Proceedings of the 9th International Symposium on
Dynamics and Control of Process Systems (DYCOPS 2010),
Leuven, Belgium, July 5-7, 2010
Mayuresh Kothare, Moses Tade, Alain Vande Wouwer, Ilse Smets (Eds.)

Multi-Objective Optimization based Robust Sensor Network Design

Prakash R. Kotecha, Mani Bhushan, Ravindra D. Gudi*

*Department of Chemical Engineering, Indian Institute of Technology Bombay,
Powai, Mumbai 400 076 (*Tel:91-22-25767204; e-mail:ravigudi@iitb.ac.in).*

Abstract: In this article, we propose an explicit integer optimization formulation for the design of reliable and robust (to uncertainty in reliability data) sensor networks. The robustness is achieved by incorporating simultaneous occurrence of different kinds of uncertainty in the failure rate data in the optimization formulation. We show the use of constraint programming to solve these combinatorial problems to global optimality and also evaluate the globally optimal pareto front between robustness and cost of these sensor networks. Such tradeoffs help the designer in making informed choices for the selection of sensor networks. The applicability of the proposed work has been demonstrated on a case study taken from literature.

Keywords: Multi-objective optimization, sensor network design, constraint programming, robust design, uncertainty.

1. INTRODUCTION

The optimal placement of sensors in a chemical plant is necessary from the viewpoints of plant safety, fault detection and diagnosis, optimal control, and plant economics. The sensor network design problem can be viewed as selection of variables to be measured so as to satisfy criteria such as cost, observability, reliability, estimation accuracy, flexibility and robustness. Initial attempts for the design of optimal sensor networks involved the design of a graph based branch and bound type algorithm (Meyer et. al., 1994) to determine the minimal cost sensor network. A tree type enumeration procedure for the design of minimal cost network with constraints on precision, availability, resilience and error detectability has also been reported (Bagajewicz, 1997). Later, this problem was posed as an explicit mixed integer non-linear programming (MINLP) optimization problem (Bagajewicz and Cabrera, 2002). This work also showed the conversion of MINLP to MILP at the cost of increasing the problem size and thus the computational burden. Nevertheless, this conversion enabled the guarantee of optimality. In sensor network literature, the concepts of reliability and network reliability were formalised in (Ali and Narasimhan, 1993). The concept of network reliability accounted for sensor failures and also indirectly incorporated the observability criterion. This work used a graph theoretic (Ali and Narasimhan, 1993,1995,1996) based greedy search algorithm to obtain the most reliable sensor network and hence did not guarantee the optimality of the solution. Moreover, these works did not consider the uncertainty in the reliability data. A duality between the reliability and the variance framework for non-redundant sensor network was shown by Kotecha & co-workers (Kotecha et. al., 2008a). This work enabled the solution of the precision problems by greedy search algorithms and the reliability problems by explicit optimization formulation. An integration of Genetic Algorithms (GA) with graph-theoretic concepts to solve the problem of optimal design of sensor network for linear processes was proposed by Sen & co-workers (Sen et. al., 1998). By the use of GA, various problems such as optimizing cost, estimation accuracy and network reliability were solved. Strategies for designing sensor networks for reliable fault diagnosis were presented by Bhushan & Rengaswamy (Bhushan and Rengaswamy, 2002). This also considered the uncertainty in the fault occurrence probability data and the sensor failure probability data.

An explicit optimization formulation based on the use of cutsets has also been proposed by Kotecha & co-workers (Kotecha et. al., 2008b) to design minimum failure rate networks for non-redundant sensor networks. These minimum failure rate networks also correspond to the maximum reliability network. This work considered the design of robust sensor networks in the presence of known and unknown levels of uncertainties in the failure rate data. However, this work did not account for the simultaneous occurrence of known uncertainty levels for some uncertain sensors and unknown uncertainty levels for other uncertain sensors. In the current article, we address this shortcoming by considering the more realistic scenario wherein some of the sensors are uncertain with their uncertainty levels known and some other uncertain sensors whose uncertainty levels are not known. Towards this end, we appropriately extend the cutset based explicit optimization idea in (Kotecha et. al., 2008b) to simultaneously incorporate different types of uncertainties in different sensors. This gives rise to a non-linear Integer Programming (IP) problem. We use Constraint Programming (CP), a domain reduction based technique, to solve these problems to global optimality. In most cases, the design of sensor networks is constrained by the monetary resources and there is a trade-off involved between the cost of the sensor network and the robustness. To study these trade-offs, we harness the ability of CP to determine all feasible solutions and generate the necessary globally pareto-optimal fronts along with their realizations.

