Proceedings of the 11th International Symposium on
Computer Applications in Biotechnology (CAB 2010),
Leuven, Belgium, July 7-9, 2010
Julio R. Banga, Philippe Bogaerts, Jan Van Impe,
Denis Dochain, Ilse Smets (Eds.)

Damping Feedback Stabilization for Cyclic Interconnections Systems: Oscillations Suppression and Synchronization

Nicolas Hudon [*] **Martin Guay** [*]

[*] *Department of Chemical Engineering, Queen's University, Kingston, ON, Canada, K7L 3N6 (e-mail:* `martin.guay@chee.queensu.ca`*)*

Abstract: This paper considers the problem of stabilizing (bio)chemical reaction networks that can be represented as cyclic interconnections. The objective of the paper is to present a constructive way to compute a dissipative potential function for the system. This potential is then used for constructing a smooth feedback stabilizing controller. We obtain a characteristic one-form for the system by taking the interior product of a non vanishing two-form with respect to the drift vector field. A homotopy operator is then constructed locally around the desired equilibrium, leading to the computation of a dissipative potential for the system. The dynamics of the system is then decomposed into an exact part and an anti-exact one. The exact part is generated by a potential, that is used to construct the smooth stabilizing feedback, under the so-called weak Jurdjevic–Quinn conditions. We consider the problems of oscillations suppression and synchronization of oscillators as illustrations of potential applications of the proposed method.

Keywords: Nonlinear control, Feedback stabilization, Cyclic interconnections, Oscillations suppression.

1. INTRODUCTION

This paper studies the feedback stabilization problem for chemical and biochemical reaction networks that can be represented as cyclic interconnection. The stability characterization problem for systems represented in this particular form has been study extensively. An example of systems with interconnections is metabolic network with feedback inhibition, studied for example in (Grognard et al., 2004). The reader is referred to the exposition by Arcak and Sontag (2006) for references on the stability characterization problem. The important contribution in (Arcak and Sontag, 2006) was to relate diagonal stability to a secant criterion in the context of interconnected system. More importantly, they gave a constructive stability proof for cyclic systems. Extensions of these results to complex (bio)chemical reaction networks are given in (Arcak and Sontag, 2008). In the general case, cyclic systems can be represented graphically as in Figure 1, where each block can be of the form

$$H_i : \begin{cases} \dot{x}_i = -f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) \end{cases} . \qquad (1)$$

This input/output representation was considered for example in (Stan et al., 2007) for the synchronization of coupled nonlinear Goodwin oscillators using incremental dissipativity.

In this paper, we look at the problem of constructing smooth feedback stabilizers for this class of systems, where a control $u(x)$ is assumed to be injected at any point in the structure. In recent years, energy-based methods were developed and studied extensively for nonlinear controller design. In particular, the representation of nonlinear systems as generalized Hamiltonian systems (see for example Ortega et al. (2002)) generated many successful controller designs strategies for systems possessing an "energy-like" function or a storage function. However, for mass balance systems, *i.e.*, for reacting networks, such generating function of the dynamics might be difficult to develop from the physics of the system. The problem of dissipative Hamiltonian representation of a reacting system was given by Otero-Muras et al. (2008). In (Ortega et al., 1999), feedback stabilization of a Lotka–Volterra system using a related passivity-based approach was considered. In that case, the authors solved a set of partial differential equations, known in the literature as the matching equations.

