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Observer-based output feedback linearizing control applied to a denitrification reactor

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Abstract: In this work a late lumping approach is used in order to design a state feedback linearizing controller for influent disturbance attenuation and regulating the nitrogen concentration at the output of a denitrification biofilter. This controller is associated to a distributed parameter observer to estimate all the states needed to compute the controlled input. It results in an output feedback nonlinear controller with stable closed-loop dynamics.

Keywords: Distributed parameter systems, linearizing control, Luenberger observer, output feedback control.

1. INTRODUCTION

Several biotechnological processes are represented by partial differential equations (PDE) describing distributed parameter systems (DPS) in both time and space. In order to control a DPS, two strategies are commonly used: early lumping approach and late lumping approach. In the first one, the model partial differential equations are dicretized to obtain a high-order ordinary differential equation (ODE) system and then, ODE-based control strategies for nonlinear or linear systems may be applied (see for example Ray [1981]). On the other hand, in the second approach, control strategies have been developed, based on the non-linear control theory, to design a controller on the PDE system so as to keep as much as possible its distributed nature (see for example Banks et al. [1996], Christofides and Daoutidis [1998]).

The research concerning control of bioprocesses was mainly focused in the early lumping approach. This is because the most important control strategies have been developed to control systems described by either linear or non-linear systems represented by ODEs. In this context, several works have been developed, for instance: Dochain et al. [1992] applied adaptive control schemes to nonlinear distributed parameter bioreactors by using an orthogonal collocation method to reduce the original PDE model to ODE equations. In Alvarez-Ramirez et al. [2001] the authors dealt with the linear boundary control problem in an anaerobic digestion process by using the solution at steady state. Torres and Queinnec [2008] proposed to control the speed rate to reject an external disturbance on a denitrification reactor by using the method of characteristics to reduce the PDE system into a high-order ODE system.

On the other hand, in the last two decades, several control strategies using a late lumping approach based on the nonlinear control theory have been proposed. Gundepudi and Friedly [1998] addressed the problem of controlling a flow system described by a set of first-order PDEs with a single characteristic variable using the inverse system. Shang et al. [2005] have designed a feedback control method over the spatial interval that yields improved performance for DPS modelled by first-order hyperbolic PDEs. Wu and Lu [2001] addressed the output regulation of flow systems described by a class of two-time-scale nonlinear PDE system using the reduced-order slow model and geometric control. More specifically about biotechnological applications, Boubaker and Babary [2003] have applied variable structure control to fixed bed reactors described by nonlinear hyperbolic PDEs. Aguilar-Garnica et al. [2009] have designed a nonlinear multivariable controller for an anaerobic digestion system described by a set of PDEs and consisting of an observer and two nonlinear control laws on the boundary conditions.

In this context, this paper presents the design of an observer-based output feedback controller by using a late lumping approach in order to regulate the nitrogen concentration at the output and to reject a disturbance (nitrate inlet) at the input of a denitrification reactor. The biofilter is modeled by a system of hyperbolic PDEs, where the diffusion phenomena has been neglected. In addition, model uncertainty and noise at the measured output must also be bypassed. As a first step, a linearizing control is developed to partially linearize (according to the system's relative degree) the closed loop dynamics and then, by using a distributed parameter observer, the state variables not available by measurements are estimated. The paper is organized as follows: in section 2, the model of the denitrification reactor is presented. In section 3, a linearizing state feedback strategy is used to control the speed rate of inlet flow by considering the system's relative degree as shown in Shang et al. [2005] and Gundepudi and Friedly [1998]. In section 4, a distributed parameter observer is designed to estimate all the states needed to compute the controlled input as proposed in Vande Wouwer and Zeitz [2003]. In section 5, the overall output feedback controller is presented. In section 6, the discussion about simulations allows us to evaluate the results of this strategy. Finally, in section 7 some conclusions about this work are presented.

