \leq 16.815\}$. After the perturbation, the calculated terminal penalty matrix changes to

$$W_\infty(k) = \begin{bmatrix} 80.896 & -9.2124 & -13.26 \\ -9.2124 & 9.4405 & 4.7057 \\ -13.26 & 4.7057 & 3.4094 \end{bmatrix}$$

and $\Omega_x(k) = \{x \in \mathbb{R}^{n_x} | (x - x^{ts}(k))^T W_\infty(k) (x - x^{ts}(k)) \leq 5.092\}$.

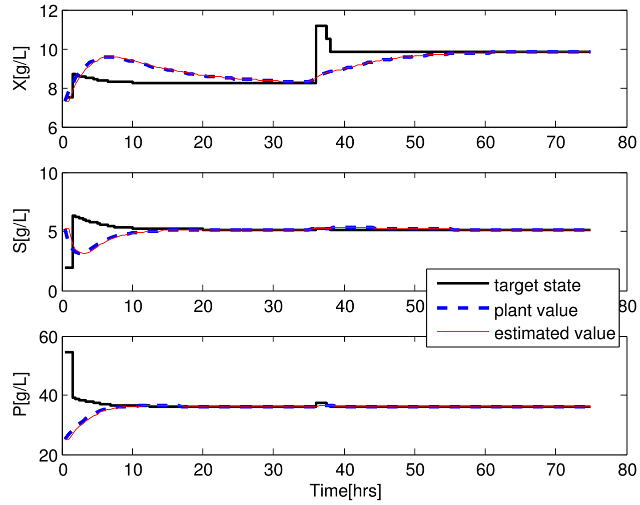

Fig. 4. State profile of scenario 2.

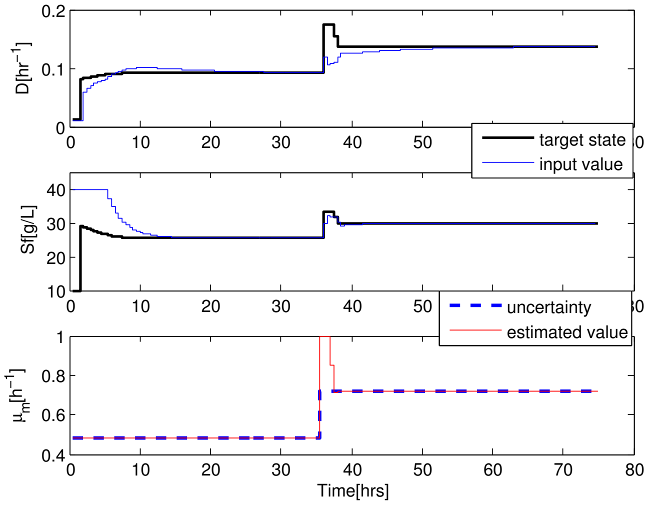

Fig. 5. Input and uncertainty profiles of scenario 2.

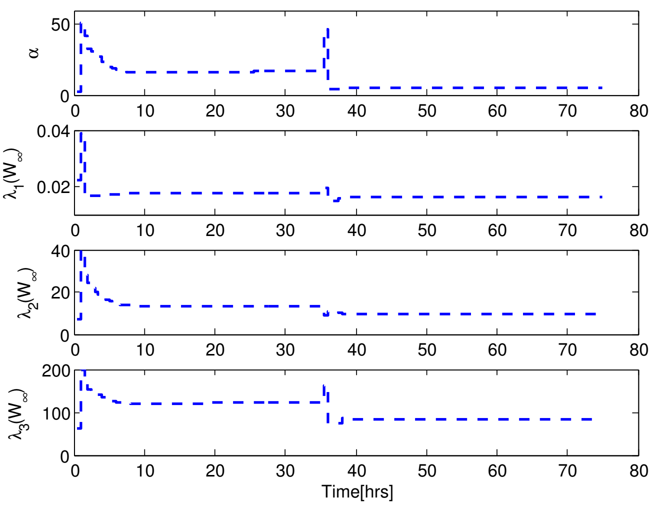

Fig. 6. Terminal region profile of scenario 2.

7. SUMMARY AND FUTURE WORK

In this work, we extend the quasi-infinite horizon NMPC scheme to deal with partial state feedback and plant-model mismatch. An adaptive extended Kalman filter based on the moving horizon framework is presented. In order to calculate the equilibrium point of the system in the presence of plant-model mismatch, two target setting optimization problems with objectives of achieving offset free output and maximizing process profit are proposed. The quasi-infinite horizon NMPC algorithm is modified based on the proposed schemes. Simulation studies of a fermentation process show that the proposed approach can track the change of uncertainty parameter and the adaptive quasi-infinite horizon NMPC is able to control the process at the desired target states, even when the plant-model mismatch is introduced.

In future, the adaptive quasi-infinite horizon NMPC will be extended to discrete-time systems. In addition, we will study the stability of the proposed approach. In particular, the effect of the estimation error on the NMPC stability in the presence of plant-model mismatch is of interest, since there are no valid separation principles.

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