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# Super-Twisting Observer-Based Output Feedback Control of a Class of Continuous Exothermic Chemical Reactors

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**Abstract:** In this paper, the problem of designing an output feedback (OF) stabilizing control scheme for (possibly open-loop unstable) exothermic reactors with temperature measurements is addressed. The proposed OF controller consists of the combination of a nonlinear state-feedback (SF) passive controller (built by passivation by backstepping) with a finite-time robustly convergent Supertwisting Observer (STO). The approach is tested with a representative example through simulations finding that the proposed controller behavior outperforms the one of its counterpart implemented with an asymptotic (infinite-time convergent) Extended Kalman Filter (EKF) [1]. 

*Keywords:* Reactor Control, Sliding Mode Observers, Output Feedback Controllers, Constructive Control

## 1. INTRODUCTION

An important class of chemical materials are produced in continuous exothermic jacketed reactors with reaction rate which grows with reactant concentration and temperature. (Lapidus *et al.* , 1977). In exothermic reactors, the interplay between heat generation-removal and reaction kinetics manifests itself as strongly nonlinear behavior, with asymmetric input-output coupling, steady-state (SS) multiplicity, and parametric sensitivity (Aris , 1969). In industry (Shinskey , 1988), volume and cascade temperature linear PI loops are employed, and the concentration is regulated by adjusting the reactant dosage via supervisory or advisory control. Thus, the objective of a process-control design scheme is to attain a closed-loop operation with an adequate compromise between safety, operability, productivity, and quality in the light of investment operation costs.

The robust Output Feedback control of continuous stirred-tank reactors with monotonic kinetics has been successfully addressed recently (Alvarez *et al.* , 2007; Diaz-Salgado *et al* , 2006) using PI controllers. The underlying idea consists in designing a robust state feedback controller that renders the closed loop system passive, is a consequence of the (robust) relative degree structure of the system. In order to construct a (robust) output feedback controller that recovers the performance of the SF, reduced order observers are designed, that are able to (asymptotically and approximately) reconstruct the state and uncertain functions, such as the kinetics, the heat transfer coefficient or the feed concentration. These estimated values are used in the SF instead of the true ones, a structure that can be implemented as a PI controller, leading to a classical structure that recovers quite well the behavior of the SF, and is considerably less model dependent due to the reconstruction of unknown and/or time-varying parameters. This state/uncertainties reconstruction is possible because of the strong observability properties of the uncertain model.

One of the limiting factors to achieve a perfect recovery of the SF performance by the OF controller is the attainable characteristics of the observer: since only an approximate estimation of the states is obtained asymptotically, due to the uncertainties and the convergence properties of a smooth observer, the control action of the OF will be only an acceptable approximation of the SF after an estimation transient time. In principle, if the states were known perfectly in a (shorter) finite time, in spite of the uncertainties/perturbations, a much better recovery of the SF properties should be obtained. This consideration constitutes the first motivation of the present work: the improvement of the reactor close-loop behavior with temperature-measurement driven control.

Sliding modes are well-known for their robustness against bounded perturbations and finite time convergence to the

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sliding surface. Sliding-Mode Observers (Utkin , 1992; Walcott and Zak , 1987) estimate robustly the state when the perturbations/measurement map is of relative degree (RD) one, but the disturbance cannot be reconstructed exactly. In order to perform this disturbance reconstruction task, a Second Order Sliding Mode algorithm, the so called Super-Twisting Algorithm (STA), has been proposed recently (Davila *et al* , 2005) for (second-order) mechanical systems. The STA robustly reconstructs, in finite-time, the states, if the perturbation is of relative degree two (RD=2), or to reconstruct the perturbation itself, when it is of relative degree one (RD=1). A Lyapunov function with an improved algorithm have been presented in (Moreno *et al.* , 2008; Moreno , 2009). The robustness (insensitivity) against bounded perturbations of the ST observer distinguishes it from other finite-time convergent observers proposed in the literature e.g. (Menold *et al.* , 2003; Sauvage *et al.* , 2007).

In this paper, we will use a Super-Twisting Observer to robustly estimate in finite time the reaction rate of continuous stirred-tank reactors, that can be considered as an uncertain quantity in the reactor model. It is well-known that the robustness of sliding mode controllers is obtained at the cost of a strong control actions, leading to chattering. The STA-based estimation scheme increase considerably the performance with respect to convergence time and robustness is traded off by a high sensitivity to sensor noise. The present paper can be considered as a first step towards the study of the applicability of these kind of algorithms in chemical process control.