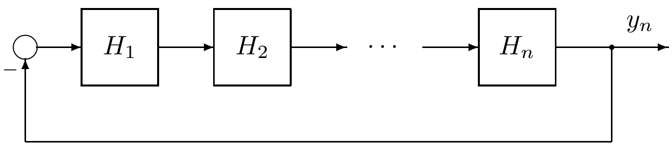

Fig. 1. Cyclic feedback system

In (De Leenheer and Aeyels, 2002), the authors considered the stabilization problem of positive systems, and in particular mass balance systems, by computing first integrals for the drift system. In the present paper, we follow this general idea using a locally defined geometric decomposition approach. We propose to use the tools of exterior calculus to construct a locally-defined dissipative function to be used to design a stabilizing controller. It is shown that a stabilizing controller can be developed using the exact part of the dynamics (the dynamics generated by the potential). More precisely, we obtain a characteristic one-form for the system by taking the interior product of a non vanishing two-form with respect to the drift vector field. A homotopy operator centered at a desired equilibrium point for the system is used to obtain an exact one form, generated by a dissipative potential, and an anti-exact form that generates the tangential dynamics. The stability argument presented in (Hudon et al., 2008) uses local equivalence between the exact part of the dynamics and a pre-defined Hamiltonian dissipative realization, viewed as a reference system to develop a local change of coordinates to derive the desired local dissipative potential for the system. In Hudon and Guay (2009), the approach is used for stabilization purposes. The problem of Lyapunov functions construction for the stabilization of time-independent nonlinear control affine systems satisfying Jurdjevic–Quinn conditions is considered (see for example Malisoff and Mazenc (2009, Chapter 4) for an extensive review of the technique). It is shown that a Lyapunov function can be computed for the closed-loop vector field subject to damping feedback control using the dissipative potential obtained from the proposed local decomposition. In this paper, we seek to apply the results from Hudon and Guay (2009) to systems presenting a cyclic interconnection structure. It will be shown through simulation, under the usual static smooth feedback stabilization conditions, that sustained oscillations can be canceled using a damping feedback controller as proposed and render the equilibrium of the system asymptotically stable. Then the problem of synchronizing two oscillators on different time scales will be considered.

The paper is organized as follows. Section 2 presents the proposed control problem and the motivating example, adapted from (Arcak and Sontag, 2008). In Section 3, the proposed control design construction is presented. Numerical applications are given in Section 4. Conclusions and future areas of investigation are outlined in Section 5.

2. PROBLEM FORMULATION

Consider the cyclic interconnection structure given in (Arcak and Sontag, 2006)

$$\dot{x}_1 = -a_1(x_1) + b_n(x_n) \tag{2}$$

$$\dot{x}_2 = -a_1(x_2) - b_1(x_1) \tag{3}$$

$$\vdots \tag{4}$$

$$\dot{x}_n = -a_n(x_n) + b_{n-1}(x_{n-1}), \tag{5}$$

where $a_i(\cdot)$ and $b_i(\cdot)$, $i = 1, \ldots, n$, are continuous functions satisfying $x_i a_i(x_i) > 0$ and $x_i b_i(x_i) > 0$ for $x_i \neq 0$. The situation considered here is the case where $a_i(\cdot)$, $i = 1, \ldots, n$ and $b_i(\cdot), i = 1, \ldots, n-1$ are increasing functions and $b_n(\cdot)$ is a decreasing function. This is a special case of the metabolic network with feedback inhibition considered in (Grognard et al., 2004).

An interesting example of such a system was given by Arcak and Sontag (2008),

$$\dot{x}_1 = -a_1 x_1 + \phi(x_3) \tag{6}$$

$$\dot{x}_2 = -a_2 x_2 + b_1 x_1 \tag{7}$$

$$\dot{x}_3 = -a_3 x_3 + b_2 x_2. \tag{8}$$

In the case where $\phi(x_3)$ is given as a function of the form $\frac{1}{1+x_3^p}$, the above cyclic interconnection is known as the Goodwin oscillator for which, depending on the value of p, a stable periodic orbit exists (see for example Stan et al. (2007) and references therein).

In the present paper, we will focus on the special case considered in (Arcak and Sontag, 2008) where $a_1 = a_2 = a_3 = 1$ and $b_1 = b_2 = 1$. In those references, the authors showed, for $\phi(x_3) = \exp(-10(x_3-1)) + 0.1\,\text{sat}(25(x_3-1))$, that the stable equilibrium $x^* = [1,\ 1,\ 1]^T$ coexists with a periodic orbit. In the present paper, we will simply use $\phi(x_3)$ as

$$\phi(x_3) = \exp(-10(x_3 - 1)). \tag{9}$$

The open-loop cycle surrounding the desired equilibrium $x^* = [1,\ 1,\ 1]^T$ is presented in Figure 3. The time trajectories of the sustained oscillations are depicted in Figure 2.

