INEQUALITIES FOR CONTINUOUS-TIME MODEL FOR SCHEDULING OF CONTINUOUS PROCESSES

Nikisha K. Shah and Marianthi G. Ierapetritou*
Rutgers, The state university of New Jersey
Piscataway, NJ 08854

Abstract

This paper presents a mathematical formulation for large-scale continuous process scheduling problem using unit-specific event-based continuous-time representation. The formulation is based on the work of Ierapetritou et al., (1999). The model considers the changeovers for multi-purpose units and tanks, setup constraints for parallel units and tanks, and includes product tanks that satisfy demand of products with intermediate due-date requirements. The main objective of the model is to satisfy the demand by respecting the due-dates and maximize the performance by minimizing changeovers and setup costs. For the large-scale scheduling problem, the resulting formulation is a complex mixed-integer linear programming model that is difficult to solve to global optimality. A set of valid inequalities are proposed that improves the computational performance of the model significantly. Applicability of the proposed valid inequalities is demonstrated by studying a case study.

Keywords

Continuous process, Continuous-time model, Short-term scheduling, Valid inequalities.

Introduction

Much of the work in the area of continuous process manufacturing has been focused on the small-scale scheduling problem but the scheduling problem of large-scale multiproduct and multipurpose continuous plant have received significant less attention. Large-scale scheduling problems arise frequently in chemical industry where the main objective is to assign sequence of tasks to processing units within certain time frame such that demand of each product is satisfied before its due date.

Over the last two decades, different mathematical formulations are proposed for continuous process scheduling on basis of time representation. In discrete-time approaches, the time horizon is divided into a number of fixed time intervals, whereas the continuous-time approaches are based on time slots/events of unknown length. Two types of continuous-time approaches are studied in the literature, where the first is based on a set of events that are used for all tasks and units (global event based models), and the second approach introduces event points based on a task (unit-specific event based models). A couple of excellent reviews can be found in the literature (Floudas and Lin, (2004); Mouret et al., (2010)) Short-term scheduling models based on unit-specific event points have received considerable attention in the literature (Ierapetritou et al., (1999); Giannelos and Georgiadis, (2002); Mendez and Cerd, (2002); Shaik and Floudas, (2007)). Shaik et al., (2009) studied large-scale scheduling models based continuous time approach, where they considered changeovers and proposed a two-level framework to effectively deal with complexity of the medium-term scheduling problem.

In our work we propose a large-scale scheduling model using unit-specific event-based continuous-time representation. The large-scale continuous process

* To whom all correspondence should be addressed. Email: marianth@soemail.rutgers.edu

scheduling problems are highly complex and to reduce the complexity, in our work we present a set of valid inequalities that reduces the complexity and improves the performance of the model.

Proposed model

In this section we present the mathematical formulation for the continuous manufacturing production plant that is an improvement over the model of Ierapetritou et al., (1999). A state-task network (STN) representation introduced by Kondili et al., (1993) is used to describe the plant operations. A detailed description of each variable and parameter used in the model can be found in the nomenclature section.

Variable recipe constraint

Constraints (1-2) enforce that the amount produced or consumed at production unit is restricted by the imposed recipe bounds. Constraint (3) satisfies material balance at each production unit. It states that the total amount of material consumed is equal to the total amount of material produced.

$$\rho_{s,i}^{p,\min} \sum_{s' \in S_i^p} bp_{s',i,j,n} \le bp_{s,i,j,n} \le \rho_{s,i}^{p,\max} \sum_{s' \in S_i^p} bp_{s',i,j,n}, \quad \forall s \in S, i \in I_s^P, j \in J_i, n \in N \tag{1}$$

$$\rho_{s,i}^{c,\min} \sum_{s' \in S_i^c} bc_{s',i,j,n} \le bc_{s,i,j,n} \le \rho_{s,i}^{c,\max} \sum_{s' \in S_i^c} bc_{s',i,j,n}, \quad \forall s \in S, i \in I_s^C, j \in J_i, n \in N \tag{2}$$

$$\sum_{s \in S_i^p} bp_{s,i,j,n} = \sum_{s \in S_i^c} bc_{s,i,j,n}, \forall j \in J, i \in I_j, n \in N \tag{3}$$

Material balance constraints

Constraints (4-5) provide the material balance for the material produced or consumed at each production units, and constraints (6a-6b) enforce the material balance over the storage tanks.

$$\sum_{i \in I_j} bp_{s,i,j,n} = \sum_{k \in K_j^{pk} \cap K_s} Kif_{s,j,k,n} + \sum_{j' \in J_j^{seq} \cap J_s^c} JJf_{s,j,j',n} + Uof_{s,j,n}, \quad \forall s \in S, j \in J_s^p, n \in N \tag{4}$$

$$\sum_{i \in I_j} bc_{s,i,j,n} = \sum_{k \in K_j^{kp} \cap K_s} Kof_{s,k,j,n} + \sum_{j' \in J_j^{seq} \cap J_s^p} JJf_{s,j',j,n} + Uif_{s,j,n}, \quad \forall s \in S, j \in J_s^c, n \in N \tag{5}$$

$$st_{s,k,n} = sto_{s,k} + \sum_{j \in J_k^{pk}} Kif_{s,j,k,n} + Rif_{s,k,n} - \sum_{j \in J_k^{kp}} Kof_{s,j,k,n} - \sum_{o \in O_s} Lf_{k,o,n}, \quad \forall s \in S, k \in K_s, n = 1 \tag{6a}$$

$$st_{s,k,n} = st_{s,k,n-1} + \sum_{j \in J_k^{pk}} Kif_{s,j,k,n} + Rif_{s,k,n} - \sum_{j \in J_k^{kp}} Kof_{s,j,k,n} - \sum_{o \in O_s} Lf_{k,o,n}, \quad \forall s \in S, k \in K_s, 1 < n \le N \tag{6b}$$