According to our simulation results with a representative example, from the perspective of a realistic industrial situation, the proposed behavior recovery scheme yields a better compromise than the one of asymptotic observer-based controller in the sense that the speed and robustness advantages of the ST observer-based controller outweights the associated increase of sensitivity to sensor noise.

## 2. PROBLEM STATEMENT

Consider the class of continuous stirred-tank reactors where a monotonic reaction takes place. Reactant is fed to the tank, and heat exchange is enabled by a cooling jacket. From standard kinetics (Aris , 1969), and viscous heat exchange considerations, the reactor dynamics are described by the following mass and energy balances:

$$\begin{aligned}
\dot{v} &= -\varepsilon_P \rho\,(c,T)\,v + q_e - q := f_v\,(c,T,v,q_e,q)\,,\\
y_v &= z_v = v\\
\dot{T} &= \Delta\rho\,(c,T) - (q_e/v)\,(T-T_e) - \xi(x)\,(T-T_j)\\
&:= f_T\,(x,T_e,q_e)\,,\\
y_T &= z_T = T\\
\dot{T}_j &= \varpi v \xi(x)\,(T-T_j) - \varpi_j q_j\,(T_j - T_{je})\\
&:= f_j\,(x,T_{je},q_j)\\
y_j &= T_j\\
\dot{c} &= -\rho\,(c,T) + (q_e/v)\,(c_e - c) := f_c\,(c,T,v,q_e,c_e)\\
z_c &= c
\end{aligned} \tag{1}$$

where $x = (v, T, T_j, c)$ and

$$c = \frac{C}{C^0},\ \Delta := \frac{Q_r C^0}{\rho c_p},\quad \xi(x) := \frac{AU(c,T,T_j)}{\rho_m v c_p},$$

$$\varpi = \frac{\rho_m c_p}{\rho_j v_j c_{pj}},\quad \varpi_j = \frac{\rho_f c_{pc}}{\rho_j v_j c_{pj}},\quad \rho\,(c,T) := R(cC^0,T)/C^0$$

$$\partial_c R > 0 \Longrightarrow \partial_c \rho(c,T) = \rho_c(c,T) > 0 := \rho \text{ c-isotonic}$$

$v$ is the volume of the reacting mixture, $T_j$ (or $T$) is the jacket (or reacting mixture) temperature, $c$ is the reactant concentration in dimensionless form (referred to reactant pure concentration $C^0$), $q_e$ (or $q$) is the feed (or exit) volumetric flowrate, $R$ is the molar reaction rate per unit volume with monotonic (growing) dependency on concentration $C$ ($\partial_c R > 0$), $\rho$ is the reaction rate expressed in terms of the dimensionless concentration, $Q_r$ is the heat of reaction per reactant mole and $\Delta$ is the corresponding adiabatic temperature rise, $U$ is the heat transfer coefficient, $A$ is the reactor mixture-jacket heat exchange area, $\rho_m$ (or $\rho_j$) (or $\rho_f$ is the reactor mixture (or jacket) (or cooling fluid) density, $c_p$ (or $c_{pj}$) (or $c_{pj}$) is the reacting mixture (or jacket) (or cooling fluid ) heat capacity per unit volume, $v_j$ is the jacket volume, and $\varepsilon_p$ is the contraction factor due to the difference between reactant and product densities. The measured outputs are: the volume $y_v$, and the jacket (or reacting mixture) temperature $y_j = T_j$ (or $y_T = T$). The measured inputs are: the jacket (or reactor) feed temperature $T_{je}$ (or $T_e$), and the feed concentration $c_e$ is an unmeasured input. Typically, the jacket-to-reacting mixture system heat capacity quotient $\varpi v$ ranges ranges from $\approx 1$ in a laboratory reactor to $\approx 5$ (or 50) in a pilot plant (or industrial) reactor, meaning that the jacket and reactor temperatures can have comparable or separated dynamics. The reactor can exhibit steady-state (SS) multiplicity, and may have to operate about an open-loop unstable SS.

