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Output-Feedback Stabilization of Continuous Bioreactors in the Presence of Biomass Decay

G. Savoglidis*, C. Kravaris* and G. Lyberatos*

**Department of Chemical Engineering, University of Patras, 26500 Patras, Greece*
(Tel: 2610-996 339; e-mail: kravaris@chemeng.upatras.gr).

Abstract: This paper studies the problem of designing output feedback controllers for enlarging the stability region of continuous stirred microbial bioreactors, in the presence of biomass decay. A specific application is in anaerobic digestion, where the stability region can be very small if the operating steady state is selected to maximize the methane production rate. A proportional output feedback control law is proposed and the size of the stability region of the closed-loop system is estimated using Lyapunov methods. The results show that, even though stability is not global, the guaranteed stability region is large enough to ensure proper operation of the reactor in the presence of physically realistic disturbances.

Keywords: Stabilization, Lyapunov methods, Bioreactor control, Nonlinear analysis, Process control.

1. INTRODUCTION

Continuous stirred microbial bioreactors, often called chemostats, cover a wide range of applications; specialised "pure culture" biotechnological processes for the production of specialty chemicals (proteins, antibiotics etc.) as well as large-scale environmental technology processes of mixed cultures such as wastewater treatment. The dynamics of the chemostat is often adequately represented by a simple dynamic model involving two state variables, the microbial biomass x and the limiting organic substrate s.

A general model for chemostat dynamics that accounts for cell mortality is of the form (see e.g. Smith and Waltman, 1995, Chapter 1; Bailey and Ollis, 1986):

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \mu(s)x - K_d x \\ \frac{ds}{dt} &= D(S_0 - s) - \frac{1}{Y_{x/s}}\mu(s)x \end{aligned} \tag{1}$$

where D is the dilution rate, S_0 is the feed substrate concentration, $Y_{x/s}$ is a biomass yield factor, K_d is the mortality rate constant and $\mu(s)$ is the specific growth rate, a given function of s. The most widely used expressions for the specific growth rate are the Andrews or Haldane equation

$$\mu(s) = \frac{\mu_{\max} s}{K_S + s + \dfrac{s^2}{K_I}} \tag{2a}$$

and the Monod equation

$$\mu(s) = \frac{\mu_{\max} s}{K_S + s} \tag{2b}$$

where $\mu_{\max}$ is the maximum specific growth rate, K_S the Monod kinetic constant and K_I the substrate inhibition kinetic constant. The Monod kinetics (2b) is a special case of Andrews kinetics (2a) corresponding to $\frac{1}{K_I} = 0$.

For the control of a chemostat, the manipulated input is the dilution rate D and the objective is to regulate the chemostat at specified design conditions. The problem of chemostat stabilization and control has received considerable attention in recent literature (see e.g. De Leenheer and Smith, 2003; Gouze and Robledo, 2006; Harmard et al., 2006; Karafyllis et al., 2008).

One important class of applications is related to anaerobic digestion, which is a key process in wastewater treatment, sludge management, energy from biomass, etc. Anaerobic digestion is a complex biochemical process, in which organic compounds are mineralised to biogas (a useful energy product), consisting primarily of methane and carbon dioxide, through a series of reactions mediated by several groups of microorganisms. Under normal (or balanced) operation, the rate of production of the intermediates is matched by their consumption rate; hence there is very little accumulation of these compounds. However, disturbances such as an increase in the concentration of organic compounds in the feed (organic overload), an increase in feed flow rate (hydraulic overload), presence of toxins in the feed, and temperature fluctuations, can cause an imbalance in the process (Switzenbaum et al., 1990), which results in accumulation of volatile organic acids. These acids cause a drop in the pH, inhibiting methanogenesis and the reactor fails. Such a failure has major consequences in the process economics since digester recovery can be a very cumbersome and costly process. For this reason, the development of appropriate control schemes for anaerobic digesters has received significant attention in the literature (Perrier and Dochain, 1993; Pullammanappallil, 1998; Antonelli and A. Astolfi, 2000; Pind et al., 2003; Mailleret et al., 2003; Syrou et al., 2004).

