Hedging Commodity Processes: Problems at the Intersection of Control, Operations, and Finance

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Abstract

The commodity markets are used to mitigate the risk associated with price uncertainty in commodity processes. The purpose of this paper is to demonstrate, with tutorial examples, opportunities to enhance financial return and further reduce financial risk due to uncertainty. The first example, the valuation of an energy swap for flex-fuel utility, demonstrates the integration of hedging with operational decisions. The second example, establishing a fair price for a process lease, illustrates the central importance of incomplete markets in process valuation.

Keywords: Process valuation, energy swaps, stochastic dominance.

Introduction

Commodity chemical and energy operations are central elements of the global economy. At a fundamental level, participants in the value chain of these industries are subject to financial risk because of uncertainty due to volatile commodity markets, unknown demand, and other future events outside the participant's control.

The purpose of this paper is to demonstrate that financial risk mitigation offers an array of engineering problems at the intersection of process control, operations, and finance. Risk assessment and mitigation, of course, are major functions of the commodity markets [2, 3]. Through futures and options trading, commodity markets provide a means for reducing exposure to price volatility.

Process design and control offer additional degrees of operational flexibility that can be used to mitigate the effects of market uncertainty. The working hypothesis of this paper is that integrating process and financial operations of complex commodity processes provides additional opportunities for enhancing returns and mitigating financial risk.

This paper introduces tutorial examples intended to illustrate the above points. The first, the valuation of an energy swap for a flexible fuel utility, considers the construction of a hedging portfolio that includes physical ownership of the underlying fuel stocks. The example demonstrates the use of process modeling and the importance of optimal process operation for effective hedging.

The second example, the valuation of a process lease when there exists significant uncertainty in the price of products and raw materials, shows the fundamental role of incomplete markets for risk mitigation for commodity processes. We propose a criterion based on second-order stochastic dominance as a framework for process valuation and risk assessment.

Example 1: Valuation of an Energy Swap for a Flex-Fuel Utility

The volatile energy markets expose utility providers to significant price uncertainty. Figure 1, for example, shows the price of the near future contract (an approximation to the 'spot' price) for natural gas (NG) and Appalachian coal (QL). We consider the case of a utility operator with the operational flexibility to use either natural gas or coal, in any mixture, to meet a known demand.

Utility operators are often thinly capitalized and work in tightly regulated retail markets. Therefore the operator will enter into an 'energy swap' with a banker who, for a fixed payment, underwrites the utility's cost of fuel. The utility operator provides a fixed payment $V(S_{NG}(t_0), S_{QL}(t_0), t_0)$ where $S_{NG}(t)$ and $S_{QL}(t)$ are spot market prices. In return, the banker pays the utility fuel costs $C(S_{NG}(t), S_{QL}(t))$. This simple type of energy swap is illustrated in Figure 2. The problem is to determine a 'fair' price for the utility operator

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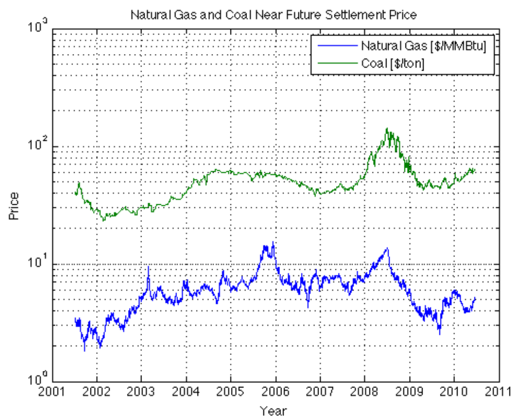


Figure 1: Natural Gas and Coal Prices (NYMEX)

$$\begin{array}{ccc} & \textit{Fixed Payment} & \\ & V(S_{NG}(t_0), S_{QL}(t_0), t_0) & \\ \textit{Utility} & \begin{array}{c}\longrightarrow \\ \longleftarrow\end{array} & \textit{Banker} \\ & \textit{Uncertain Fuel Cost} & \\ & C(S_{NG}(t), S_{QL}(t)) & \end{array}$$

Figure 2: Illustration of a simple energy swap. The utility provides a fixed payment to an energy banker at time t_0. In return, the banker accepts responsibility for paying the utilities cost of fuel

to pay the energy banker for the obligation to finance the utilty's energy costs.