### *2.1 Non-linear FF-SF control*

Assuming that the detailed reactor model (1) is available and its state and inputs are known, the reactor nonlinear feedforward (FF) state-feedback (SF) control problem can be addressed via passivation by backstepping. The 3-input ($\mathbf{u}$) 3-output ($\mathbf{z}$) reactor system (1) does not have RD's, meaning that the FF-SF problem cannot be solved with static control. From the application of the dynamic extension procedure (Isidori , 1995), the reactor system (1) augmented with the dynamic extension (2a) has RD vector $\kappa$ (2b) with respect to the control input-regulated output pair $(\mathbf{u}, \mathbf{z})$

$$\dot{q}_e = v_{q_e},\quad \kappa = (\kappa_v^c, \kappa_c^c, \kappa_T^c) = (1,2,2),\quad \kappa_v^c + \kappa_c^c + \kappa_T^c = 5 \tag{2}$$

if, in a compact set about the nominal operation:

$$\text{(i) } c \neq c_e,\ \text{(ii) } f_T : T_j - monotonic,\ \text{(iii) } T_{je} \neq T_j. \tag{3}$$

Conditions (3) are always met because they say that: (i) the reactant is part of the reacting mixture, (ii) there is reactor-jacket heat exchange, and (iii) the heat exchange rate is uniquely determined by the jacket temperature. The related zero-dynamics are trivially given by the nominal SS $\left(\bar{v}, \bar{T}, \bar{T}_j, \bar{c},\right)$.

On the basis of the inverse optimality approach (Sepulchre *et al.* , 1997) the relative degree obstacle of the geometric controller can be removed by passivation via backstepping. For this aim introduce the quadratic candidate Lyapunov function

$$V = \left(e_c^2 + e_T^2 + e_j^2 + e_v^2\right)/2 > 0,\quad e_j^* = T_j - T_j^* \tag{4}$$

where $T_j^*$ is a jacket temperature virtual control to be determined, and express the function $f_T$ (5) as follows

$$f_T = f_T^* + \bar{a}_T e_j^*, \quad f_T^* = f_T(x, T_e, q_e) - \bar{a}_T(T_j - T_j^*) \tag{5}$$

$$\bar{a}_T = \partial_{T_j} f_T(\bar{c}, \bar{T}, \bar{T}_j) > 0 \tag{6}$$

Write the corresponding dissipation

$$\dot{V} = e_v f_v + e_T(f_T^* + \bar{a}_T e_j^*) + e_j^*(f_j - \dot{T}_j^*) + e_c f_c$$

Perform *backstepping* by moving $\bar{a}_T e_j^* e_j$ from the third to the fourth term

$$\dot{V} = e_v f_v + e_T f_T^* + e_j^*(f_j + \bar{a}_T e_T - \dot{T}_j^*) + e_c f_c$$

and enforce the implicit control expressions

$$\begin{array}{c} f_v = -k_v e_v, \quad f_T = -k_T e_T, \\ f_j + \bar{a}_T e_T - \dot{T}_j^* = -k_j e_j^*, \quad f_c = -k_c e_c \end{array} \tag{7}$$

to obtain the negative dissipation

$$\dot{V} = -(k_v e_v^2 + k_T e_T^2 + k_j e_j^{*2} + k_c e_c^2)$$

with the *closed loop stable dynamics* $\dot{e}_v = -k_v e_v$, $\dot{e}_T = -k_T e_T + \bar{a}_T e_j^*$, $\dot{e}_j^* = -k_j e_j^* - \bar{a}_T e_T$, $\dot{e}_c = -k_c e_c$. The (unique) solution for $(q, T_j^*, q_j, q_e)$ of (7) yields the *stabilizing passive FF-SF controller*

$$q = k_v(v - \bar{v}) + q_e - \varepsilon_P \rho(c, T) v \tag{8a}$$

$$T_j^* = T_j + \frac{\left[\begin{array}{r} -k_T(T - \bar{T}) - \Delta\rho(c, T) + \\ +\frac{q_e}{v}(T - T_e) + \xi(x)(T - T_j) \end{array}\right]}{\bar{a}_T} \tag{8b}$$

$$q_j = \frac{\left[\begin{array}{r} +k_j(T_j - T_j^*) + \varpi v \xi(x)(T - T_j) + \\ +\bar{a}_T(T - \bar{T}) - \dot{T}_j^* \end{array}\right]}{(T_j - T_{je})\varpi_j} \tag{8c}$$