The paper is organized as follows: The following section gives the proposed mathematical formulation followed by a section briefly explaining constraint programming and the determination of pareto optimal fronts. A case study from the literature is then presented and the results are discussed. We conclude the article by summarizing the developments in this article and present possible future extensions of this work.

2. MATHEMATICAL FORMULATION

2.1 Minimum failure rate networks

The following formulation determines the most reliable network by minimizing the network failure rate.

$$\min \quad \hat{\lambda} \tag{1}$$

$$\text{Subject to } \hat{\lambda} = \max\left(\lambda_1, \lambda_2, ..., \lambda_n\right) \tag{2}$$

$$\sum_{i \in E'} x_i \le |S| - 1, \quad \forall S \subset V, |S| > 2 \tag{3}$$

$$\sum_{i \in N} \left(1 - x_i\right) = n - m \tag{4}$$

$$\lambda_i = \left(1 - x_i\right)\lambda_i + x_i \sum_{k=1}^{|C^i|} \left[\left(\sum_{\substack{j \in C_k^i, \\ j \ne i,}} \lambda_j\right)\left(\prod_{\substack{j \in C_k^i, \\ j \ne i}} (1 - x_j)\right)\right] \forall\, i \in N \tag{5}$$

$$x_j \in \{0, 1\} \quad j \in N \tag{6}$$

$$\sum_{i \in C^k} x_i \ge 1 \quad \forall k \in C^T \tag{7}$$

In the above formulation, $\hat{\lambda}$ corresponds to the network failure rate defined as the maximum failure rate amongst all variables as given in (2). It was shown in (Kotecha et. al., 2008b) that minimizing $\hat{\lambda}$ is exactly equivalent to maximizing network reliability which is defined as the minimum reliability amongst all variables (Ali and Narasimhan, 1993). Constraint (3) ensures that the unmeasured variables do not form a cycle. S denotes a subset of the set of vertices V subject to the criterion that the cardinality of S (number of nodes in S) is greater than 2. E' denotes the set of edges connecting the nodes present in S. Constraint (3) is written for all subsets of V with cardinality greater than 2. Constraint (4) ensures that the number of unmeasured variables is equal to the total number of independent mass balance equations. Constraints (3) and (4) together ensure that the unmeasured variables form a spanning tree in the graph G (Deo, 1974) thereby ensuring the observability of all variables for a non-redundant sensor network design (Ali and Narasimhan, 1993). The failure rate of a measured variable should be equal to the failure rate of the sensor used for measuring it whereas the failure rate of an unmeasured variable should be equal to the sum of the failure rates of the sensors used for estimating the failure rate. Constraints (5) determine the nominal failure rate of a variable without considering any uncertainty. The first term on the RHS of this equation corresponds to direct measurement of the variable (corresponding to $x_i = 0$), while the second term (corresponding to $x_i = 1$) incorporates all possible ways of indirectly estimating the i^{th} variable based on other measurements. For the latter case, since there is only one cutset involving variable which will have all other variables (other than i) measured, only one term in the summation of the second term on RHS will be active. Constraints (7) ensure that at least one variable in each cutset is not measured. These are redundant as the problem is defined completely even without the inclusion of these cuts. However, these have been included as they were observed to substantially reduce the computational burden while solving the above integer nonlinear programming formulation using Constraint Programming (CP). The inclusion of these cuts does not involve any additional effort as the cutsets have to be anyway determined for writing the expressions for the failure rates in (5). The algorithm for determining all the cutsets in a graph is presented in (Kotecha et. al., 2008b).