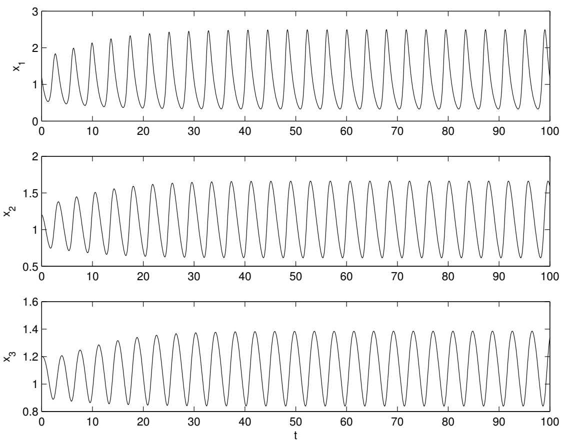

Fig. 2. Open-loop trajectories — $x_0 = [1.2,\ 1.2,\ 1.2]^T$

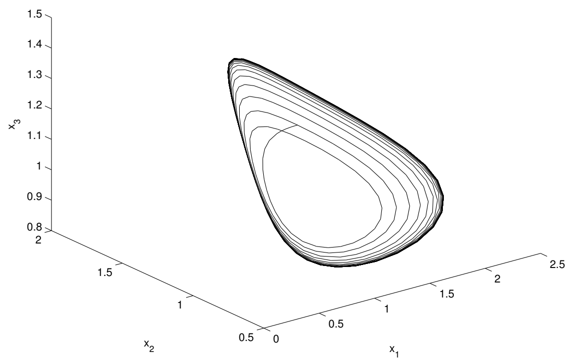

Fig. 3. Open-loop stable cycle — $x_0 = [1.2,\ 1.2,\ 1.2]^T$

In the sequel, we study two problems related to this system. First, it is desired to stabilize the equilibrium x^* using smooth damping feedback, *i.e.*, we will suppose that it is possible to add a feedback control signal $u(x)$ in the loop, *i.e.*, the system is modified to be

$$\dot{x} = f(x) + ku \tag{10}$$

where the k is a constant $n \times 1$ vector. The problem is to design a damping controller u to cancel the oscillations using a damping controller of the Jurdjevic–Quinn type,

$$u = -\nabla_x^T \psi(x) \cdot k \tag{11}$$

where $\psi(x)$ is a dissipative potential for the uncontrolled dynamics to be constructed in Section 3.2.

The second problem considers is to synchronize the above system with another cyclic system using the same type of damping controller with a different dissipative potential that is constructed in Section 3.3.

3. FEEDBACK CONTROLLER CONSTRUCTION

We propose a feedback controller construction strategy for both problems depicted in Section 2. The construction of a local dissipative potential based on a homotopy decomposition is presented in Section 3.1. In Section 3.2, we present the construction of a damping feedback control based on the Jurdjevic–Quinn approach. The problem of synchronization of two cyclic interconnection systems is presented in Section 3.3. Due to space limitations, we omit reviews of exterior calculus on $\mathbb{R}^n$. The reader is referred to the general presentation from (Edelen, 2005), summarized in Hudon et al. (2008) and Hudon and Guay (2009). We denote a smooth vector field as

$$X(x) = \sum_{i=1}^{n} v^i(x)\partial_{x_i} \tag{12}$$

and a smooth differential one-form as

$$\omega(x) = \sum_{i=1}^{n} \omega_i(x) dx_i, \tag{13}$$

where $v^i(x)$ and $\omega_i(x)$ are smooth functions on $\mathbb{R}^n$. The space of k-forms over $\mathbb{R}^n$ is denoted $\Lambda^k(\mathbb{R}^n)$.

Consider a control affine system

$$\dot{x} = f(x) + \sum_{k=1}^{m} u_k g_k(x), \quad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^m, \tag{14}$$

for some $f, g_1 \cdots, g_m \in \mathcal{C}^\infty(\mathbb{R}^n;\mathbb{R}^n)$ and assume that $f(0) = 0, \quad k = 0,\ldots,m$. Moreover, assume that for every $x \in \mathbb{R}^n \setminus \{0\}$,

$$\text{span}\{f(x), \text{ad}_f^k g(x),\ k \in \mathbb{N}\} = \mathbb{R}^n. \tag{15}$$

Consider the feedback law $u = (u_1,\ldots,u_m)^T$ defined by

$$u_k = -\nabla^T \psi \cdot g(x)(x), \quad \forall\, k \in 1,\ldots,m, \tag{16}$$

with $\psi(x)$ a weak Jurdjevic–Quinn function (Malisoff and Mazenc, 2009, Chapter 4), *i.e.* such that $\psi(x) > 0$ and $(\nabla^T\psi \cdot g)(x) < 0$ for all x in a neighborhood $\mathcal{O} \subset \mathbb{R}^n \setminus \{0\}$, $\psi(0) = 0$ and $(\nabla^T\psi \cdot g)(0) = 0$.