2. DENITRIFICATION MODEL

The denitrification process under study is a biofilter (a tubular reactor) filled with a porous pouzzolane material which allows the removal of nitrate and nitrite from influent wastewater. The influent conditions involve a sufficiently high ratio C/N such as to ensure that carboneous component does not become the limiting source for the growth. Denitrification is a two-stage reaction performed in anaerobic conditions. The first stage is the denitration which transforms nitrate (NO_3) into nitrite (NO_2) while the second phase transforms nitrite into gaseous nitrogen (N_2). The same micro-organism population (bacteria) is involved in both stages, with a carbon source as co-substrate. This biomass accumulates on the solid media surface thanks to filtration of bacteria present in the feeding water (if any) and to net growth. Thus, the biomass forms a biofilm around the filter particles, which thickens with time. One can then consider that all the biomass is fixed and does not move along the reactor. On the contrary, the soluble compounds (nitrate, nitrite and ethanol) are transported along the biofilter. It has been previously shown in Bourrel et al. [2000] that, except during the initial colonization step, the biomass concentration remains almost constant at a value X_{amax} along the biofilter and homogeneously distributed, even after a washing out. The dynamics of the biomass concentration are then cancelled and it is assumed this concentration remains constant at X_{amax}. Moreover, in the denitrification reactor model considered in this work, the diffusion phenomena has been neglected, resulting in the following quasi-linear hyperbolic PDE system:

$$\frac{\partial x_1(z,t)}{\partial t} = -\frac{v}{\epsilon}\frac{\partial x_1(z,t)}{\partial z} - \frac{1-Y_{h_1}}{1.14 Y_{h_1}\epsilon}\mu_1(x_1,x_3)X_{amax} \tag{1}$$

$$\frac{\partial x_2(z,t)}{\partial t} = -\frac{v}{\epsilon}\frac{\partial x_2(z,t)}{\partial z} + \frac{1-Y_{h_1}}{1.14 Y_{h_1}\epsilon}\mu_1(x_1,x_3)X_{amax} - \frac{1-Y_{h_2}}{1.71 Y_{h_2}\epsilon}\mu_2(x_2,x_3)X_{amax} \tag{2}$$

$$\frac{\partial x_3(z,t)}{\partial t} = -\frac{v}{\epsilon}\frac{\partial x_3(z,t)}{\partial z} - \frac{1}{Y_{h_1}\epsilon}\mu_1(x_1,x_3)X_{amax} - \frac{1}{Y_{h_2}\epsilon}\mu_2(x_2,x_3)X_{amax} \tag{3}$$

for $0 < z \leq L$, where z is the axial space variable. $x_1(z,t)$, $x_2(z,t)$ and $x_3(z,t)$ represent the nitrate ($g[N]/m^3$), nitrite ($g[N]/m^3$) and ethanol ($g[DCO]/m^3$) concentrations, respectively. v, Y_{h_1}, Y_{h_2}, μ_1 and μ_2 represent the flow speed m/h (the ratio between the feeding rate (m^3/h) at reactor input and the biofilter transverse surface (m^2)), micro-organisms yield coefficients and population specific rates which transform nitrate into nitrite, then nitrite into gas nitrogen ($1/h$).

The nitrate and nitrite specific growth rates are described by the model of Monod with two substrate limitations:

$$\mu_1(x_1,x_3) = \eta_g \mu_{1_{max}} \frac{x_1}{x_1 + K_{NO_3}} \frac{x_3}{x_3 + K_C}$$

$$\mu_2(x_2,x_3) = \eta_g \mu_{2_{max}} \frac{x_2}{x_2 + K_{NO_2}} \frac{x_3}{x_3 + K_C}$$

where η_g, $\mu_{1_{max}}$, $\mu_{2_{max}}$, K_{NO_3}, K_{NO_2} and K_C are the correction factor for the anaerobic growth, the maximum specific growth rates of biomass on nitrate and nitrite and the affinity constants with respect to nitrate, nitrite and ethanol, respectively.