For the description of the dynamics of anaerobic digestion, the mathematical model (1) can be used. This system of equations describes methanogenesis, the ultimate step in anaerobic digestion, which is rate limiting and is usually the most sensitive step. In other words, it is assumed that the bioconversion of organics into fatty acids (hydrolysis and acidification) has fast kinetics.
The measured output of the system is the methane production rate

$$Q = Y_m \mu(s) x \tag{3}$$

where Y_m is the yield coefficient for methane production.

The purpose of this work is to study the problem of output-feedback stabilization of a bioreactor whose dynamics follows (1) with measured output of the form (3), the motivation coming from control problems for anaerobic digestion processes. Section 2 examines the equilibrium and stability properties of the open-loop system (1), calculates the optimal operating conditions where the system must be regulated, and explains the nature of the control problem. In Section 3, a simple proportional output feedback controller is studied and the stability properties of the resulting closed-loop system are established via Lyapunov analysis (Khalil, 1996).

2. OPEN-LOOP SYSTEM PROPERTIES AND OPTIMAL OPERATING CONDITIONS

Consider the dynamic system (1), with $\mu(s)$ given by (2a) or (2b), where the dilution rate D is the input variable of the system and $S_0, K_d, Y_{x/s}, \mu_{\max}, K_S, K_I$ are constant parameters. The following assumption will be made throughout this paper:

$$K_d < \mu(S_0) \tag{H}$$

2.1 Equilibrium curve

The system steady states can be calculated from the set of equations:

$$x_s = Y_{x/s}(S_0 - s_s)\left(1 - \frac{K_d}{\mu(s_s)}\right) \qquad D_s = \mu(s_s) - K_d \tag{5}$$

Only positive steady states $(x_s > 0,\ s_s > 0,\ D_s > 0)$ are physically meaningful and need to be considered.

Assumption (H) is sufficient to guarantee the existence of positive steady states. In particular, under assumption (H), the equilibrium curve (locus of points (x_s, s_s) with $x_s > 0,\ s_s > 0,\ D_s > 0$) has the shape shown in Figure 1.

In Figure 1, s_* represents the smallest root of the equation $\mu(s) = K_d$:

$$s_* = K_S \cdot \frac{2\frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) + \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)^2 - 4\left(\frac{K_s}{K_I}\right)\left(\frac{K_d}{\mu_{\max}}\right)^2}} \tag{6a}$$

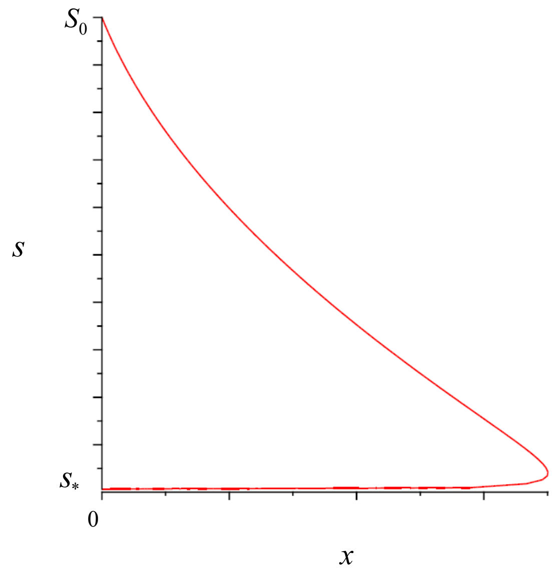

Figure 1. Equilibrium curve of the open-loop system

In the special case of Monod kinetics $\left(\frac{1}{K_I} = 0\right)$,

$$s_* = K_S \cdot \frac{\frac{K_d}{\mu_{\max}}}{1 - \frac{K_d}{\mu_{\max}}} \tag{6b}$$

is the unique root of $\mu(s) = K_d$.

<u>Remark</u>: For $\frac{1}{K_I} \neq 0$, the equation $\mu(s) = K_d$ has two roots, which are both real and positive; the smallest root s_* is given by (6a) and the largest root is

$$s^* = K_S \cdot \frac{2\frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) - \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)^2 - 4\left(\frac{K_s}{K_I}\right)\left(\frac{K_d}{\mu_{\max}}\right)^2}} \tag{7}$$

Assumption (H) implies that

$$s_* < \min\left\{S_0, \sqrt{K_s \cdot K_I}\right\} \quad \text{and} \quad s^* > \max\left\{S_0, \sqrt{K_s \cdot K_I}\right\}$$

For $\frac{1}{K_I} = 0$ (Monod kinetics), assumption (H) implies that the unique root s_* of $\mu(s) = K_d$ satisfies $s_* < S_0$.