$$= \mu_j(\hat{T}, y_j, y_v, c_e, T_e, q, T_j, T_j^*)$$

$$q_e = v[-k_c(c - \bar{c}) + \rho(c, T)]/(c_e - c) \tag{8d}$$

$$\dot{T}_j^* = \nu^*(x, q, q_e, T_e, T_{je}, c_e, \dot{T}_e, \dot{q}_e), \tag{8e}$$

needs $(\rho_c, \rho_T, \xi_c, \xi_T)$

Our *problem* consists in designing an observer for concentration and reaction rate values, and use this estimates and the measurements $(y_v, y_T, y_{T_j})$, in the stabilizing passive FF-SF controller to regulate the reactor operation about a (possibly open-loop unstable) nominal SS. We are interested in: (i) a closed-loop robustly stable functioning, and (ii) the attainment, as much as possible, of robustness and modeling independency features.

From this desired features, our choice is a sliding-mode observer because of its natural robust properties. Such mixture of the the FF-SF controller and sliding mode observers has never been performed for this reactor, so it represents the main contribution of this paper.

## 3. SUPER-TWISTING OBSERVER (STO)

According to the heat balance (1) the reactor temperature $T$ and reaction rate $\rho$ are related by (1)

$$\dot{T} = \Delta\rho(c, T) - (q_e/v)(T - T_e) - \xi(T - T_j) \ ,$$

where for simplicity $\xi$ is assumed to be constant and known. In this case the unknown term is $\rho$ and $T$ is the measured variable. It is evident that the RD between them is one so the structural requirement to implement a sliding-mode observer. Consider the following one

$$\begin{aligned} \dot{\hat{T}} &= -k_1 \left|\tilde{T}\right|^{1/2} \operatorname{sign}\left(\tilde{T}\right) - k_2 \tilde{T} + \Delta\hat{\rho} \\ &\quad - (q_e/v)(T - T_e) - \xi(T - T_j) \ , \\ \dot{\hat{\rho}} &= -k_3 \operatorname{sign}\left(\tilde{T}\right) - k_4 \tilde{T} \\ \hat{c} &= \rho^{-1,c}(\hat{\rho}, T) \end{aligned} \tag{9}$$

where $\tilde{T} = \hat{T} - T$ is the measurement error, and $\rho^{-1,c}$ is the inverse of $\rho$ with respect to $c$; $k_1$-$k_4$ are adjustable gains. This observer is a modification of the classical Super-Twisting observer (Davila *et al* , 2005) proposed in (Moreno *et al.* , 2008) and its aim is the estimation of the reaction rate $\rho$ by means of the measurement of the temperature $T$. The concentration is estimated by inverting the relation between concentration and rate $\rho(c, T)$, for a given temperature $T$, what is possible for monotonic kinetics. If $\rho(c, T)$ is perfectly known, then the estimation of $c$ will be independent of the unknown input $c_{in}$. The estimation error ($\tilde{T} = \hat{T} - T$, $\tilde{\rho} = \hat{\rho} - \rho$) dynamics are given by

$$\begin{aligned} \dot{\tilde{T}} &= -k_1 \left|\tilde{T}\right|^{1/2} \operatorname{sign}\left(\tilde{T}\right) - k_2 \tilde{T} + \Delta\tilde{\rho} \\ \dot{\tilde{\rho}} &= -k_3 \operatorname{sign}\left(\tilde{T}\right) - k_4 \tilde{T} + \delta(t) \end{aligned} \tag{10}$$

where

$$\delta(t) = \dot{\rho}(c, T) = \frac{\partial\rho(c, T)}{\partial c}\left[-\rho(c, T) + (q_e/v)(c_e - c)\right] + \\ + \frac{\partial\rho(c, T)}{\partial T}\left[\Delta\rho(c, T) - (q_e/v)(T - T_e) - \xi(T - T_j)\right]$$

In a compact region of the state space, as for example the physical operation region, this function is bounded by some constant$|\delta(t)| \leq \delta$. Using a Lyapunov function, and following the arguments in (Moreno , 2009), it can be shown that positive gains $k_1, \cdots, k_4$ can be chosen, so that $\tilde{T} \to 0$ and $\tilde{\rho} \to 0$ converge to zero in finite time, despite of the perturbation $\delta(t)$.