Associated to the dynamic equations for the denitrification process, appropriate initial and boundary conditions are given by:

- Initial spatial profile at $t = 0$ for $0 \leq z \leq L$:

$$x_1(z, t=0) = 0 \; g[N]/m^3 \tag{4}$$

$$x_2(z, t=0) = 0 \; g[N]/m^3 \tag{5}$$

$$x_3(z, t=0) = 0 \; g[COD]/m^3 \tag{6}$$

- Dirichlet boundary conditions at $z = 0$ (input) for $t > 0$:

$$x_1(z=0,t) = x_{1,in} = 16.93 \; g[N]/m^3 \tag{7}$$

$$x_2(z=0,t) = x_{2,in} = 0 \; g[N]/m^3 \tag{8}$$

$$x_3(z=0,t) = x_{3,in} = 101.5 \; g[COD]/m^3 \tag{9}$$

Remark 1. Initial conditions express that the initial state corresponds to the instant after a wash out when the liquid in the biofilter is only clean water without nutrients.

The system (1)-(3) can be rewritten in matrix form as:

$$\frac{\partial x}{\partial t} = A\frac{\partial x}{\partial z} + f(x) \tag{10}$$

where $x = [x_1 \; x_2 \; x_3]^T$ is the state vector, matrix A is a diagonal square matrix $\in \mathbb{R}^{n \times n}$ which diagonal elements are denoted by a_{ii}, $f(x)$ is a vector of non-linear functions $\in \mathbb{R}^n$ and $n = 3$.

We are interested in regulating the nitrogen concentration at the reactor output. An output function is then defined as the sum of nitrate and nitrite concentrations at the reactor output:

$$y(t) = h(x) = x_1(z,t)|_{z=L} + x_2(z,t)|_{z=L} \tag{11}$$

This problem was addressed in Bourrel et al. [2000], where the authors first imposed the desired closed-loop dynamics and then the linearizing feedback controller was designed. However, as it will be shown, the closed-loop dynamics depend on the order of derivation of the output which is needed to derive an input-output map (i.e. on the relative degree of the system).

3. STATE FEEDBACK LINEARIZING CONTROL

Differentiating $y(t)$ with respect to time and by considering v as the control variable, it is obtained:

$$\dot{y}(t) = L_f h(x)|_{z=L} - \frac{v(t)}{\epsilon} \left. \frac{\partial h(x)}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L} \quad (12)$$

where $L_f h(x)$ is the Lie derivative of $h(x)$ with respect to $f(x)$.

Since for all $t > 0$

$$\left. \frac{\partial h(x)}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L} \neq 0$$

the relative degree of the system (10)-(11) is $r = 1$.

In order to feedback linearize the system (10)-(11), a new system of coordinates can be introduced (see Isidori [1989]):

$$\Phi(x) = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

with $\xi_1 = y(t)$.

Furthermore, because r is strictly less than n, it is always possible to find $n - r = 2$ additional functions ξ_2, ξ_3 such that (see Levine [1996]):

$$\left. \frac{\partial \xi_i}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L} = 0$$

for $i = 2, 3$. In this way, ξ_2 and ξ_3 can be obtained by solving the following two PDE:

$$\left. \frac{\partial \xi_2}{\partial x_1} \frac{\partial x_1}{\partial z} \right|_{z=L} + \left. \frac{\partial \xi_2}{\partial x_2} \frac{\partial x_2}{\partial z} \right|_{z=L} + \left. \frac{\partial \xi_2}{\partial x_3} \frac{\partial x_3}{\partial z} \right|_{z=L} = 0 \quad (13)$$

$$\left. \frac{\partial \xi_3}{\partial x_1} \frac{\partial x_1}{\partial z} \right|_{z=L} + \left. \frac{\partial \xi_3}{\partial x_2} \frac{\partial x_2}{\partial z} \right|_{z=L} + \left. \frac{\partial \xi_3}{\partial x_3} \frac{\partial x_3}{\partial z} \right|_{z=L} = 0 \quad (14)$$

It must be pointed out that solving the two PDEs above is a hard task because they depend on the solution of the state equations.