The above basic formulation does not consider uncertainty in the failure rate data. However failure rate (or reliability) data is rarely precisely known. Two different scenarios of uncertainty in the failure rate data of a given sensor: known and unknown levels of uncertainty were considered in (Kotecha et. al., 2008b). In the known scenario, it was assumed that the worst case (highest) failure rate value is available, while in the unknown case the worst case value is not known. In both cases, however, a nominal (most likely) value is assumed to be known. In the case of unknown level of uncertainty, the amount of uncertainty a network can tolerate was maximized and in the case for known level of uncertainties, the worst case network failure rate was minimized. However, their work did not consider the practical scenario of both known and unknown levels of uncertainty occurring simultaneously, i.e. when the worst case failure rates of some sensors are known whereas for some other sensors these are unknown. The following formulation considers this scenario by defining a measure for robustness in the presence of simultaneous uncertainty.

2.2 Maximization of Robustness Index

The robustness index RI is defined as

$$RI = \left(e - \hat{\lambda}_U\right) \tag{8}$$

Where e is the uncertainty that can be tolerated by the network in each uncertain sensor when the uncertainty level is not known. $\hat{\lambda}_U$ is the network failure rate while considering the worst case failure rates for sensors with known levels of uncertainty and also simultaneously assuming that the nominal failure rate of all sensors with unknown levels of uncertainty have increased by e. In case of only one type of uncertainty, (Kotecha et. al., 2008b) considered either minimization of $\hat{\lambda}_U$ (when uncertainty levels in uncertain sensors were known) or maximization of e (when uncertainty levels in uncertain sensors were now known). However, in presence of simultaneous uncertainties just focusing on minimizing $\hat{\lambda}_U$ or maximizing e will not be sufficient as a sensor network with low values of $\hat{\lambda}_U$ may not result in high values of e and vice-versa. Hence in this article we combine these two objectives as in (8). The resulting optimization formulation then is,

$$\max \quad RI \tag{9}$$

$$\text{Subject to } \sum_{i\in E'} x_i \le |S|-1, \quad \forall S\subset V, |S|>2 \tag{10}$$

$$\sum_{i\in N}(1-x_i) = n-m \tag{11}$$

$$\hat{\lambda}_i = (1-x_i)\lambda_i + x_i \sum_{k=1}^{|C^i|}\left[\left(\sum_{\substack{j\in C_k^i,\\ j\ne i,}}\lambda_j\right)\left(\prod_{\substack{j\in C_k^i,\\ j\ne i}}(1-x_j)\right)\right] \forall i\in N \tag{12}$$

$$\hat{\lambda}_i^U = (1-x_i)\lambda_i + x_i \sum_{k=1}^{|C^i|}\left[\begin{pmatrix}\sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\in U}}(\lambda_j+e) + \\ \sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\in K}}(\lambda_j^U) + \sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\notin U\\ j\notin K}}\lambda_j\end{pmatrix}\left(\prod_{\substack{j\in C_k^i,\\ j\ne i}}(1-x_j)\right)\right] \forall i\in N\setminus(U\cup K) \tag{13}$$

$$\hat{\lambda}_i^U = (1-x_i)\lambda_i^U + x_i \sum_{k=1}^{|C^i|}\left[\begin{pmatrix}\sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\in U}}(\lambda_j+e) + \\ \sum_{\substack{j\in CC_k^i,\\ j\ne i,\\ j\in K}}(\lambda_j^U) + \sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\notin U\\ j\notin K}}\lambda_j\end{pmatrix}\left(\prod_{\substack{j\in C_k^i,\\ j\ne i}}(1-x_j)\right)\right] \quad \forall i\in K \tag{14}$$

$$\hat{\lambda}_i^U = (1-x_i)(\lambda_i+e) + x_i \sum_{k=1}^{|C^i|}\left[\begin{pmatrix}\sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\in U}}(\lambda_j+e) + \\ \sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\in K}}(\lambda_j^U) + \sum_{\substack{j\in C_k^i,\\ j\ne i,\\ j\notin U\\ j\notin K}}\lambda_j\end{pmatrix}\left(\prod_{\substack{j\in C_k^i,\\ j\ne i}}(1-x_j)\right)\right] \forall i\in U \tag{15}$$