With this feedback, one has for all $x \in \mathbb{R}^n \setminus \{0\}$

$$\frac{d\psi}{dt}(x) = f(x) \cdot \nabla\psi(x) - \sum_{k=1}^{m} (g_k(x) \cdot \nabla^T \psi(x))^2 < 0. \tag{17}$$

Therefore, the origin $0 \in \mathbb{R}^n$ is asymptotically stable in closed-loop. The function $\psi(x)$ is not a control Lyapunov function (CLF) in general. In (Hudon and Guay, 2009), a deformation approach of the function $\psi(x)$ was presented. We refer to (Malisoff and Mazenc, 2009, Definition 2.2) for CLF construction methods based on the prior knowledge of a function $\psi(x)$ satisfying the weak Jurdjevic–Quinn conditions. In the next section, we seek to use the drift vector field $f(x)$ structure to design a dissipative potential.

3.1 Construction of a Potential

We first show how to construct a radial homotopy operator $\mathbb{H}$, *i.e.*, a linear operator on elements of one forms on $\mathbb{R}^n$ that satisfies the identity

$$\omega = d(\mathbb{H}\omega) + \mathbb{H}d\omega, \tag{18}$$

for a given differential form ω.

The first step in the construction of a homotopy operator is to define a star-shaped domain on $\mathbb{R}^n$. An open subset S of $\mathbb{R}^n$ is said to be star-shaped with respect to a point $p^0 = (x_1^0,\ldots,x_n^0) \in S$ if the following conditions hold:

- S is contained in a coordinate neighborhood U of p^0.
- The coordinate functions of U assign coordinates $(x_1^0,\ldots,x_n^0)$ to p^0.
- If p is any point in S with coordinates $(x_1,\ldots,x_n)$ assigned by functions of U, then the set of points $(x + \lambda(x - x^0))$ belongs to S, $\forall\lambda \in [0,1]$.

A star-shaped region S has a natural associated vector field $\mathfrak{X}$, defined in local coordinates by

$$\mathfrak{X}(x) = (x_i - x_i^0)\partial_{x_i}, \quad \forall x \in S. \tag{19}$$

For a differential form ω of degree k on a star-shaped region S centered at an equilibrium x^0, the homotopy operator is defined, in coordinates, as

$$(\mathbb{H}\omega)(x) = \int_0^1 \mathfrak{X}(\lambda(x_i - x_i^0)) \lrcorner\, \omega(x_i^0 + \lambda(x_i - x_i^0))\lambda^{k-1} d\lambda, \tag{20}$$

where $\omega(x_i^0 + \lambda(x_i - x_i^0))$ denotes the differential form evaluated on the star-shaped domain in the local coordinates defined above.

The important properties of the homotopy operator that are used in the sequel are the following:

(i) $\mathbb{H}$ maps $\Lambda^k(S)$ into $\Lambda^{k-1}(S)$ for $k \geq 1$ and maps $\Lambda^0(S)$ identically to zero.
(ii) $d\mathbb{H} + \mathbb{H}d =$ identity for $k \geq 1$ and $(\mathbb{H}df)(x) = f(x) - f(x_0)$ for $k = 0$.
(iii) $(\mathbb{H}\mathbb{H}\omega)(x_i) = 0, \quad (\mathbb{H}\omega)(x_i^0) = 0$.
(iv) $\mathfrak{X} \lrcorner \mathbb{H} = 0, \quad \mathbb{H}\mathfrak{X}\lrcorner = 0$.

The first part of the right hand side of (18), $d(\mathbb{H}\omega)$, is obviously a closed form, since $d \circ d(\mathbb{H}\omega) = 0$. Since by property (i) of the homotopy operator, for $\omega \in \Lambda^k(S)$, we have $(\mathbb{H}\omega) \in \Lambda^{k-1}(S)$, $d(\mathbb{H}\omega)$ is also exact on S. We denote the exact part of ω by $\omega_e = d(\mathbb{H}\omega)$ and the anti-exact part by $\omega_a = \mathbb{H}d\omega$. It is possible to show that ω vanishes on $\mathbb{R}^n$ if and only if ω_e and ω_a vanish together Edelen (2005).