According to (12) and denoting:

$$a(\xi) = L_f h(x)|_{z=L}$$

$$b(\xi) = \frac{-1}{\epsilon} \left. \frac{\partial h(x)}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L}$$

the following representation is obtained:

$$\frac{d\xi_1}{dt} = \frac{\partial \xi_1}{\partial x} \frac{\partial x}{\partial t} = a(\xi) + b(\xi) v(t) \quad (15)$$

Because ξ_2 and ξ_3 have been chosen so that

$$\left. \frac{\partial \xi_i}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L} = 0$$

one has,

$$\begin{aligned} \frac{d\xi_i}{dt} &= \frac{\partial \xi_i}{\partial x} \left. \left(f(x) - \frac{v(t)}{\epsilon} \frac{\partial x}{\partial z} \right) \right|_{z=L} \\ &= L_f \xi_i|_{z=L} - \frac{v(t)}{\epsilon} \left. \frac{\partial \xi_i}{\partial x} \frac{\partial x}{\partial z} \right|_{z=L} \\ &= L_f \xi_i|_{z=L} \end{aligned}$$

By setting:

$$q_i(\xi) = L_f \xi_i|_{z=L} \quad (16)$$

for $i = 2, 3$, the state space description of the original system (10)-(11) in the new coordinates may then be written as:

$$\begin{aligned} \dot{\xi}_1 &= a(\xi) + b(\xi) v(t) \\ \dot{\xi}_2 &= q_2(\xi) \\ \dot{\xi}_3 &= q_3(\xi) \end{aligned} \quad (17)$$

The objective is to build a control law $v(t)$ which stabilizes the closed loop system and such that the output $y(t)$ tracks a given constant reference y_r while limiting as much as possible the activity of the control input. Define the tracking error e_0 like $y(t) - y_r$. If the original system (10)-(11) is locally exponentially minimum phase and $\alpha_0 > 0$ then the state feedback control law:

$$\begin{aligned} v(t) &= \frac{1}{b(\xi)} \left(-a(\xi) - \alpha_0 e_0 \right) \\ &= \left. \left(\frac{-\epsilon}{\frac{\partial x_1}{\partial z} + \frac{\partial x_2}{\partial z}} \right) \right|_{z=L} \times \\ &\quad \left. \left(\frac{1 - Y_{h_2}}{1.71 Y_{h_2} \epsilon} \mu_2(x_2, x_3) X_{amax} - \alpha_0 \left(x_1 + x_2 - y_r \right) \right) \right|_{z=L} \end{aligned} \quad (18)$$

linearizes partially the original system and results in a (locally) exponentially stable closed loop system (see Sastry [1999]). Thus, by inspecting (17) the resulting closed loop dynamics is given by:

$$\dot{y}(t) = -\alpha_0 \left(y(t) - y_r \right) \quad (19)$$

because it was only necessary to differentiate once the output function to see explicity the control input.

The value of α_0 has to be sufficiently small to reject the influence of the x_1 and x_2 derivatives at the reactor output in the output dynamics but large enough to bypass the model uncertainties, especially those that come from

$\mu_2(x_2, x_3)$. In addition of this source of uncertainty, one could also consider parameter uncertainties (on X_{amax}, ϵ, Y_{h_2}), but in some sense, such uncertainties are hidden in that one of μ_2 and therefore they are not directly considered in the following.

4. DISTRIBUTED PARAMETER OBSERVER

In order to implement the control law (18) it is necessary to know the nitrite and ethanol concentrations to compute $\mu_2(x_2, x_3)$ and to approximate the spatial derivatives of both the nitrate and the nitrite concentrations at the reactor output. Nitrate and nitrite concentrations are available by measurements at the output of the biofilter. In addition, nitrite at the input is known to be zero. In order to design an observer with the minimum of information it is necessary to measure the nitrate at the input. Thus, the measured output is defined as:

$$y_m = [x_1(z=0,t)\ x_2(z=0,t)\ x_1(z=L,t)\ x_2(z=L,t)]^T \qquad (20)$$

A nonlinear distributed parameter observer (DPO), with a formulation analog to the Luenberger observer, is then designed so as to assign the error dynamics as proposed in Vande Wouwer and Zeitz [2003] to estimate the concentrations not accessible by measurements:

$$\frac{\partial \hat{x}}{\partial t} = A\frac{\partial \hat{x}}{\partial z} + f(\hat{x}) + \Gamma(\hat{x})\,(y_m - \hat{y}_m) \qquad (21)$$

with initial condition represented by:

$$\hat{x}(z, t=0) = \hat{x}(z, 0) \qquad (22)$$

where $\hat{x} = [\hat{x}_1\ \hat{x}_2\ \hat{x}_3]^T$ is the estimated state vector and $\Gamma(\hat{x}) \in \mathbb{R}^{3\times 3}$ is the correction term.