This is a surprising result for an observer for two reasons: i) the finite time convergence is a property not achieved by the usual smooth (differentiable) observers considered in the literature. For the observer (9) this is possible due to the non locally Lipschitz terms $|\tilde{T}|^{1/2}\operatorname{sign}(\tilde{T})$, and $\operatorname{sign}(\tilde{T})$ introduced in the correction terms of the observer and ii) the exact convergence, despite of the presence of persistently acting "perturbation" term $\delta(t)$ in the observation error requires the action of a discontinuous correction term, provided in the observer (9) by the signum function $\operatorname{sign}(\tilde{T})$, with gain $k_3 > \delta$. These beneficial effects are only met by the use of this kind of terms that have strong effects in the neighborhood of the origin of the error space, i.e. (i) the "gain" of the function $\operatorname{sign}(\tilde{T})$ is infinity at $\tilde{T} = 0$, and (ii) the derivative of the function $|\tilde{T}|^{1/2}\operatorname{sign}(\tilde{T})$ is also infinity at $\tilde{T} = 0$. Thus, in a small neighborhood of zero error the observer is of high gain.

In a finite-time way that is analogous to the asymptotic (infinite-time) estimation of the heat transfer coefficient $\xi(x)$ (Gonzalez *et al.* , 2004), it is possible to estimate $\xi$ from the measurement of $T_j$ and $\varepsilon_P$ from the measurement of $v$. However, in this paper, for simplicity reasons, we restrict ourselves to the presented case, since it contains the basic issues.

## 4. OF CONTROLLER

In this section, the behavior of the exact model-based passive nonlinear controller (8) is recovered via the coupling of a concentration observer and the stabilizing passive FF-SF controller, with emphasis on the attainment of robustness, and model independency features.

### *4.1 OF Control with supertwisting observer*

The combination of the nonlinear passive SF controller (8) with the ST observer (9) yields the proposed OF controller

$$\begin{aligned} \dot{\hat{T}} &= -k_1 \left|\tilde{T}\right|^{1/2} \operatorname{sign}\left(\tilde{T}\right) - k_2\tilde{T} + \Delta\hat{\rho} \\ &\quad - (q_e/v)(T - T_e) - \xi (T - T_j) \ , \\ \dot{\hat{\rho}} &= -k_3 \operatorname{sign}\left(\tilde{T}\right) - k_4\tilde{T} \\ \hat{c} &= \rho^{-1,c}(\hat{\rho}, T) \end{aligned} \tag{11}$$

$$q_e = y_v(-k_c(\hat{c} - \bar{c})) + \hat{\rho}/(c_e - \hat{c}) \tag{12}$$

$$q = -k_v(y_v - \bar{v}) + q_e - \varepsilon_P \hat{\rho} v \tag{13}$$

$$q_j = \mu_j(\hat{T}, y_j, y_v, c_e, T_e, q, T_j, T_j^*, \dot{T}_j^*) \tag{14}$$

Note that by estimating $\rho$ the control scheme avoids the knowledge of its model and therefore obtains complete robustness to uncertainty on the parameters involved. That is, the proposed controller is less interactive and model dependent

### *4.2 OF Control with EKF*

To compare the proposed scheme (11-14) with a standard observer approach, a reduced-order EKF for the reactor system (1) is set, and its combination with the nonlinear FF-SF passive controller(8) yields the *EKF-based OF controller.*

$$\dot{\hat{c}} = -\rho(\hat{c}, \hat{T}) + [q_e(c_e - \hat{c})]/y_v - \frac{\sigma_{12}}{r_{11}}\tilde{T}$$

$$\dot{\hat{T}} = \Delta\rho(\hat{c}, \hat{T}) + [q_e(T_e - \hat{T})]/y_v - \xi(\hat{T} - T_j) - \frac{\sigma_{22}}{r_{11}}\tilde{T}$$

$$\dot{\sigma}_{11} = 2\big[(-\rho_c(\hat{c}, \hat{T}) + \frac{q_e}{y_v})\sigma_{11} - \rho_T(\hat{c}, \hat{T})\sigma_{12}\big] + q_{11} - \frac{\sigma_{12}^2}{r_{11}}$$