Drawing on the literature for financial derivatives, the standard approach to this problem is to determine the initial capital needs for the energy banker to set up and operate a risk-free hedge. The hedge consists of portfolio of cash invested in risk-free assets and ownership of natural gas and coal inventories. The banker extracts money from the portfolio to cover the utility's fuel costs. Given stochastic models for the evolution of energy prices, the stochastic control problem is to manage the portfolio to pay the fuel costs of utility while minimizing or eliminating risk. If risk can be completely eliminated, then the no-arbitrage price of the energy swap is the value of the capital required to set up the hedge in the first place.

The self-financing hedging portfolio V is composed of cash, physical ownership of θ_{NG} units of natural gas, θ_{QL} units of coal, and the fuel financing obligation

$$\begin{aligned} dV &= \underbrace{r(V - \theta_{NG}S_{NG} - \theta_{QL}S_{QL})dt}_{Return\,on\,Cash} + \underbrace{\theta_{NG}dS_{NG} + \theta_{QL}dS_{QL}}_{Return\,on\,Hedging\,Position} \\ &\quad \underbrace{-C(S_{NG}, S_{QL})dt}_{Fuel\,Cost} + \underbrace{\theta_{NG}y_{NG}dt + \theta_{QL}y_{QL}dt}_{Convenience\,Yield} \end{aligned}$$

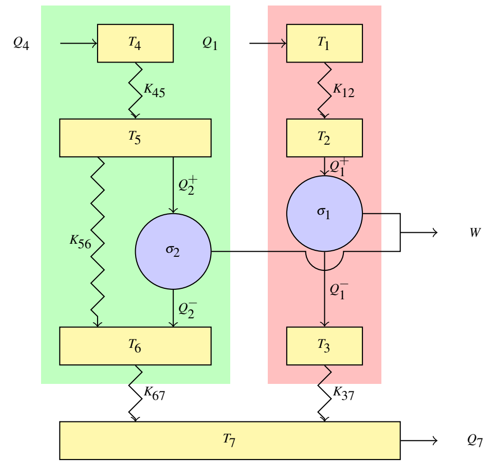


Figure 3: Diagram corresponding to a thermodynamically consistent model of a flex-fuel generation facility. The left path corresponds plant performance when coal-fired, the right path corresponds to the more efficient natural gas generating capability of the plant. For details, see [5].

The convenience yields y_{NG} and y_{QL} are common features of commodity price models which are not shared by other financial instruments. The convenience yield is the net return to the portfolio attributable to physical ownership of the underlying commodity. The convenience yield is positive if the owner places a high intrinsic value on physical ownership, such as the avoidance plant shutdowns in the event of raw material shortages. The convenience yield may be negative if the cost-of-carry of the physical inventory is large.

Prices are modeled as general Itô processes

$$\begin{aligned} dS_{NG} &= a_{NG}(S_{NG})\,dt + \sigma_{NG}S_{NG}dZ_{NG} \\ dS_{QL} &= a_{QL}(S_{QL})\,dt + \sigma_{QL}S_{QL}dZ_{QL} \end{aligned}$$

where dZ_{NG} and dZ_{QL} are correlated IID stochastic processes. For commodities, the deterministic portion of the returns, $a_k(S_k)$, typically exhibit mean-reversion.

A necessary input to this model is the cost of fuel required to meet the utilities production requirement. Process flexibility provides the operator with the ability to respond to market prices. The banker, of course, assumes the operator will respond by implementing a minimum cost strategy. In prior work we presented a framework for constructing heat rate models for complex utilities that are consistent with first and second laws of thermodynamics [5, 8, 9]. Following that framework, given $S_{NG}(t)$, $S_{QL}(t)$, and a heat rate model, the minimum fuel cost $C(S_{NG}(t), S_{QL}(t))$ is found by the solution of a bilinear optimization problem.