From the decomposition outlined above, we have

$$\omega - \omega_a = \omega_e. \tag{21}$$

Taking the exterior derivative on both sides and using the fact that ω_e is closed, we have

$$d(\omega - \omega_a) = d(\omega_e) = 0. \tag{22}$$

In the sequel, we apply the homotopy operator on one-forms.

First, we define a non-vanishing closed two-form $\Omega(x)$ on $\mathbb{R}^n$ as

$$\Omega = \sum_{1 \le i < j \le n} dx_i \wedge dx_j. \tag{23}$$

In the present paper, the non-vanishing two-form Ω is not necessarily defined in a canonical way, since the objective is ultimately to compute an admissible dissipative potential (and not a minimal one). For example, if $n = 3$, we would have,

$$\Omega = dx_1 \wedge dx_2 + dx_1 \wedge dx_3 + dx_2 \wedge dx_3. \tag{24}$$

The orientation of the two-form will be fixed, if necessary, by checking the sign of the obtained dissipative function, $\psi(x)$.

We obtain a first one-form associated to the system by contracting this two-form with respect to the drift vector field,

$$\omega_0(x) = (f \lrcorner \Omega)(x). \tag{25}$$

From the above discussion, we know that we can locally construct a homotopy operator on $\mathbb{R}^n$ such that $\omega_0 = \omega_{0,e} + \omega_{0,a}$. Since $\omega_{0,e}$ is exact, it is given as the exterior derivative of a potential function and we rewrite

$$\omega_0 = -d\psi + \omega_{0,a}. \tag{26}$$

We assume that $\psi(x)$, obtained after application of the homotopy operator (*i.e.*, $\psi(x) = -(\mathbb{H}\omega_{0,e}))(x)$, is such that $\nabla^T \psi(x) \cdot f < 0$ for $x \in \mathbb{R}^n \setminus \{0\}$. In practice, one may use an integrating factor $\gamma(x)$ to guarantee that

$$\psi(x) = -\left(\mathbb{H}(\gamma\omega_0)\right)(x) \tag{27}$$

has the desired properties. In the present paper, the anti-exact part that does not contribute locally to the dissipative dynamics is not taken into account for the design. In practice, a feedback gain κ is used to dominate the tangential dynamics, *i.e.*, we construct the damping feedback controller

$$u_k(x) = -\kappa(\nabla^T \psi \cdot g)(x). \tag{28}$$

However, if one was considering the problem of deforming $\psi(x)$ to derive a control Lyapunov function, it was shown in (Hudon and Guay, 2009) that $\omega_{0,a} \equiv 0$ has to hold locally by building an integrating factor. Essentially, this last condition is equivalent to the equality of mixed partial derivatives for the construction of a storage function for dissipative systems.

We now turn our attention to the cyclic interconnection system structure.

3.2 Stabilization of a Desired Equilibrium

In this section, we decompose specialize the above construction to cyclic interconnection systems. We take advantage of the structure, defining Ω as

$$\Omega = \sum_{i=1}^{n} dx_1 \wedge dx_2 + \ldots + dx_i \wedge dx_{i+1} + \ldots + dx_n \wedge dx_1. \tag{29}$$

Contracting this two-form with respect to the drift vector field denoted $f_1\partial_{x_1} + f_2\partial_{x_2} + \ldots + f_n\partial_{x_n}$, we obtain the one-form

$$\omega_0 = (f_n - f_2)dx_1 + (f_1 - f_3)dx_2 + \ldots (f_{n-1} - f_1)dx_n. \tag{30}$$

A potential for damping stabilization design can be obtained by applying a locally defined homotopy operator centered at an admissible equilibrium point x^* of the dynamics,

$$\psi(x) = -\sum_{i=1}^{n} \mathbb{H}\omega_0 \tag{31}$$

$$= \int_0^1 \left((f_{i-1} - f_{i+1})|_{x^*+\lambda(x-x^*)}\lambda(x_i - x_i^*)\right) d\lambda, \tag{32}$$

with f_{i-1} for $i = 1$ being f_n and f_{i+1} for $i = n$ being f_i.