$$\sum_{i\in N} c_i(1-x_i) \le C^* \tag{16}$$

$$\max\left(\hat{\lambda}_1,\hat{\lambda}_2....\hat{\lambda}_n\right) \le \hat{\lambda}_c \tag{17}$$

$$\max\left(\hat{\lambda}_1^U,\hat{\lambda}_2^U....\hat{\lambda}_3^U\right) \le \hat{\lambda}_c^U \tag{18}$$

$$\sum_{i\in C^k} x_i \ge 1 \quad \forall k\in C^T \tag{19}$$

$$x_j \in \{0,1\} \quad j\in N \tag{20}$$

Constraints (10) to (12) are as explained earlier. Constraints (13), (14), and (15) calculate the failure rates of variables in the presence of uncertainties. In particular, (13) is written for the variables whose sensors have no uncertainty in their failure rate data. Constraint (14) is written for the variables which have sensors with known levels of uncertainty whereas (15) is written for the variables which have sensors with unknown levels of uncertainty. Constraint (16) ensures that the cost of the sensor network is below the available monetary resources. Constraint (17) allows for the selection of only those networks whose nominal failure rate is below a critical threshold value, and (18) ensures that the worst case network failure rate does not exceed the threshold value of $\hat{\lambda}_c^U$. Constraints (17) and (18) can be viewed as performance constraints. For a feasible network, the value of $\hat{\lambda}_c^U$ cannot be less than $\hat{\lambda}_c$. Similarly, the value of $\hat{\lambda}_c$ cannot be smaller than $\hat{\lambda}^*$.

Remark: It should be noted that RI will always be negative since the tolerable uncertainty is always bounded between 0 and $\hat{\lambda}^* - \lambda_{small}$ whereas the worst case network failure rate $\left(\hat{\lambda}_U\right)$ cannot be less than $\hat{\lambda}^*$. Hence, if no uncertain sensors are selected then $RI = -\lambda_{small}$.

2.3 Minimization of the cost

The objective function for the minimization of the cost of sensor network is given by the following expression

$$\min \quad \sum_{i\in N} c_i(1-x_i) \tag{21}$$

Subject to Constraints (10) to (20)

For the sake of brevity, we restrict with the above two objectives of cost minimization and RI maximization. Several other objectives as shown in (Kotecha et. al., 2008b) can also be considered.

Like most other engineering problems, the sensor network design problem is very often associated with multiple objectives. In this case, the multiple objectives can be considered to be maximization of robustness index and the minimization of the sensor network cost. A key feature of multi-objective optimization problems that differentiates them from single objective optimization problems, is that it is often not possible to simultaneously optimize all the objectives, instead these problems are usually characterized by a set of optimal solutions characterizing the trade offs. The trade-off solutions are typically represented as a pareto-optimal front consisting of the set of non-dominated solutions (Deb, 2001). A solution is said to be non-dominated if it is feasible and there is no other feasible solution which has better values for all the objectives.

In this article, we have used constraint programming to determine the multiple solutions for the single objective problem and the determination of pareto fronts for the multi-objective problems. In the subsequent section we briefly explain constraint programming.

3. CONSTRAINT PROGRAMMING

Constraint Programming (CP) is an intelligent enumeration based search technique that uses constraint propagation as its inference engine and continuously reduces the domain of its variables to reach feasible solutions. It was developed in the Computer Science and Artificial Intelligence community and has been widely used to solve combinatorial feasibility problems. However, it solves an optimization problem by transforming it into a feasibility problem. The important merits of CP include (i) easier modeling because of its expressive power (ii) guarantee of global optimality even for non-convex problems and (iii) ease of determination of multiple optimal solutions. The constraint programming technique can be considered to consist of two distinct parts namely constraint propagation and a search technique. Constraint propagation is used to reduce the domain of variables till all the variables are assigned a unique value from their domains. If any of the variable domain becomes empty, it means that there is no feasible solution. However, in the absence of a unique value for the variables from their domains, a non deterministic choice is made and constraint propagation is carried further. If this choice does not yield a solution, backtracking is done to make another choice and the procedure is repeated. The process of making a non deterministic choice is called as choice point selection and reaching infeasibility for a particular choice point is known as failure. A detailed algorithm on the working of CP can be obtained from literature (Kotecha et. al., 2008b). We next briefly discuss CP based strategies for determining multiple solutions of a single objective optimization problem and the determination of pareto optimal solutions for a multi-objective optimization problem.