$$\begin{aligned} \dot{\sigma}_{22} &= 2\big[\Delta\rho_c(\hat{c}, \hat{T})\sigma_{12} + (\Delta\rho_T(\hat{c}, \hat{T}) - \frac{q_e}{y_v} - \xi)\sigma_{22}\big] \\ &\quad + q_{22} - \frac{\sigma_{22}^2}{r_{11}} \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{\sigma}_{12} &= \Delta\sigma_{11}\rho_c(\hat{c}, \hat{T}) + [-\rho_c(\hat{c}, \hat{T}) + \frac{q_e}{y_v} + \Delta\rho_T(\hat{c}, \hat{T}) \\ &\quad - \frac{q_e}{y_v} - U]\sigma_{12} - \rho_T(\hat{c}, \hat{T})\sigma_{22} - \frac{\sigma_{12}\sigma_{22}}{r_{11}} + q_{21} \end{aligned}$$

$$q_e = y_v(-k_c(\hat{c} - \bar{c})) + \rho(\hat{c}, \hat{T})/(c_e - \hat{c}) \tag{16}$$

$$q = -k_v(y_v - \bar{v}) + q_e - \varepsilon_P \rho\left(\hat{c}, \hat{T}\right) v \tag{17}$$

$$q_j = \mu_j(\hat{T}, y_j, y_v, c_e, T_e, q, T_j, T_j^*, \dot{T}_j^*) \tag{18}$$

where $\sigma_{nn}$ are the elements of the error covariance matrix. In sum, this controller: (i) consists of 5 ordinary differential equations (ODE's) and 5 algebraic ones (AE's), and (ii) needs the detailed model (1).

The EKF based controller is strongly interactive, that is, the combined temperature-concentration controller depends on the reaction rate and heat transfer coefficient functions.

Table 1. Steady states

| $k = e^{25}, \gamma = 1e4, \xi = c_e = \varpi_j = 1, T_e = 350, \varepsilon_P = 0.15$ <br> $\Delta = 200, \varpi = 10, q_e = q = 1, q_j = 8.7719, T_{je} = 293$ | | | |
|---|---|---|---|
| steady-state | $S_E$ | $U$ | $S_I$ |
| concentration [mol/L] | 0.993 | 0.500 | 0.030 |
| temperature [K] | 332.71 | 400.00 | 463.94 |
| jacket temperature [K] | 314.15 | 350.00 | 384.06 |
| volume [L] | 1.0 | 1.0 | 1.0 |
| local condition | stable | unstable | stable |

## 5. CLOSED-LOOP STABILITY

The stability analysis of the control algorithm with the ST observer is very simple, at least in the nominal case: since the observer converges in finite time, from this moment on the asymptotically stable closed-loop dynamics are exactly the ones of the exact model-based nonlinear state feedback controller. The only issue to consider here is to assure that the controller does not escape to infinity before observer has converged. This is however simple to solve by, for example, saturating the controller.

In our approach the temperature control component, which performs the key stabilizing task and a great deal of the concentration regulation task, is independent of the reaction rate-heat transfer function pair, and the component which regulates the concentration depends only on the reaction rate.

## 6. APPLICATION EXAMPLE

Model functions and parameter values listed in Table 1 were adapted from an experimental catalytic reactor (Baratti *et al.* , 1993). The reaction rate function is given by:

$$\rho(c, T) = (cke^{-(\frac{\gamma}{T})}) \quad [mol/L \cdot \min]$$

The reactor has three SS's (listed in Table 1), two of them corresponding to extinction ($S_E$) and ignition ($S_I$) stable operations, and one being unstable (U). To subject the controller to a severe test the reactor must be operated about the unstable SS The initial conditions for closed-loop testing were about the unstable SS: $x(0) = [0.45, 397, 353, 0.9]^T$

*Behavior testing strategy*

Two observer-controller schemes are compared in the closed loop behavior: (i) the proposed STO-based controller, and (ii) the EKF-based controller. Using the same control gains for both controllers a nominal and a robust case were considered. For the nominal case the controllers use the exact model functions of the reactor and a known and constant value of all inputs. Even though a nominal case is wanted, it is considered a step change of the not measured input $c_e$ at $t = 3$ of $-10\%$ of its nominal value.