Using the assumptions along the lines of those used in (Arcak and Sontag, 2006, Sections 5 and 6), it is possible to show that $\psi(x)$ is locally a weak Jurdjevic–Quinn function. Due to space limitations, we omit the explicit computations. But to summarize, if one first expands the terms $(f_{i-1} - f_{i+1})x_i$, one sees that the terms $b_i(x_i)x_i$, by assumption positive semi-definite appear. Then, using the assumption that there exists $\gamma_i > 0$, such that $\frac{b_i(x_i)}{a_i(x_i)} \le \gamma_i$, for $x \ne 0$, one can shows that the positive terms $a_i(x)x_i > 0$ dominate the negative terms in the explicit expression for $\psi(x)$. To show that $(\nabla^T\psi \cdot f)(x) < 0$, one need essentially to assume some "secant criterion" on the γ_k for the explicit expression of $(\nabla^T\psi \cdot f)(x) < 0$ to be negative definite. An application of this construction is presented in Section 4.1.

3.3 Synchronization

We now turn our attention to the synchronization of two cyclic systems originally on different time scales. To be more precise, it is desired here to force one oscillating system to follow a copy of the system oscillating at a different frequency, a problem referred to phase entrainment by some authors. Consider, for example, the problem of synchronizing two systems:

$$\dot{x}_{1,1} = -a_1(x_{1,1}) + b_n(x_{n,1}) + u_1 \tag{33}$$

$$\dot{x}_{2,1} = -a_1(x_{2,1}) - b_1(x_{1,1}) + u_2 \tag{34}$$

$$\vdots \tag{35}$$

$$\dot{x}_{n,1} = -a_n(x_{n,1}) + b_{n-1,1}(x_{n-1,1}) + u_n, \tag{36}$$

and

$$\tau\dot{x}_{1,2} = -a_1(x_{1,2}) + b_n(x_{n,2}) \tag{37}$$

$$\tau\dot{x}_{2,2} = -a_1(x_{2,2}) - b_1(x_{1,2}) \tag{38}$$

$$\vdots \tag{39}$$

$$\tau\dot{x}_{n,2} = -a_n(x_{n,2}) + b_{n-1,2}(x_{n-1,2}). \tag{40}$$

We assume that the functions $a_{i,j}$ and $b_{i,j}$ are identical with the properties given in Section 2. We consider two differential one-forms obtained using $f_1(x_{i,1})$ and $f_2(x_{i,2})$, the drift vector fields from the two systems:

$$\eta_1 = f_1(x_{i,1}) \lrcorner \Omega_1 \tag{41}$$

$$\eta_2 = f_2(x_{i,2}) \lrcorner \Omega_2. \tag{42}$$

Since both systems have the same drift structure, we have that $\Omega_2 = \delta\Omega_1$, where $\delta = \tau^2$. We hence defined a closed one-form for the error as

$$\omega_0 = (f_1(x_{i,1}) - \delta f_2(x_{i,2})) \lrcorner \Omega_1. \tag{43}$$

Then, applying a homotopy centered at the origin for this dynamics, we have

$$\psi(x_{i,1}, x_{i,2}) = \mathbb{H}\omega_0 = \int_0^1 \omega_0(\lambda(x_{i,1} - x_{i,2}))\lambda(x_{i,1} - x_{i,2}) d\lambda. \tag{44}$$

Assuming that all the reference trajectories $x_{i,2}(t)$ are available, the damping controllers for the system are given by

$$u_k = -k_k \nabla^T_{x_{i,1}} \psi \cdot g_k. \tag{45}$$

Using the damping method from above, stabilization of the slave system to the reference trajectories follows. Application of this construction is demonstrated in Section 4.2.

4. NUMERICAL APPLICATION

This section presents the application of the constructions presented in Section 3 on the example presented in Section 2 through numerical simulations.

4.1 Suppression of Oscillations

We here assume that the full feedback controller affects only the first state variable, *i.e.*, the modified system is given by

$$\dot{x}_1 = -a_1 x_1 + \phi(x_3) + u \tag{46}$$

$$\dot{x}_2 = -a_2 x_2 + b_1 x_1 \tag{47}$$

$$\dot{x}_3 = -a_3 x_3 + b_2 x_2, \tag{48}$$

where u is given as

$$u(x) = -\nabla^T \psi(x) \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = -k \frac{\partial \psi}{\partial x_1}(x). \tag{49}$$

The stabilization of the desired equilibrium is presented in Figure 4.