The bilinear optimization model is formulated as follows. Given prices $S_{NG}(t)$ and $S_{QL}(t)$, the task is minimize the cost

$$C(S_{NG}, S_{QL}) = \min_{T,\sigma}(S_{NG}q_{NG} + S_{QL}q_{QL})$$

which depends on the nodal temperatures $T = [T_1, \dots, T_N]$, heat inputs $q = [q_{NG}, q_{QL}]$, work outputs w, and entropy flux $\sigma = [\sigma_{NG}, \sigma_{QL}]$. Subject to a fixed work output, the utility is modeled by the bilinear relationships

$$\begin{aligned} q &= \left(K + \sum_k E_k \sigma_k\right) T \\ w &= \left(\sum_k W_k \sigma_k\right) T \end{aligned}$$

Matrix parameters K, E_k, and W_k are constructed from the energy conversion network diagram, an example of which is shown in Figure 3.

The control task is to manage the hedging portfolio in order to minimize risk. Substituting the price model into expression for dV, using Itô's lemma, and choosing $\theta_{NG} = \frac{\partial V}{\partial S_{NG}}$ and $\theta_{QL} = \frac{\partial V}{\partial S_{QL}}$ produces a risk-free portfolio. (This is a standard technique in the finance literature, for details refer to [2]). As typical for the Hamilton-Jacobi-Bellman formation of stochastic control, the parameters θ_{NG} and θ_{QL} are functions of time t, and of the prices $S_{NG}(t)$ and $S_{QL}(t)$.

$$\begin{aligned} -\frac{\partial V}{\partial t} &= \frac{\sigma_{NG}^2 S_{NG}^2}{2}\frac{\partial^2 V}{\partial S_{NG}^2} + \rho S_{NG} S_{QL}\frac{\partial^2 V}{\partial S_{NG}\partial S_{QL}} + \frac{\sigma_{QL}^2 S_{QL}^2}{2}\frac{\partial^2 V}{\partial S_{QL}^2} + \\ &\quad (rS_{NG} - y_{NG})\frac{\partial V}{\partial S_{NG}} + (rS_{QL} - y_{QL})\frac{\partial V}{\partial S_{QL}} - rV + C(S_{NG}, S_{QL}) \end{aligned}$$

subject to boundary conditions $V(S_{NG}, S_{QL}, T) = 0$, $V(0, S_{QL}, t \le T) = 0$, $V(S_{NG}, 0, t \le T) = 0$ and no curvature for large S_{NG} and S_{QL}.

The functions $\theta_{NG}(S_{NG}(t), S_{QL}(t), t)$ and $\theta_{QL}(S_{NG}(t), S_{QL}(t), t)$ implement the optimal hedging as a open loop control policy measuring current spot prices.

Typical solutions are shown in the accompanying figures. The figures demonstrate solution of the linear HJB equation that incorporate realistic price and utility models. The hedging position of the energy banker is determined by the feedback control law.

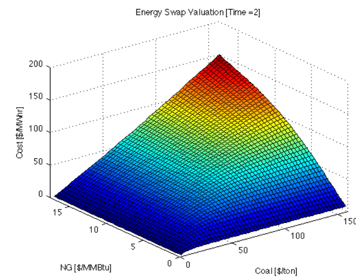


Figure 4: A typical solution for $V(S_{NG}, S_{QL}, t)$ with parameters $r = 0.1$, $y_{NG} = -0.2$, $y_{QL} = 3$

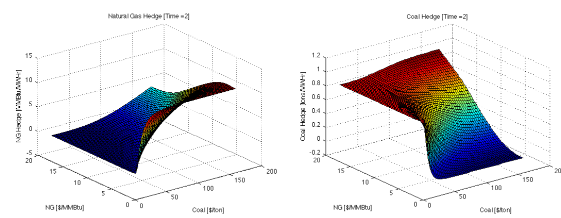


Figure 5: Values of the feedback control for natural gas $\theta_{NG} = \frac{\partial V}{\partial S_{NG}}$ and coal $\theta_{QL} = \frac{\partial V}{\partial S_{QL}}$

The cost of operation is $C = \$5/\text{ton}$ of P produced. The lease provides the operator with the right, but not the obligation, to produce one ton of P at time T_1. What is a fair price for the lease?