The design of operator Γ is based on the estimation error equations $e(z,t) = \hat{x}(z,t) - x(z,t)$. It is then obtained:

$$\frac{\partial e}{\partial t} = A\frac{\partial e}{\partial z} + f(\hat{x}) - f(x) + \Gamma(\hat{x})\,(y_m - \hat{y}_m) \qquad (23)$$

$$e(z, t=0) = \hat{x}(z,0) - x(z,0) \qquad (24)$$

The linearization of $f(x)$ along the estimated trajectory $\hat{x}(z,t)$ can be done to obtain (see Vande Wouwer and Zeitz [2003]):

$$\frac{\partial e}{\partial t} = A\frac{\partial e}{\partial z} + \left.\frac{\partial f(x)}{\partial x}\right|_{\hat{x}} e + \Gamma(\hat{x})\,(y_m - \hat{y}_m) \qquad (25)$$

This linearization is justified by the fact that the estimation error is assumed sufficiently small, i.e.:

$$\|e(z, t=0)\| = \|\hat{x}(z,0) - x(z,0)\| << 1 \qquad (26)$$

In order to stabilize the closed-loop dynamics and to cancel the nonlinear term, physical knowledge about the system is used to design the correction term $\Gamma(\hat{x})\,(y_m - \hat{y}_m)$. Considering the i^{th} PDE, the i^{th} correction term γ_i is constructed in terms of error profile $e(z,t)$ and a tuning parameter row vector $\alpha_i \in \mathbb{R}^{1\times 2}$, i.e.:

$$\gamma_i^T\,(y_m - \hat{y}_m) = -\left[\left(\left.\frac{\partial f_i(x)}{\partial x_1}\right|_{\hat{x}} + \alpha_{i1}\right)\ \left(\left.\frac{\partial f_i(x)}{\partial x_2}\right|_{\hat{x}} + \alpha_{i2}\right)\ 0\right] e(z,t) \qquad (27)$$

for $i = 1, 2, 3$. A rough initial profile $\hat{x}_0(z)$ as well as error profile $e(z,t)$ along the space in equations above are constructed by linear interpolation of known values at the measurement points.

Remark 2. The correction term (27) is used to compensate the nonlinearities of the i^{th} equation. The resulting observer system is asymptotically stable as soon as $\alpha_{i,j}$ are positive elements high enough. Since measurements about ethanol are not available inside the reactor, their error profile cannot be calculated. Therefore, the error related to this variable is not considered.

5. OUTPUT FEEDBACK CONTROL

At this moment a feedback linearizing controller and a distributed parameter observer have been developed by using a late lumping approach over the hyperbolic PDE system (1)-(3). The overall control law is then the aggregation of the feedback linearizing control with the distributed parameter observer, approximating the spatial derivatives by finite differences, that is:

$$v(t) = \left.\left(\frac{-\epsilon}{\frac{\Delta \hat{x}_1}{\Delta z} + \frac{\Delta \hat{x}_2}{\Delta z}}\right)\right|_{z=L} \times \left.\left(\frac{1 - Y_{h_2}}{1.71 Y_{h_2}\epsilon}\mu_2(\hat{x}_2, \hat{x}_3)X_{amax} - \alpha_0\,(\hat{x}_1 + \hat{x}_2 - y_r)\right)\right|_{z=L} \qquad (28)$$

with $v(0) = 4m/h$ and $\hat{x}(z,t)$ estimated by using (21).