It should be noted that the damping can be adjusted, *i.e.*, the convergence rate can be adjusted arbitrarily for the controller structure proposed here, as depicted in Figure 5 to suppress oscillations.

Finally, one can check that the controller structure $k = [1,\ 0,\ 0]^T$ fulfills the rank condition stated in Section 3.

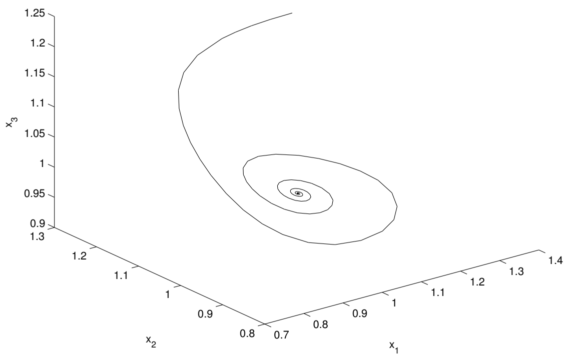

Fig. 4. Stabilization of the equilibrium — $k = 1$

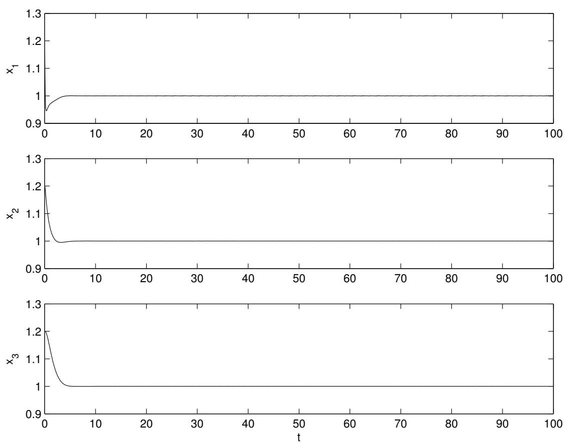

Fig. 5. State Trajectories — $k = 10$

4.2 Synchronization

We now apply the construction of Section 3.2. We consider the synchronization of the original system with full feedback

$$\dot{x}_1 = -a_1 x_1 + \phi(x_3) + u_1 \tag{50}$$

$$\dot{x}_2 = -a_2 x_2 + b_1 x_1 + u_2 \tag{51}$$

$$\dot{x}_3 = -a_3 x_3 + b_2 x_2 + u_3 \tag{52}$$

with a reference system on a different time scale

$$\sigma \dot{x}_{1r} = -a_1 x_{1r} + \phi(x_{3r}) \tag{53}$$

$$\sigma \dot{x}_{2r} = -a_2 x_{2r} + b_1 x_{1r} \tag{54}$$

$$\sigma \dot{x}_{3r} = -a_3 x_{3r} + b_2 x_{2r}. \tag{55}$$

In this paper, controller action is assumed to be applicable directly to each state, forcing each state to synchronize independently. If only one state is required to be synchronized, that structure can obviously be modified. In the simulations presented in Figure 6, the control $u_i(x) = -k_i \frac{\partial \psi}{\partial x_i}$ are set to zero for $t < 20$. Both systems are initialized at $x_0 = [1.2,\ 1.2,\ 1.2]^T$.

The results show that the original system synchronizes with the reference dynamics. Current studies seek to extend the procedure formally to the synchronization and observer design problem, as proposed in (Nijmeijer and Mareels, 1997).