Example 2: Lease for a Simple Process

In this next example we consider a very different problem in which a process operator wishes to establish a fair price for leasing a unit of process capacity. At time $t = T_0$, an operator is offered a lease for a unit of capacity for a process converting A into P according to the stoichiometry

$$2A \longrightarrow P$$

Case 1: Future Prices Known at time T_0

If the prices of A and P are known and fixed at T_0

$$\begin{aligned} S_A &= 30 \\ S_P &= 80 \end{aligned}$$

then the profit at time T_1 is given by

$$\begin{aligned} \text{Profit} &= S_P - 2 \times S_A - C \\ &= 80 - 2 \times 30 - 5 \\ &= 15 \end{aligned}$$

There is no risk, so the operator should be indifferent to leasing the process or purchasing a risk-free bond. Thus the present value for a risk-free bond with $r_f = 5\%$

$$V(T_0) = \frac{15}{1 + r_f} = 14.29$$

Either investment will return 15 at time T_1.

Case 2: Uncertain Price for P at T_0

As before, we assume current prices of A and P are 30 and 80. In this case, however, the price for P at time T_1 is uncertain. We consider two scenarios

$$S_P(T_1|T_0) = \begin{cases} 60 & \text{Scenario 1} \\ 100 & \text{Scenario 2} \end{cases}$$

Profit assuming optimal operation at T_1

$$\text{Profit} = \begin{cases} \max(0, 60 - 2 \times 30 - 5) = 0 & \text{Scenario 1} \\ \max(0, 100 - 2 \times 30 - 5) = 35 & \text{Scenario 2} \end{cases}$$

This is not risk-free, therefore a risk-free bond is not a satisfactory pricing benchmark.

The process value at T_0 is established by constructing a pricing benchmark – a 'replicating portfolio' – that produces outcomes identical to the outcomes as the process lease under all scenarios. Using a risk-free bond and P as assets, the matrix of asset payoffs is

$$\begin{array}{cc} & \begin{array}{cc} \text{Bond} & P \\ \downarrow & \downarrow \end{array} \\ A = & \left[\begin{array}{cc} 1.05 & 60 \\ 1.05 & 100 \end{array} \right] \begin{array}{l} \leftarrow \text{Scenario 1} \\ \leftarrow \text{Scenario 2} \end{array} \end{array}$$

A 'replicating portfolio' x consists of risk-free bonds and contracts for P with same payoff as the process.

$$\underbrace{\left[\begin{array}{cc} 1.05 & 60 \\ 1.05 & 100 \end{array} \right]}_{A\text{: asset payoffs}} \underbrace{\left[\begin{array}{c} x_{Bond} \\ x_P \end{array} \right]}_{x} = \underbrace{\left[\begin{array}{c} 0 \\ 35 \end{array} \right]}_{b\text{: process payoff}} \begin{array}{l} \leftarrow \text{Scenario 1} \\ \leftarrow \text{Scenario 2} \end{array}$$

Solving for the replicating portfolio

$$\left[\begin{array}{c} x_{Bond} \\ x_P \end{array} \right] = \left[\begin{array}{cc} 1.05 & 60 \\ 1.05 & 100 \end{array} \right]^{-1} \left[\begin{array}{c} 0 \\ 35 \end{array} \right] = \left[\begin{array}{c} -50 \\ 0.875 \end{array} \right]$$

Value of the replicating portfolio at current prices –

$$V(T_0) = \underbrace{\left[\begin{array}{cc} 1.00 & 80 \end{array} \right]}_{S^T} \left[\begin{array}{c} -50 \\ 0.875 \end{array} \right] = 20$$

An operator can exactly reconstruct the process payoff given \$20 at time T_0. This establishes a 'no-arbitrage' price for the process lease.