6. SIMULATIONS AND RESULTS

In order to simulate the closed loop system, the original PDE system (1)-(3) and the observer PDE system (21), associated to the initial and boundary conditions given in (4)-(9), are solved by the Method of Lines (ML) and the spatial derivatives are approximated by fourth-order finite differences (FDM). $N = 151$ discretization points uniformly distributed throughout the reactor are sufficient to correctly solve the PDEs. A sample period $T = 1\ min.$ is used, whereas estimation starts after fifteen minutes and the control action starts after stabilizing the observer (after thirty minutes). To simulate the nominal "real" biofilter behavior, the values in Table 1 are considered (borrowed from Bourrel et al. [2000]).

As mentioned before, we are interested in evaluating the sensitivity of the closed-loop system with respect to the state estimation errors but also to the uncertainties of the model parameters. Since the variations of each parameter is aggregated as a variation of the overall μ_1 and μ_2 values, one considers, for the simulation of the closed-loop system, modified values for the maximum specific growth rates as follows: $\mu_{1_{max}} = 0.414\ 1/h$ and $\mu_{2_{max}} = 0.368\ 1/h$.

Parameter	Value	Parameter	Value
Y_{h_1}	0.56	Y_{h_2}	0.54
$\mu_{1_{max}}$	0.36 $1/h$	$\mu_{2_{max}}$	0.32 $1/h$
K_{NO_3}	1.5 $g[N]/m^3$	K_{NO_2}	1.0 $g[N]/m^3$
K_C	40 $g[COD]/m^3$	X_{amax}	800 $g[COD]/m^3$
n_g	0.8	ϵ	0.52
v	4 m/h	L	2.1 m

Table 1. Parameters considered to simulate the biofilter.

We are interested in regulating the output $y(t)$ less or equal than the European norm $(5.65g[N]/m^3)$ when the system is submitted to a varying influent nitrate concentration. The disturbance influent profile shown in figure 1 intends to illustrate a periodic varying disturbance but non-periodic or slowly varying disturbance could also be considered. In this application a reference $y_r = 5.0$ is considered. In order to obtain correct estimation and well rejection of disturbance and noise. In addition, to keep the distributed parameter observer dynamics faster than the linearizing feedback controller ones, the elements of the matrix α are proposed by trial and error, large enough:

$$\alpha = \begin{bmatrix} 110 & 0 \\ 100 & 110 \\ 100 & 100 \end{bmatrix}$$

The nitrate and the nitrite spatial derivatives at the reactor output necessary to compute the control law (28) are approximated thanks to finite differences. Besides the measurements available at the output $(z = L)$, the estimates of the concentration at the last point before the output is used. The location of this point depends of the number N of discretization points. However, such an influence is limited as soon as the variation of the concentrations remains smooth enough at the end of the biofilter. In the present configuration with $N = 151$, it is the estimations of nitrate and nitrite concentrations at $z = 2.086m$ which are used to compute the spatial derivatives.

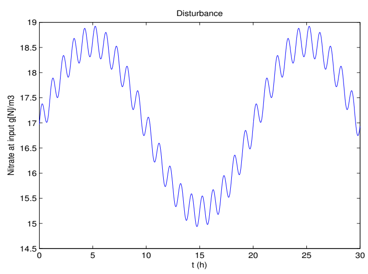

Fig. 1. Disturbance influent at the reactor input.

Figures 2 and 3 show both the nitrate and the nitrite derivatives, in solid blue the real value and in dashed green the estimated one. It can be observed that the estimated values follow correctly the real ones with an expected error because the estimated values and, the uncertainties and noise influence. Since the concentrations at the reactor output are less than at points before, negative slope values are computed.

Figure 4 shows in blue the ethanol real value and in black the estimated one at the reactor output. This value is needed to compute the growth term $\mu_2(x_2, x_3)$ at the reactor output. In this case, the bias in the estimation is related to the model uncertainties, as ethanol measurements are not available.

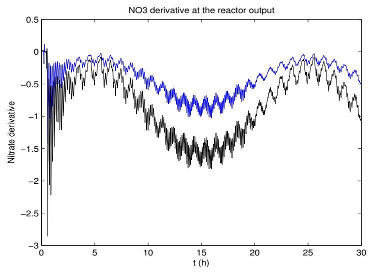

Fig. 2. Nitrate derivative estimated at the reactor output. In blue the real derivative and in black the estimated one.