Finally, it is important to point out that in practice, $\frac{K_d}{\mu_{\max}} \ll 1$ (usually significantly less than 0.1), therefore, in practice, *s_* is a small fraction of K_s*. Typically, s_* is a couple of orders of magnitude smaller than S_0.

2.2 Optimal steady state for methane production

For a given feed, there is a value of the dilution rate that maximises the methane production rate. The steady state that corresponds to the maximization of methane production rate, i.e. $Q_s = Y_m \mu(s_s) x_s = Y_m Y_{x/s}(\mu(s_s) - K_d)(S_0 - s_s)$, draws technical interest. The methane production rate is maximized when:

$$\frac{dQ_s}{ds_s} = 0 \Leftrightarrow \frac{d\mu(s_s)}{ds}(S_0 - s_s) = \mu(s_s) - K_d$$

Substituting $\mu(s)$ from (2a) to the above expression, leads to

$$\frac{\mu_{\max}\left(K_s - \frac{s_s^2}{K_I}\right)}{\left(K_s + s_s + \frac{s_s^2}{K_I}\right)^2}(S_0 - s_s) = \frac{\mu_{\max} s_s}{K_s + s_s + \frac{s_s^2}{K_I}} - K_d$$

For $\frac{1}{K_I} \neq 0$, the above is a quartic equation. Only one of its roots corresponds to a positive steady state ($x_s > 0,\ s_s > 0$) and it represents the optimal steady state value s_s^{opt}.

In the special case of Monod kinetics $\left(\frac{1}{K_I} = 0\right)$, the above simplifies to a quadratic equation and the optimal steady state value is given by

$$s_s^{opt} = \frac{S_0 + K_s \frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) + \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)\left(1 + \frac{S_0}{K_s}\right)}} \qquad (8)$$

For the following values of the parameters:

$S_0 = 10000\ mg/l,\ Y_{x/s} = 0.05\ mg/mg,\ K_d = 0.05\ d^{-1}$

$\mu_{\max} = 0.5\ d^{-1},\ K_S = 100\ mg/l,\ K_I = 4000\ mg/l$

the optimal steady state is $s_s^{opt} = 519.148\ mg/l$. This corresponds to $x_s = 411.355\ mg/l$ and $D_s = 0.328097\ d^{-1}$.

The above numerical values of the parameters and the resulting optimal steady state conditions will be used in the numerical calculations throughout this paper.

2.3 Local asymptotic stability

The eigenvalues of the linearization of the open-loop system (1) are the roots of the quadratic polynomial

$$\lambda^2 + \left(1 - \frac{K_d}{\mu(s_s)}\right)\left(\mu(s_s) + (S_0 - s_s)\frac{d\mu}{ds}(s_s)\right)\lambda + \left(1 - \frac{K_d}{\mu(s_s)}\right)\mu(s_s)(S_0 - s_s)\frac{d\mu}{ds}(s_s)$$

Since every positive steady state satisfies the inequalities $\left\{\begin{matrix}\mu(s_s) > 0 \\ K_d < \mu(s_s) \\ s_s < S_0\end{matrix}\right\}$, it is the sign of $\frac{d\mu}{ds}(s_s)$ that determines its local stability characteristics. In particular, a positive equilibrium will be

- locally asymptotically stable if $\frac{d\mu}{ds}(s_s) > 0$
- unstable if $\frac{d\mu}{ds}(s_s) < 0$

Note that, because $\frac{d\mu(s_s^{opt})}{ds}(S_0 - s_s^{opt}) = \mu(s_s^{opt}) - K_d > 0$, the optimal steady state is always locally asymptotically stable.

2.4 The need for control

Figure 2 depicts the phase portrait of the system dynamics under constant dilution rate *D*, in particular for $D = D_s = 0.328097\ d^{-1}$, which is the optimal steady state value. In the diagram, the points S and U represent the corresponding stable and the unstable steady states of the system, which are the solutions of equations (5). Notice that the optimal steady state S is locally asymptotically stable but the stability region is very small. This makes the optimal operation of the biochemical reactor very sensitive to disturbances.