3.1 Determination of Multiple Solutions

CP can be used to determine the multiple solutions (or realizations; solutions with equal objective function values) of an optimization problem by using a two step strategy without the addition of any traditional cuts. The first step is the solution of the problem (say P) to determine the optimal solution. The second step involves the determination of all solutions of a feasibility problem with an additional constraint which ensures that only the optimal solutions of P are feasible. A detailed description for the determination of all multiple solutions can be obtained from (Kotecha et. al., 2008b).

3.2 Determination of Pareto-Optimal front

CP can be used to determine the globally optima pareto front because it does not suffer from the drawbacks of the traditional methods used for solving multi objective techniques. In particular, it does not require any a priori knowledge about weights on the individual objectives or the precedence of the various objectives or the performance level in the various objectives. A detailed discussion on various aspects of this procedure is elaborated in [Kotecha at. al., 2010].

4. CASE STUDY

In this section, we demonstrate the use of the proposed formulations on a widely discussed case study taken from the literature (Bagajewicz, 1997, Bagajewicz & Cabrera, 2002). The flowsheet in Figure 1 has 24 variables and 11 equations (overall mass balances corresponding to 11 units). There are ${}^{24}C_{11}$ combinations for choosing 13 variables to be measured for a non-redundant sensor network design. However, all these combinations do not form observable networks.

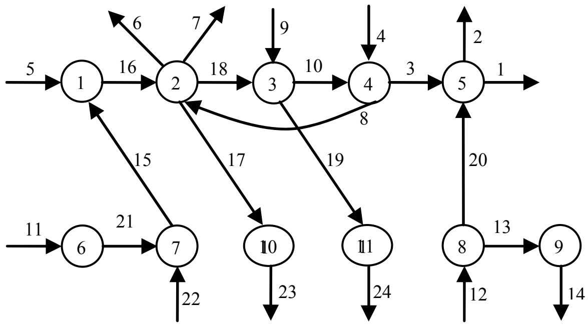

Fig. 1. Schematic of the Case Study

The failure rates and cost of the sensors used in this article are given in Table 1 (Kotecha et. al., 2008b)a. The number of tree constraints is 2570 and the number of distinct cutsets is 148. The number of cutsets for each variable is given in Table 1. The following three different scenarios are considered for the uncertain sensors.

Scenario I: An unknown uncertainty level is considered in the sensors for measuring variables 1, 2, 3, and 4 whereas known levels of uncertainty was considered in the sensors for measuring variables 9, 10 and 20. The levels of uncertainty in these sensors were 3, 2 and 6 times their nominal failure rates. The value for the critical worst case failure rate $\hat{\lambda}_c^U$ is taken to be 120.

Scenario II: This scenario is similar to that of Scenario I except that the critical worst case failure rate is 150 instead of 120.

Scenario III: Here, we consider the sensors for variables 5 and 11 to be uncertain with known levels of uncertainty. The worst case failure rate for these sensors is 3 and 5 times greater than their nominal value in Table 1. We also consider variables 2, 9, and 14 as uncertain with unknown level of uncertainties. The value for the critical worst case failure rate $\hat{\lambda}_c^U$ is considered to be 120.

4.1 Results:

We first present results for the single objective optimization of minimizing the cost and maximizing the robustness index. This is followed by the evaluation of trade-off between these two objectives simultaneously. The results reported are generated using ILOG CP Solver (ILOG, 2003).