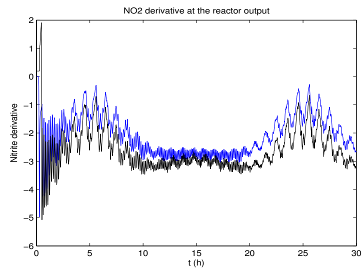

Fig. 3. Nitrite derivative estimated at the reactor output. In blue the real derivative and in black the estimated one.

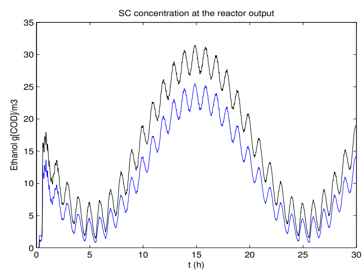

Fig. 4. Ethanol estimated at the output reactor. In blue the real concentration and in black the estimated one.

On the other hand, to calibrate the linearizing controller, a gain $\alpha_0 = 10$ was first proposed by using also a trial and error strategy. Under the hypothesis that the system is certain, the output converges quickly towards the reference and the disturbance shown in figure 1 is correctly rejected. However, the controller is strongly susceptible to model uncertainties. α_0 was then increased so as to reduce the influence of model uncertainties. $\alpha_0 = 90$ is then proposed to get robustness over uncertainties in the original model growth terms μ_1 and μ_2 without degradating too much the closed loop dynamics. A tolerance upon 15% of error on the prediction of μ_1 and μ_2 was observed. It must be pointed out that parametric uncertainties of the biofilter model is the most important problem to bypass by the linearizing feedback controller, when the state estimation is sufficiently accurate. In addition, a noise level upon 2% was correctly filtered.

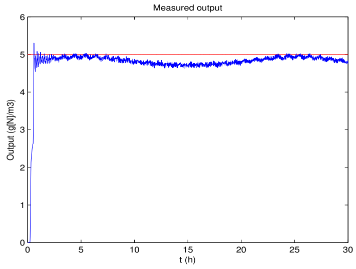

Fig. 5. Nitrogen concentration at the reactor output in closed-loop. In red the output reference and in blue the measured output.

Figure 5 shows the output reference in red and the nitrogen concentration at reactor output in blue according to the controlled input shown in figure 6 computed by using the observed-based linearizing controller designed. It can be seen a transition period of more or less three hours before tracking the reference and well rejecting the disturbance shown in figure 1.

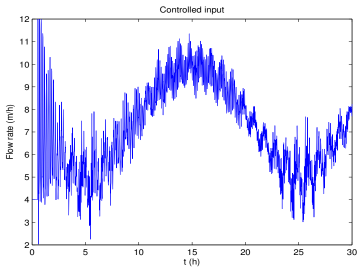

Fig. 6. Time-evolution of the flow rate along the biofilter.

7. CONCLUSIONS

This paper presents the design of a linearizing feedback controller to track, around a reference, the nitrogen concentration at the output of a denitrification reactor modeled by an hyperbolic PDE system. The flow rate $v(t)$ is choosen as the control input. This controller is complemented by a distributed parameter observer to estimate the overall set of states needed. In order to keep the system's distributed nature in the controller design, a late lumping approach has been considered.

The main idea behind the controller is to linearize the closed loop dynamics. In this way, the system's relative degree $r = 1$ indicates that the system is partially linearizable by representing it in a new system of coordinates of same dimension like the original. The first equation is constructed directly from the original system and the remaining equations are proposed to assure locally exponentially minimum phase of the system. However, this property is difficult to demonstrate, therefore a pragmatic version of the linearizing method is applied. Since the observer-based controller expressions are based on the original model, they are very sensitive to the model parameter uncertainties. In order to compensate them, a balance between the response and the robustness was found. In addition, it was demonstrated that the disturbance nitrate at the input is correctly rejected. However, it must be pointed out that a more theoretical robust analysis over the system dynamics is needed in order to correcly support its closed-loop performance. It is considered as a perspective of this work. Another perspective would be to combine a feedforward action with the feedback system to exploit the evident correlation between the disturbance and the control action.

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