The goal of control is the stabilization of the system in the sense of enlargement of the stability region.

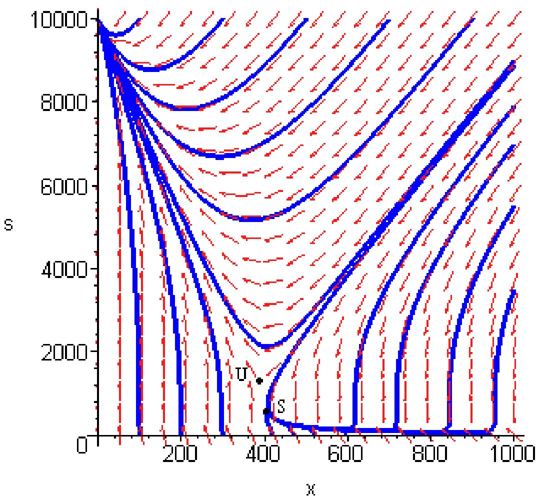

Figure 2. Phase portrait of the open-loop dynamics

3. OUTPUT FEEDBACK CONTROL – ESTIMATION OF THE STABILITY REGION

3.1 Control law in the absence of biomass decay

Consider the dynamic system (1), with measured output being given by (3). The control objective is to stabilize the system at a given design steady state, e.g. the optimal steady state that corresponds to maximal methane production rate.

Previous work in the literature (Mailleret and Bernard, 2001; Syrou et al., 2004; Karafyllis et al., 2008) has studied the special case of K_d=0. In particular, in Syrou et al. (2004), a control-Lyapunov function approach was formulated, that led to the following control law:

$$D = \frac{1}{Y_m Y_{x/s}(S_0 - s_s^{des})} Q \qquad (9)$$

where s_s^{des} is the value of the substrate concentration at the design steady state. It was shown that the resulting closed loop system is asymptotically stable, with stability region the entire open first quadrant. Moreover, it was shown that the control law (9) is robust with respect to bounded errors in S_0 (Karafyllis et al., 2008).

Figure 3 depicts the phase portrait of the closed loop system for K_d =0 and all the other parameter values as in Section 2.2, with the design steady state being the optimal steady state.

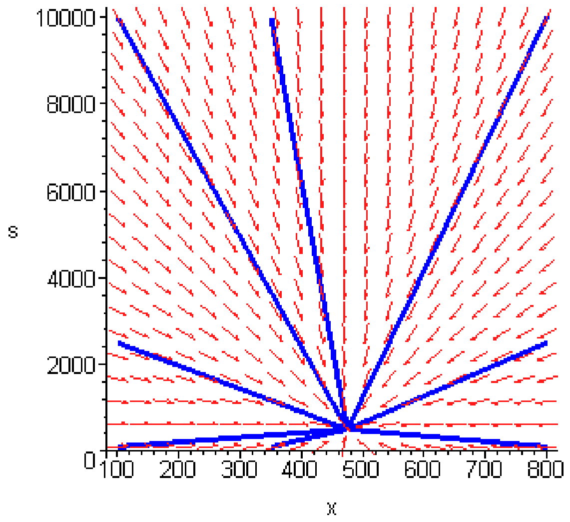

Figure 3. Phase portrait of closed-loop dynamics for K_d =0

When $K_d \neq 0$, it is still possible to derive stabilizing controllers for the system (1) with stability region the entire open first quadrant, but the necessary control law is state feedback (Karafyllis et al., 2009).

Because of the significant practical advantages of output feedback, the focus of the present work will be to examine whether output feedback is capable of guaranteeing a large enough stability region. In what follows, the same output feedback control law (9) will be studied, in the presence of death rate $K_d \neq 0$.