The robust case tries to reflect a realistic worst case situation. The robust conditions considered for both controllers are: (i) errors in the mathematical model parameters: $k_{err} = k_{nom} - 9.5\%$, $\gamma_{err} = \gamma_{nom} + 0.5\%$, $\Delta_{err} = \Delta_{nom} + 5\%$, $\varepsilon_{Perr} = \varepsilon_{Pnom} + 0.5\%$, (ii) gaussian measurement noises injected to the measured variables with standard deviations corresponding to commercial "cheap" instruments, ($\pm 3$ Kelvin and $\pm 0.3$ liters), (iii) oscillatory changes in the measured inputs with a frequency close to the natural resonance mechanism [$T_e = 350 + 2\sin(4\pi t)$, $T_{je} = 293 + 2\cos(4\pi t)$], and (iv) the same step change in $c_e$ as in the nominal case.

*Tuning.*

The ST-observer-based controller was tuned following the guidelines given in (Moreno , 2009): $k_1 = 56$, $k_2 = 10$, $k_3 = 210$, $k_4 = 10$, and its initial conditions are $\hat{T}(0) = 410$ (as in the EKF), $\hat{\rho}(0) = 0.737$.

The EKF-based controller (15-18) was tuned with the method from (Baratti *et al.* , 1993): $q_{11} = 7.590 \times 10^{-3}, q_{22} = 6.006 \times 10^{-7}, q_{21} = 0, r_{11} = 2.376 \times 10^{-7}, \hat{c}(0) = 0.5, \hat{T}(0) = 410, \sigma_{11}(0) = \sigma_{12}(0) = \sigma_{22}(0) = 0$.

*Nominal Behavior.* With the actual parameter values, the proposed OF control with ST-Observer (11-14) and the EKF-based (15-18) controllers were applied to the reactor, and the result behaviors are presented in Fig. 1.

The simulation results show that both controllers: (i) stabilize the reactor, and (ii) present an overall similar behavior. In the case of the ST observer the controller gets saturated in some periods of time since negative flow rates are of course not allowed. In the case of the EKF-based controller, the control actions occurs in a coordinated manner away from saturation. The convergence of the estimation errors is similar but it can be seen that the EKF estimates $T$ and $\rho$ in a slightly faster way.

In Fig. 1 a standard drawback of sliding modes appears, that is the high-frequency switching, in this case in the estimation of $T$ and $\rho$. In order to deal with it, actuator filters have been introduced in the simulation in order to smooth the control inputs of the system. The response is presented in Fig. 2 and one can see that the performance is a little bit affected but the flow rates $q$, $q_e$, $q_j$ are smoother. The introduced filters are $T_f(s) = \frac{1}{0.0265s+1}$, that is, with a cutoff frequency of 37.7 $(rad/min)$.

*Robust Behavior.* The proposed OF control with ST-Observer-based (11-14) and the EKF-nonlinear (15,16-18) controllers were tested under the four previously described perturbed conditions. The corresponding closed-loop responses are presented in Fig. 3.

The robust behavior shows that both controllers achieve regulation around the desired set point achieving: (i) practical stability with respect to noisy measurements, initial condition errors and known input persistent oscillation ($t < 3$) with a response time similar to the one presented in the nominal case and a concentration regulation offset much greater for the EKF-based controller compared to the ST-observer-based one; the offset was smaller for volume and temperature regulation (model independent control components); and (ii) perturbation rejection ($t > 3$) with respect to known input persistent oscillation, noisy measurements and not measured input step change.

The estimation of $T$ is satisfactory with both observers whereas for $\rho$ the difference is significative: while the EKF presents a mean error of around 20% caused by the parameter uncertainty, the ST-observer reconstruct $\rho$ perfectly since no model is needed to do it. This allow the ST-observer-controller scheme to obtain a much better concentration regulation offset.

*General comparison.* That the EKF based controller performs better when the model is perfect is consistent with the passivability and strong observability properties of the reactor (Lopez *et al.* , 2004). This means that the employment of the detailed model favors the EKF over the STO (intended for robust purposes at the cost of reconstruction quality and performance at the transient period) In the robust case the STO outperforms the EKF, as expected from the fact that the STO is intended to cope with uncertainties and finite-time convergence. From a practical perspective the interesting case is the robust one.

## 7. CONCLUSIONS

A passivation by backstepping nonlinear state-feedback controller has been implemented with a robustness-oriented finite-time convergent ST observer. The behavior of the resulting OF controller has been favorably compared with the one of the same state-feedback controller with an asymptotically convergent EKF, in terms of a suitable compromise between regulation speed, robustness, and control effort. The results can be seen as a first step towards the consideration of sliding-mode, robust, finite-time observers for chemical processes in general.