Case 3: Uncertain Future Prices for A and P

Now suppose the price of both A and the price of P are uncertain at T_1. Assume an asset 'payoff' matrix

$$\begin{array}{cc} & \begin{array}{ccc} \text{Bond} & A & P \\ \downarrow & \downarrow & \downarrow \end{array} \\ A = & \left[\begin{array}{ccc} 1.05 & 25 & 60 \\ 1.05 & 25 & 100 \\ 1.05 & 35 & 60 \\ 1.05 & 35 & 100 \end{array} \right] \begin{array}{l} \leftarrow \text{Scenario 1} \\ \leftarrow \text{Scenario 2} \\ \leftarrow \text{Scenario 3} \\ \leftarrow \text{Scenario 4} \end{array} \end{array}$$

with current prices

$$\begin{array}{cc} & \begin{array}{ccc} \text{Bond} & A & P \\ \downarrow & \downarrow & \downarrow \end{array} \\ S = & \left[\begin{array}{ccc} 1.00 & 30 & 80 \end{array} \right] \end{array}$$

The process payoff is no longer in the range space of the asset payoffs.

$$\underbrace{\left[\begin{array}{ccc} 1.05 & 25 & 60 \\ 1.05 & 25 & 100 \\ 1.05 & 35 & 60 \\ 1.05 & 35 & 100 \end{array} \right]}_{A\text{: Asset payoff}} \underbrace{\left[\begin{array}{c} x_{Bond} \\ x_A \\ x_P \end{array} \right]}_{x} = \underbrace{\left[\begin{array}{c} 5 \\ 45 \\ 0 \\ 25 \end{array} \right]}_{b\text{: Process payoff}}$$

This situation is called an 'incomplete market'. The range of marketed assets is not sufficient to replicate the uncertain payoff. The best we can do is find no-arbitrage bounds on the price.

For this purpose, we introduce the concept of stochastic dominance (see, for example, [1, 10]). Portfolio x_A *stochastically dominates* portfolio x_B to zeroth order if the payoffs for x_A are greater than the payoffs for x_B for each scenario. A rational investor will always pay more for a dominating portfolio.

> *Super-Replicating Portfolio* The minimum cost portfolio dominating the process payoff
>
> $$V_{sup}(T_0) = \min_x S^T x \qquad \text{subject to} \qquad Ax \geq b$$
>
> This establishes an upper bound on the value of a process lease. A rational investor would never pay more for the process lease.
>
> *Sub-Replicating Portfolio* The maximum cost portfolio dominated by the process payoff
>
> $$V_{sub}(T_0) = \max_x S^T x \qquad \text{subject to} \qquad Ax \leq b$$
>
> This establishes a lower bound on the value of a process lease. A rational investor would pay at least this much for the process lease.

Applying these definitions to the example problem described above,

$$V_{sup}(T_0) = \min_x \underbrace{\left[\begin{array}{ccc} 1.00 & 30 & 80 \end{array}\right]}_{S^T} \underbrace{\left[\begin{array}{c} x_{Bond} \\ x_P \\ x_A \end{array}\right]}_{x}$$

subject to

$$\underbrace{\left[\begin{array}{ccc} 1.05 & 25 & 60 \\ 1.05 & 25 & 100 \\ 1.05 & 35 & 60 \\ 1.05 & 35 & 100 \end{array}\right]}_{A\text{: Asset payoff}} \underbrace{\left[\begin{array}{c} x_{Bond} \\ x_A \\ x_P \end{array}\right]}_{x} \geq \underbrace{\left[\begin{array}{c} 5 \\ 45 \\ 0 \\ 25 \end{array}\right]}_{b\text{: Process payoff}}$$

$V_{sup}(T_0) = 20.9524$ with $(S \circ x_{sup})^T = [30.9524, -60, 50]$ where $\circ$ denotes the element-by-element Hadamard product of two vectors.

A sub-replicating portfolio is also found by solving linear programming problem. Leaving out the details for brevity, $V_{sup}(T_0) = 15.9524$ with $(S \circ x_{sup})^T = [-19.0476, -15, 50]$.