3.2 Stability analysis in the presence of biomass decay

Application of control law (9), or equivalently

$$D = \frac{1}{Y_{x/s}(S_0 - s_s^{des})} \mu(s) x$$

to the dynamic system (1) results in the following closed-loop system:

$$\begin{aligned} \frac{dx}{dt} &= \left(1 - \frac{x}{Y_{x/s}(S_0 - s_s^{des})}\right)\mu(s)x - K_d x \\ \frac{ds}{dt} &= \frac{(s_s^{des} - s)\mu(s)x}{Y_{x/s}(S_0 - s_s^{des})} \end{aligned} \qquad (10)$$

The closed-loop system (10) has a unique positive equilibrium

$$\begin{cases} x_s = x_s^{des} = Y_{x/s}(S_0 - s_s^{des})\left(1 - \dfrac{K_d}{\mu(s_s^{des})}\right) \\ s_s = s_s^{des} \end{cases} \qquad (11)$$

which is exactly the design steady state for the bioreactor. The question concerns its asymptotic stability and moreover, obtaining an estimate of the size of the stability region. We will prove the following

<u>*Proposition*</u>*: Assume that* $\mu(s)$ *given by (2a) or (2b) and* $K_d < \mu(S_0)$. *Then, the set*

$$I = \left\{(x,s) \in \mathbb{R}^2 \,|\, x > 0,\ \mu(s) > K_d\right\}$$

is contained in the stability basin of (10).

Note that for Andrews kinetics (2a), the above set is the strip

$$I = \left\{(x,s) \in \mathbb{R}^2 \,|\, x > 0,\ s_* < s < s^*\right\}$$

where s_*, s^* are the roots of $\mu(s) = K_d$, given by (6a), (7) respectively, whereas for Monod kinetics (2b), it is

$$I = \left\{(x,s) \in \mathbb{R}^2 \,|\, x > 0,\ s > s_*\right\}$$

where s_* is the unique root of $\mu(s) = K_d$, given by (6b).

The proof of the Proposition will be based on two Lemmas:

<u>*Lemma 1*</u>*: The positive definite quadratic function*

$$V(x,s) = \frac{1}{2}\left[\left(\frac{x}{Y_{x/s}(S_0 - s_s^{des})} - \left(1 - \frac{K_d}{\mu(s_s^{des})}\right)\right)^2 + \left(\frac{s}{s_s^{des}} - 1\right)^2\right] \qquad (12)$$

satisfies

$$\dot{V}(x,s) < 0 \quad \textit{for every } (x,s) \in E - \left\{\left(x_s^{des}, s_s^{des}\right)\right\}$$

where

$$E = \left\{(x,s) \in \mathbb{R}^2 \,|\, x > 0,\ s > \bar{s}\right\}$$

with $\bar{s} = K_S \cdot \dfrac{\dfrac{K_d}{\mu_{\max}}}{\dfrac{K_d}{\mu_{\max}} \cdot \dfrac{s_s^{des}}{K_I} + 2}$.

Proof of Lemma 1: The equations (10) of the closed loop system can be rearranged to

$$\begin{aligned} \frac{dx}{dt} &= -\left(\frac{x}{Y_{x/s}(S_0 - s_s^{des})} - \left(1 - \frac{K_d}{\mu(s_s^{des})}\right)\right)\mu(s)x - K_d\left(1 - \frac{\mu(s)}{\mu(s_s^{des})}\right)x \\ \frac{ds}{dt} &= -(s - s_s^{des})\frac{\mu(s)x}{Y_{x/s}(S_0 - s_s^{des})} \end{aligned}$$

Computing the time derivative of V along the trajectories of the system, we find

$$\dot{V}(x,s) = -\frac{\mu(s)x}{Y_{x/s}(S_0 - s_s^{des})}\left[\left(\frac{x}{Y_{x/s}(S_0 - s_s^{des})} - \left(1 - \frac{K_d}{\mu(s_s^{des})}\right)\right)^2 + \left(\frac{s}{s_s^{des}} - 1\right)^2 \right.$$
$$\left. + K_d\, s_s^{des} \frac{\frac{1}{\mu(s)} - \frac{1}{\mu(s_s^{des})}}{s - s_s^{des}}\left(\frac{x}{Y_{x/s}(S_0 - s_s^{des})} - \left(1 - \frac{K_d}{\mu(s_s^{des})}\right)\right)\left(\frac{s}{s_s^{des}} - 1\right)\right]$$