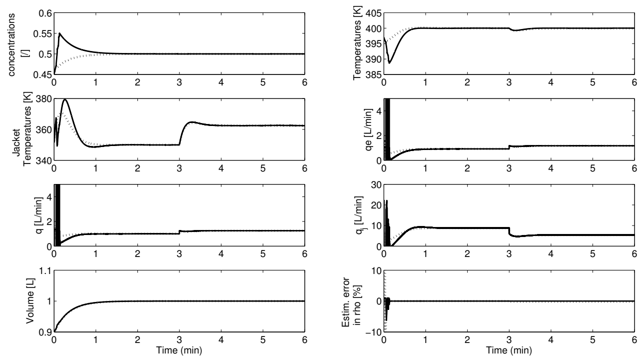


Fig. 1. Closed-loop nominal reactor behavior with ST (cont. line) and EKF - observer based (dotted line) controllers without actuator filters.

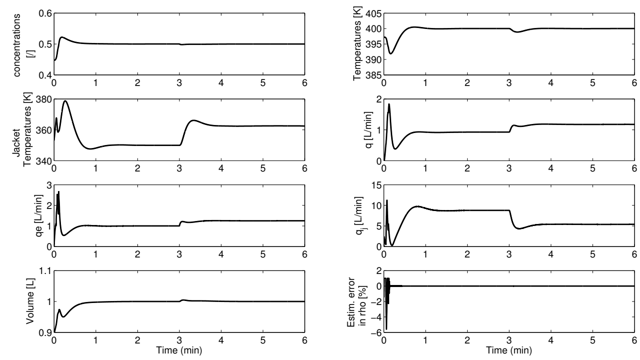


Fig. 2. Closed-loop nominal reactor behavior with ST observer based controllers with actuator filters.

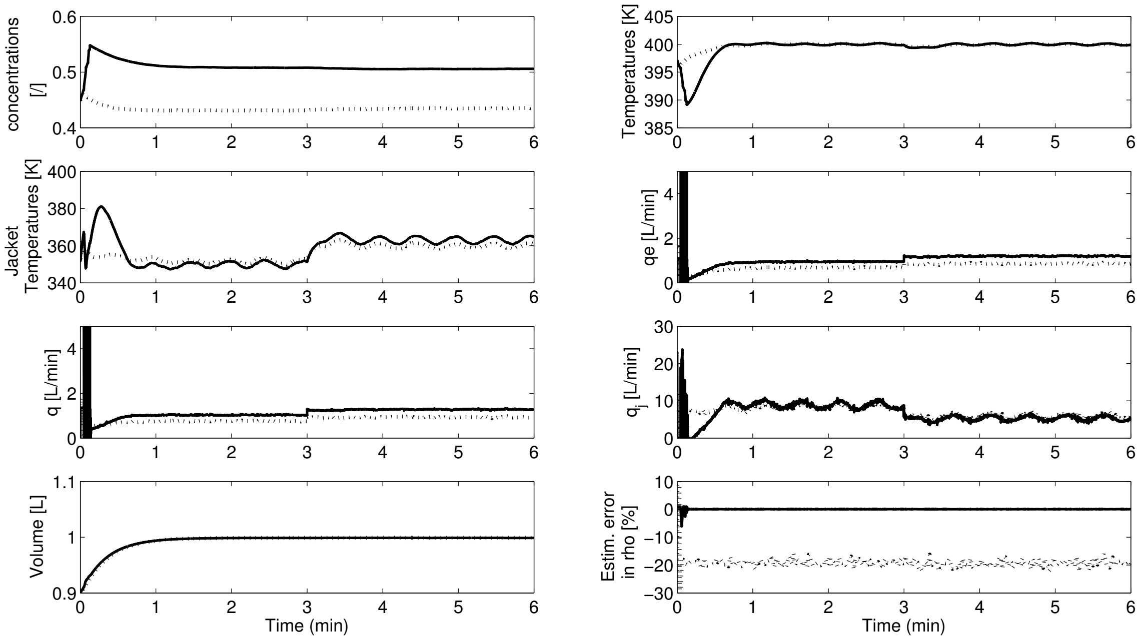


Fig. 3. Closed-loop robust reactor behavior with ST (cont. line) and EKF - observer based (dotted line) controllers without actuator filters.

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