Risk-Aversion

Second-Order Stochastic Dominance (SSD) as a Measure Risk-Aversion

Given a process lease, a no-cost hedging portfolio x can improve financial behavior. For example, to reduce financial risk, we propose a formulation

$$\max_{z,x} z$$

subject to

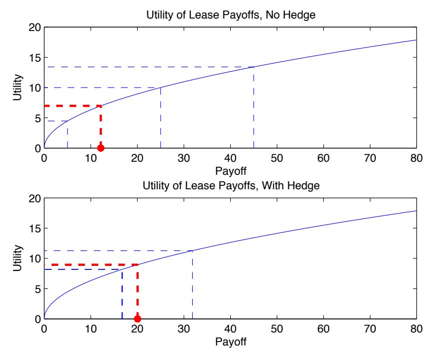
$$S^T x \leq 0$$
$$Ax + b \geq z$$

As before, the elements of x denote the quantities of each asset that will held in the hedging portfolio. The constraint $S^T x \leq 0$ means the portfolio can be purchased at a cost less than or equal to zero. The constraint $Ax + b \geq z$ means the sum of the portfolio payoff and the process payoff will be greater than z for all scenarios. Maximizing z means that we're maximizing the worst-case return that can be obtained under all scenarios with a no-cost portfolio.

Applying this criterion to the example described above yields a hedging portfolio $S \circ x = [35, 15, -50]^T$. Comparing payoffs

$$b = \left[\begin{array}{c} 5 \\ 45 \\ 0 \\ 25 \end{array}\right] \qquad Ax + b = \left[\begin{array}{c} 16.75 \\ 31.75 \\ 16.75 \\ 16.75 \end{array}\right]$$

These payoffs have the same no-arbitrage price bounds, but neither is zero-order dominant. A risk-averse operator, however, will always prefer the hedged payoff. To see this, we introduce a non-decreasing, concave utility function $u(x)$ to express the operator's degree of risk aversion. As shown in the following figure for a particular choice of utility function, assuming scenarios are equi-probable, a risk-averse investor would prefer the hedged portfolio.



For this case, it turns out that the expected utility of the hedged portfolio is greater than the unhedged portfolio for all admissible utility functions. This property, when it exists, is called second-order stochastic dominance.

Properties of Second-Order Stochastic Dominance (SSD)

The random variable X with probability density function (p.d.f.) $f_X(x)$ is stochastically dominant to second-order with respect to random variable Y with p.d.f. $f_Y(y)$, denoted $X \succeq_{SSD} Y$, if

$$E_X[u(\cdot)] \geq E_Y[u(\cdot)]$$

for any non-decreasing concave utility function u.

- All risk-averse investors (in the sense of a non-decreasing concave utility function) prefer X to Y if $X \succeq_{SSD} Y$
- SSD creates a partial ordering of distributions [1, 4].

Quantile functions are a useful test for second order stochastic dominance. Given a random variable X with p.d.f. $f_X(x)$, the cumulative distribution function (c.d.f.) is

$$F_X(x) = \int_{-\infty}^{x} f(\xi)\, d\xi$$

The quantile function is the 'inverse' of the c.d.f. For $p \in (0, 1)$

$$F_X^{(-1)}(p) = \inf\{x : F_X(x) \geq p\}$$

The second quantile function

$$F_X^{(-2)}(p) = \int_0^p F_X^{(-1)}(\psi)\, d\psi$$

Note that $F_X^{(-2)}(1) = E_X[X]$. A computationally important property of the second quantile function is stated as follows ([11, 7]): $X \succeq_{SSD} Y$ if and only if $F_X^{(-2)}(p) \geq F_Y^{(-2)}(p)$ for all $p \in (0,1)$.

Relationship to Value at Risk

Value at Risk (VaR) is a commonly used measure of the distributions of losses. VaR is expressed as a quantile of the distribution of losses. $VaR_\alpha(X)$ is the negative of the $1-\alpha$ quantile of expected payoff [10]

$$VaR_\alpha(X) = -F_X^{(-1)}(1-\alpha)$$

Conditional Value at Risk[1](CVaR) is a 'coherent' risk measure of expected loss at a specified quantile [7].

$$CVaR_\alpha(X) = -\frac{F_X^{(-2)}(1-\alpha)}{1-\alpha}$$

$X \succeq_{SSD} Y$ if and only if $CVaR_\alpha(X) \leq CVaR_\alpha(Y)$ for all $\alpha \in (0,1)$.