Now for $\mu(s)$ given by (2a),

$$K_d s_s^{des} \frac{\frac{1}{\mu(s)} - \frac{1}{\mu(s_s^{des})}}{s - s_s^{des}} = \frac{K_d}{\mu_{\max}}\left(\frac{s_s^{des}}{K_I} - \frac{K_S}{s}\right) = \frac{K_d}{\mu_{\max}}\left(\frac{K_S}{\bar{s}} - \frac{K_S}{s} - 2\right),$$

from which $-2 < K_d s_s^{des} \frac{\frac{1}{\mu(s)} - \frac{1}{\mu(s_s^{des})}}{s - s_s^{des}} < 0$ on the set E, hence the quantity inside the brackets is positive on $E - \left\{\left(x_s^{des}, s_s^{des}\right)\right\}$, hence the result.

<u>Lemma 2</u>: *For every ε with $0 < \varepsilon < \mu(s_s^{des}) - K_d$, the set*

$$I_\varepsilon = \left\{(x,s) \,\middle|\, x \ge \gamma\varepsilon,\ \mu(s) \ge K_d + \varepsilon\right\}$$

where $\gamma = \dfrac{Y_{x/s}(S_0 - s_s^{des})}{\mu_{\max}}$ *, is positively invariant.*

Proof of Lemma 2: Considering the flow on the boundaries of I_ε,

a) For $x = \gamma\varepsilon$, $\mu(s) \ge K_d + \varepsilon$,

$$\left.\frac{dx}{dt}\right)_{x=\gamma\varepsilon} = \left[\left(1 - \frac{\varepsilon}{\mu_{\max}}\right)\mu(s) - K_d\right]\gamma\varepsilon$$

But since

$$\left(1 - \frac{\varepsilon}{\mu_{\max}}\right)\mu(s) - K_d \ge \left(1 - \frac{\varepsilon}{\mu_{\max}}\right)(K_d + \varepsilon) - K_d$$
$$= \varepsilon - \frac{\varepsilon}{\mu_{\max}}(K_d + \varepsilon) = \varepsilon\left(1 - \frac{K_d + \varepsilon}{\mu_{\max}}\right) > 0$$

it follows that $\left.\frac{dx}{dt}\right)_{x=\gamma\varepsilon} > 0$.

b) For $x \ge \gamma\varepsilon$, $s = s_{\varepsilon*}$, where $s_{\varepsilon*}$ is the smallest root of $\mu(s) = K_d + \varepsilon$,

$$\left.\frac{ds}{dt}\right)_{s=s_{\varepsilon*}} = \frac{\left(s_s^{des} - s_{\varepsilon*}\right)\mu(s_{\varepsilon*})x}{Y_{x/s}(S_0 - s_s^{des})}$$

And since $\varepsilon < \mu(s_s^{des}) - K_d$, it follows that $s_{\varepsilon*} < s_s^{des}$,

therefore $\left.\frac{ds}{dt}\right)_{s=s_{\varepsilon*}} > 0$.

c) For $x \ge \gamma\varepsilon$, $s = s_\varepsilon^*$, where s_ε^* is the largest root of $\mu(s) = K_d + \varepsilon$,

$$\left.\frac{ds}{dt}\right)_{s=s_\varepsilon^*} = \frac{\left(s_s^{des} - s_\varepsilon^*\right)\mu(s_\varepsilon^*)x}{Y_{x/s}(S_0 - s_s^{des})}$$

And since $\varepsilon < \mu(s_s^{des}) - K_d$, it follows that $s_\varepsilon^* > s_s^{des}$,

therefore $\left.\frac{ds}{dt}\right)_{s=s_\varepsilon^*} < 0$.

Proof of the Proposition: For every initial condition $(x(0), s(0)) \in I$, there is an $\varepsilon > 0$ such that $(x(0), s(0)) \in I_\varepsilon$, hence, by Lemma 2, the system trajectory is wholly contained in $I_\varepsilon \subset I$. Hence I is a positively invariant set. Moreover, because

$$s_* = K_s \cdot \frac{2\frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}}\right) + \sqrt{\left(1 - \frac{K_d}{\mu_{\max}}\right)^2 - 4\left(\frac{K_s}{K_I}\right)\left(\frac{K_d}{\mu_{\max}}\right)^2}}$$
$$\ge K_s \cdot \frac{\frac{K_d}{\mu_{\max}}}{1 - \frac{K_d}{\mu_{\max}}} > K_s \cdot \frac{K_d}{\mu_{\max}} > K_s \cdot \frac{\frac{K_d}{\mu_{\max}}}{2 + \frac{s_s^{des}}{K_I} \cdot \frac{K_d}{\mu_{\max}}} = \bar{s}$$

it follows that $I \subset E$. Hence, by Lemma 1, the positive definite quadratic function V given by (12) has negative definite $\dot{V}$ on the positively invariant set I, hence the result.