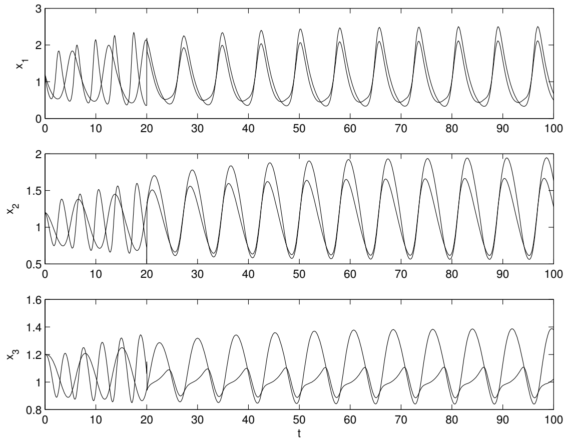

Fig. 6. Synchronization simulations — $\sigma = 2$

5. CONCLUSION

This paper presented an approach for the construction of smooth damping controllers for a class of cyclic systems. Taking the interior product of a non vanishing two-form with respect to the drift vector field, we first obtained a (possibly) non-closed characteristic one-form for the system. Constructing a locally defined homotopy operator on a star-shaped domain centered at the desired equilibrium point, we presented how to decompose locally the obtained form into an exact and an anti-exact one-forms. From (Hudon et al., 2008), we know that the exact part is associated to a dissipative (stable) potential. The obtained anti-exact form is associated to a non dissipative potential which generated tangential dynamics that do not contribute to the value of the dissipative potential locally on the star-shaped domain. A stabilizing controller was designed using the Jurdjevic–Quinn approach, following the construction developed in (Hudon and Guay, 2009). Application of the technique for stabilization of the desired equilibrium was presented as well as the application of the technique for oscillator synchronization. Future research will focus on extensions to more general networks and to time-dependent feedback design in cases where static stabilization might fail. The synchronization idea presented here leads to the natural extension to observer design (Nijmeijer and Mareels, 1997), and eventually, to adaptive control. Finally, application to stabilization and synchronization of circadian model such as the one presented in (Ito, 2007) will be considered.

REFERENCES

Arcak, M. and Sontag, E. (2006). Diagonal Stability of a Class of Cyclic Systems and its Connection with the Secant Criterion. *Automatica*, 42, 1531–1537.

Arcak, M. and Sontag, E. (2008). A Passivity-Based Stability Criterion for a Class of Interconnected Systems and Applications to Biochemical Reaction Networks. *Mathematical Biosciences and Engineering*, 5, 1–19.

De Leenheer, P. and Aeyels, D. (2002). Stabilization of Positive Systems with First Integrals. *Automatica*, 38, 1583–1589.

Edelen, D. (2005). *Applied Exterior Calculus.* Dover Publications Inc., Mineola, NY.

Grognard, F., Chitour, Y., and Bastin, G. (2004). Equilibria and Stability Analysis of a Branched Metabolic Network with Feedback Inhibition. In *Proceedings of the 9th International Symposium on Computer Applications in Biotechnology.*

Hudon, N. and Guay, M. (2009). Construction of Control Lyapunov Functions for Damping Stabilization of Control Affine Systems. In *Proceedings of the 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 8008–8013. Shanghai, China.

Hudon, N., Höffner, K., and Guay, M. (2008). Equivalence to Dissipative Hamiltonian Realization. In *Proceedings of the 47th IEEE Conference on Decision and Control*, 3163–3168.

Ito, H. (2007). Asymmetric iISS and ISS Properties Yielding Stability of Interconnected Systems and an Application to a Feedback Model. In *Proceedings of the 2007 American Control Conference*, 3041–3046.

Malisoff, M. and Mazenc, F. (2009). *Construction of Strict Lyapunov Functions.* Communications and Control Engineering Series. Springer-Verlag, London.

Nijmeijer, H. and Mareels, I. (1997). An Observer Looks at Synchronization. *IEEE Transactions on Circuits and Systems*, 44(10), 882–890.

Ortega, R., Astolfi, A., Bastin, G., and Rodrigues-Cortes, H. (1999). Output Feedback Control of Food-Chain Systems. In H. Nijmeijer and T. Fossen (eds.), *New Directions in Nonlinear Observer Design*, volume 244 of *Lecture Notes in Control and Information Sciences*, 291–310. Springer-Verlag, London.

Ortega, R., van der Schaft, A., Maschke, B., and Escobar, G. (2002). Interconnection and Damping Assignment Passivity-Based Control of Port-Controlled Hamiltonian Systems. *Automatica*, 38(4), 585–596.

Otero-Muras, I., Szederkényi, G., Alonso, A., and Hangos, K. (2008). Local Dissipative Hamiltonian Description of Reversible Reaction Networks. *Systems and Control Letters*, 57, 554–560.

Stan, G.B., Hamadeh, A., Sepulchre, R., and Gonçalves, J. (2007). Output Synchronization in Networks of Cyclic Biochemical Oscillators. In *Proceedings of the 2007 American Control Conference*, 3973–3978. New York, NY.