SSD Pricing

What is the minimum cost for a portfolio exhibiting SSD dominance over process payoffs?

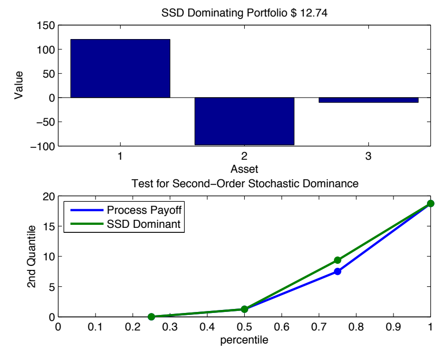
$$V_{sup}^{(2)}(T_0) = \min_x \underbrace{\begin{bmatrix} 1.00 & 30 & 80 \end{bmatrix}}_{S^T} \underbrace{\begin{bmatrix} x_{Bond} \\ x_P \\ x_A \end{bmatrix}}_{x}$$

subject to

$$\underbrace{\begin{bmatrix} 1.05 & 25 & 60 \\ 1.05 & 25 & 100 \\ 1.05 & 35 & 60 \\ 1.05 & 35 & 100 \end{bmatrix}}_{A:\text{ Asset payoff}} \underbrace{\begin{bmatrix} x_{Bond} \\ x_A \\ x_P \end{bmatrix}}_{x} \succeq_{SSD} \underbrace{\begin{bmatrix} 5 \\ 45 \\ 0 \\ 25 \end{bmatrix}}_{b:\text{ Process payoff}}$$

Any risk-averse investor would prefer that portfolio to the process payoff. Kopa [7, 6]has shown the solution to this problem is given by a linear program. For brevity, we have to leave out the computational details,

[1] also called Expected Shortfall (ES), Average Value at Risk (AVaR), and Expected Tail Loss (ETL)



$$V_{sup}^{(2)}(T_0) = 12.73$$

$$S \circ x_{SSD} = \begin{bmatrix} 120.24 \\ -97.5 \\ -10 \end{bmatrix}$$

SSD Hedging increases SSD Valuation

A no-cost hedge can be found to improve SSD Valuation. Setting up

$$\max_x \sum_{m=1}^{M} \left[F_{Ax+b}^{(-2)}\left(\frac{m}{M}\right) - F_b^{(-2)}\left(\frac{m}{M}\right) \right]$$

subject to

$$\underbrace{\begin{bmatrix} 1.00 & 30 & 80 \end{bmatrix}}_{S^T} \underbrace{\begin{bmatrix} x_{Bond} \\ x_P \\ x_A \end{bmatrix}}_{x} \leq 0$$

$$\underbrace{\begin{bmatrix} 1.05 & 25 & 60 \\ 1.05 & 25 & 100 \\ 1.05 & 35 & 60 \\ 1.05 & 35 & 100 \end{bmatrix}}_{A:\text{ Asset payoff}} \underbrace{\begin{bmatrix} x_{Bond} \\ x_A \\ x_P \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 5 \\ 45 \\ 0 \\ 25 \end{bmatrix}}_{b} \succeq_{SSD} \underbrace{\begin{bmatrix} 5 \\ 45 \\ 0 \\ 25 \end{bmatrix}}_{b:\text{ Process payoff}}$$

$$V_{sup,hedged}^{(2)}(T_0) = 19.88$$

The value of the SSD hedged lease is between the no-arbitrage bounds on the price of the lease. This establishes more realistic value for the process lease

Concluding Remarks

Using tutorial examples, this paper illustrates how financial and process operations may be combined to determine values for energy swaps, and to value simple process leases. We propose second-order stochastic dominance as criterion for the valuation of commodity process operations. Extending these ideas to more realistic process models, and to additional sources of process flexibility and uncertainty, are significant research challenges.

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