The following Figure depicts the phase portrait of the closed loop system, for the parameter values and optimal design steady state of Section 2.2.

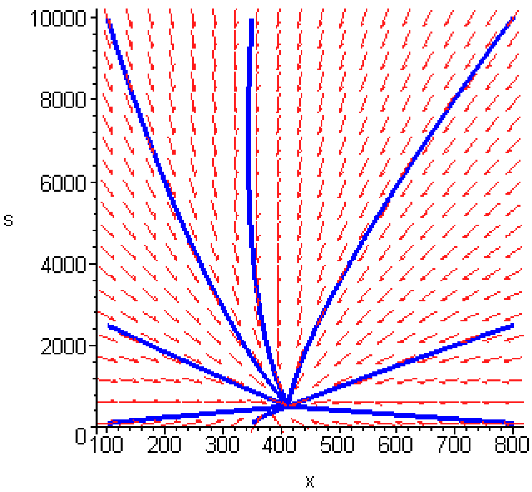

Figure 4. Phase portrait of closed-loop dynamics for $K_d \neq 0$

At a first glance, the phase portrait of Figure 4 looks quite similar to the one of Figure 3, and it seems to suggest that closed-loop stability has been achieved over the entire first quadrant. However, constructing a phase portrait in a region close to the origin (see Figure 5), shows that this is not the case. In particular, we see from Figure 5 that for small enough $s(0)$, trajectories terminate on the s-axis, instead of being directed to the design steady state. In other words, the result is death of the cells, when the initial substrate concentration is too small.

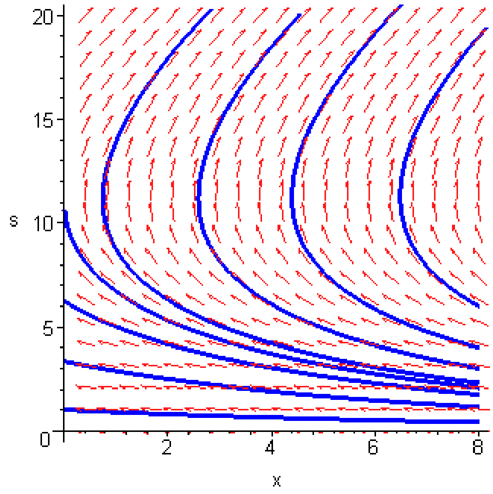

Figure 5. Detail of phase portrait of Figure 4

Note that, for the parameters used, $s_* = 11.114\, mg/l$, therefore the previously derived stability region estimate

$$I = \left\{ (x,s) \in \mathbb{R}^2 \,\middle|\, x > 0,\ s_* < s < s^* \right\}$$

is not conservative on the low end.

Similar behavior to Figure 5 is observed for substrate initial conditions larger than s^*, with trajectories terminating on the s-axis (phase portrait not shown because of space limitations). For the parameters used, $s^* = 35988.89\ mg/l$, which is significantly larger than S_0.

5. CONCLUSIONS

The present work studied the problem of designing an output feedback controller for the purpose of enlarging the stability region of continuous stirred microbial bioreactors, in the presence of biomass decay. The theory was motivated by application problems in anaerobic digestion, where the stability region can be very small if the operating steady state is selected to maximize the methane production rate. For output measurement proportional to the biomass growth rate, the proposed control law is a simple proportional output feedback controller. A non-conservative estimate of the stability region of the closed-loop system under the proposed controller was derived using Lyapunov methods.

The conclusion from the stability analysis is that, although the proposed output feedback controller does not stabilize the system globally over the entire 1st quadrant, the region of stability is very large from a practical point of view. Instability can only occur under extreme conditions that are highly unlikely to occur in practice.

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