Table 1. Data for case study

Var i	Failure rate λ_i	Sensor cost c_i	Number of cutsets $\lvert C_i \rvert$
1	13	32	84
2	19	16	84
3	21	10	30
4	25	7	108
5	10	50	93
6	20	15	120
7	15	27	120
8	19	23	36
9	12	35	90
10	10	46	54
11	11	44	33
12	19	20	58
13	16	25	30
14	17	24	30
15	19	19	33
16	28	5	33
17	14	31	61
18	19	17	50
19	14	29	46
20	11	42	30
21	20	12	33
22	22	8	64
23	11	38	61
24	16	26	46

4.2 Determination of minimum cost network:

Table 2 shows the minimum cost networks for all the above three scenarios. For Case I, the minimum cost network is 261 and is characterized by 10 realizations. All these 10 realizations have -111 as their robustness index. The designer can choose any of these solutions based on some other criteria. For Case II, the minimum cost network is 261 but is characterized by 38 solutions. This is because the value of $\hat{\lambda}_c^U$ can increase till 150 whereas in Case I it was restricted to 120. Hence, there are more number of optimal solutions. Of these 38 solutions, 36 solutions have a maximum robustness of -111. For Case III, the minimum cost network is 261 with 23 realizations. Of these, 7 realizations have a maximum robustness index of -87.

4.3 Determination of maximum robustness index:

The result for maximizing robustness for the three scenarios is presented in Table 3. The maximum achievable robustness for Case I and II is -81. This is not surprising since an increase in $\hat{\lambda}_c^U$ (as in Case II) does not affect the maximum robustness as $\hat{\lambda}_u$ is to be minimized. However, the number of optimal solutions can increase as $\hat{\lambda}_c^U$ is increased. However, for the current scenario the number of optimal solutions for both the cases remains the same.

Table 2. Results for the minimum cost followed by maximum robustness index

	Min Cost	Realizations	Max RI	Realizations
Case I	261	10	-111	10
Case II	261	38	-111	36
Case III	261	23	-87	7

Of these realizations, the minimum cost network is 288 units. The maximum robustness index for Case III is -10. The value -10 is the maximum possible value $(-\lambda_{small})$ that robustness index can have and is characterized by networks which do not employ any of the 5 uncertain sensor networks.

Table 3. Results for the maximum robustness cost followed by minimum cost

	Max RI	Realizations	Min Cost	Realizations
Case I	-81	72	288	1
Case II	-81	72	288	1
Case III	-10	60	289	1

4.4 Determination of Pareto front between RI and cost:

In this section, we evaluate the trade offs between the objectives of cost and robustness. Figure 2 shows the pareto front for all the three scenarios mentioned above.

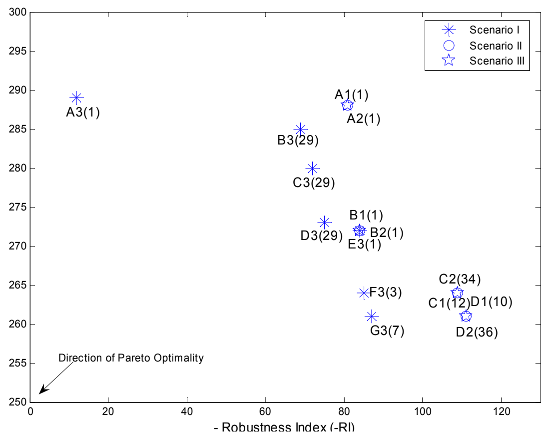

Fig. 2. Pareto front between Robustness Index (RI) and cost

For easy interpretation, the pareto fronts are plotted between $-RI$ and cost instead of RI and cost. Since $-RI$ is to be minimized, the direction of pareto points is towards the origin. The pareto front for Case I is characterized by 24 pareto points. However, only 4 of these points are distinct (A1-D1) and others are the realizations of these pareto points.

Of the four pareto points, only two are characterized by realizations and the remaining two have single realizations each. The designer can choose to operate at any of these four points and can select a realization depending on other criteria. Similarly, the pareto front for Case II is also characterized by (A2-D2) exactly the same four pareto points. However, the points C2 and D2 have more realizations than their counterparts in C1 and D1. For Case III, the pareto front has 7 distinct pareto points and a total of 99 pareto points. It can be seen that point E3 coincides with B1(and B2). As expected, it can be seen that the points in Table 2 and Table 3 for each of the case form the extreme points of the pareto front. Thus these trade offs can guide the designer in selecting a suitable network. Moreover, the realizations provide additional flexibility to the designer as they allow the consideration of other objectives other than the two presented in this article.

5. CONCLUSIONS

In this work, we have presented an explicit optimization formulation for the design of robust reliable sensor networks with simultaneous occurrence of known and unknown uncertainty levels in the failure rate data. We have shown the efficiency of CP in solving these combinatorial problems to global optimality and the ease in obtaining the realizations. Also, we have presented the trade off between the robustness index and cost of the sensor network in terms of the globally optimal pareto front to aid in selection of a suitable network.

NOMENCLATURE

N set of sensors
U set of sensors with unknown levels of uncertainty
K set of sensors with known levels of uncertainty
G graph with V nodes and E edges
V, E set of all nodes and edges in graph G
C^T set of all the cutsets
$S, E^{'}$ set of nodes $S \subset V$, edges in S
C^i set of all cutsets involving the i^{th} variable
c_i, C^* cost of sensor, total monetary allowance
m total number of process units without the inclusion of the environment node
n total number of variables in the process plant (cardinality of the set N)
λ failure rate of sensor at time t
λ_i^U maximum failure rate for the i^{th} uncertain sensor
$\hat{\lambda}_c$ critical/threshold nominal network failure rate
$\hat{\lambda}_c^U$ critical/threshold worst case network failure rate in the presence of failure uncertainties
λ_{small} minimum nominal failure rate among the uncertain sensors with unknown level of uncertainty
x_i binary variable, equals one to indicate a sensor not being selected and zero otherwise
e uncertainty in each of the sensors with unknown level of uncertainty
$\hat{\lambda}_i$ nominal failure rate of the i^{th} variable
$\hat{\lambda}_i^U$ failure rate of the i^{th} variable in the presence of uncertainty
$\hat{\lambda}^*$ optimal nominal network failure rate
$\hat{\lambda}^{U*}$ optimal worst case network failure rate

REFERENCES

Ali, Y., Narasimhan, S., (1993), Sensor network design for maximizing reliability of linear processes, AIChE Journal, 39, 820-828.

Ali, Y., Narasimhan, S., (1995), Redundant sensor network design for linear processes, AIChE Journal, 41, 2237-2249.

Ali, Y., Narasimhan, S., (1996), Sensor network design for maximizing reliability of bilinear processes, AIChE Journal, 42, 2563-2575.

Bagajewicz, M. J., (1997), Design and retrofit of sensor networks in process plants, AIChE Journal, 43, 2300-2306.

Bagajewicz, M. J., Cabrera, E., (2002), New MILP formulation for instrumentation network design and upgrade, AIChE Journal, 48, 2271-2282.

Bhushan, M., Rengaswamy, R., (2002), Comprehensive Design of a Sensor Network for Chemical Plants Based on Various Diagnosability and Reliability Criteria. 1. Framework, Industrial Engineering and Chemistry Research, 41, 1826-1839.

Deb, K., (2001), Multi-Objective Optimization Using Evolutionary Algorithms, John Wiley & Sons, Inc., New York, NY, USA.

Deo, N., (1974), Graph Theory with Applications to Engineering and Computer Science, Prentice-Hall, Inc., Upper Saddle River, NJ, USA.

ILOG, (2003) OPLStudio 3.7, User'sManual.

Kotecha, P. R., Bhushan, M., Gudi, R.D., Keshari, K.M., (2008a), A duality based framework for integrating reliaility and precision for sensor network design, Journal of Process Control, 18, 189-201.

Kotecha, P. R., Bhushan, M., Gudi, R.D., (2008b), Design of Robust, Reliable Sensor Networks Using Constraint Programming, Computers & Chemical Engineering, 32, 2030-2049.

Kotecha, P. R., Bhushan, M., Gudi, R.D., (2010), Efficient Generation of Globally Optimal Pareto Fronts Using Constraint Programming, AIChE Journal, 56, 387-404.

Meyer, M., Le Lann, J. M., Koehret, B., Enjalbert, M., (1994), Optimal selection of sensor location on a complex plant, using a graph oriented approach, Computers & Chemical Engineering, 18, 535-540.

Sen, S., Narasimhan, S., Deb, K., (1998), Sensor network design of linear processes using genetic algorithms, Computers & Chemical Engineering, 22, 385-390.