Capacity constraints for production units

Constraint (7a) enforces that the material processed by unit j performing task i is bounded by the maximum and minimum rate of production. Constraint (8) provides upper bounds on the total amount of the material processed at each unit over the entire time horizon.

$$R_{ij}^{\min}\left(Tf_{ijn} - Ts_{ijn}\right) \le \sum_{s \in S_i^p} bp_{sijn} \le R_{ij}^{\max}\left(Tf_{ijn} - Ts_{ijn}\right), \quad i \in I, j \in J_i, n \in N \tag{7}$$

$$\sum_{s \in S_i^p} bp_{sijn} \le UH\left(R_{ij}^{\max} wv_{ijn}\right), \quad \forall i \in I, j \in J_i, n \in N \tag{8}$$

Capacity constraints for storage tanks

Constraints (9-11) define the binary variables associated with the flow in and out of the tanks. Constraint (12) enforces the maximum capacity of the tank and constraints (13a-13c) define the binary variable associated with material present in the tank.

$$Kif_{s,j,k,n} \le M \cdot in_{s,j,k,n}, \quad \forall j \in J, k \in K_j^{pk}, n \in N \tag{9}$$

$$Kof_{s,j,k,n} \le M \cdot out_{s,j,k,n}, \quad \forall j \in J, k \in K_j^{kp}, n \in N \tag{10}$$

$$Lf_{s,k,o,n} \le M \cdot l_{k,o,n}, \quad \forall k \in K_P, s \in S_k, o \in O_s, n \in N \tag{11}$$

$$st_{s,k,n} \le V_k^{\max} y_{s,k,n}, \quad \forall s \in S, k \in K_s, n \in N \tag{12}$$

$$sto_{s,k} + \sum_{j \in J_k^{pk}} Kif_{s,j,k,n} + Rif_{s,k,n} \le M \cdot y_{s,k,n}, \quad \forall s \in S, k \in K_s, n = 1 \tag{13a}$$

$$st_{s,k,n-1} + \sum_{j \in J_k^{pk}} Kif_{s,j,k,n} + Rif_{s,k,n} \le M \cdot y_{s,k,n}, \quad \forall s \in S, k \in K_s, n > 1 \tag{13b}$$

$$\sum_{j \in J_k^{kp}} Kof_{s,k,j,n} + \sum_{o \in O_s} Lf_{k,o,n} \le M \cdot y_{s,k,n}, \quad \forall k \in K_P, s \in S_k, n \in N \tag{13c}$$

The maximum and minimum unloading rate for product storage tanks must be bounded as specified by constraint (14).

$$U_k^{\min}\left(Tof_{k,o,n} - Tos_{k,o,n}\right) \le Lf_{k,o,n} \le U_k^{\max}\left(Tof_{k,o,n} - Tos_{k,o,n}\right), \forall k \in K_p, o \in O_{s \in S_k}, n \in N \tag{14}$$

Demand constraints

The plant has two types of products; type A can be stored in tank and has intermediate due dates and type B that is not stored in tank and whose demand should be satisfied based on the production unit. Constraint (15) guarantees that sufficient amount of product will be available to meet the demand.

$$D_{o,s}^- + r_s^- - dg_o^l - rg_s^l + \le \sum_{k,n} Lf_{k,o,n} + \sum_{j,n} Uof_{s,j,n} \le D_{o,s}^+ + r_s^+ + dg_o^u + rg_s^u, \left\{\forall s \in S_b, o \in O_s\right\} \cup \left\{\forall s \in S_f\right\} \tag{15}$$

Due to production capacity limitation, sometimes the demand order of finished product cannot be satisfied within its due date. To obtain a feasible solution positive artificial variables associated with over and under production are introduced and they are penalized in the objective function to minimize quantity giveaway.

Allocation constraints

Constraints (16-18) express the requirements that each multipurpose production unit and storage tank can only perform one task at any given event point. Constraint (21) restricts that the product tank can satisfy at most one demand order at any event point.

$$\sum_{i \in I_j} wv_{i,j,n} \le 1, \quad \forall j \in J, n \in N \tag{16}$$

$$\sum_{s \in K_s} y_{s,k,n} \le 1, \quad \forall k \in K^m, n \in N \tag{17}$$

$$\sum_{o \in O_{s \in S_k}} l_{k,o,n} \le 1, \quad \forall k \in K_p, n \in N \tag{18}$$

Set-up constraints

Set-up variables are 0-1 continuous variables defined by constraints (19a-19b) for production unit and constraints (20a-20b) for storage tanks. In our work, we include set-up constraints only for parallel units and tanks.

$$\beta_{k,n} \le \sum_{s \in S_k} y_{s,k,n} + \sum_{s \in S_k, n' < n} y_{s,k,n'}, \quad \forall k \in K^h, n \in N, \sum_s yo_{s,k} = 0 \tag{19a}$$

$$\beta_{k,n} \ge \sum_{s \in S_k} y_{s,k,n} - \sum_{s' \in S_k, n' < n} y_{s',k,n'}, \quad \forall k \in K^h, n \in N, \sum_s yo_{s,k} = 0 \tag{19b}$$

$$\alpha_{j,n} \le \sum_{i \in J_i} wv_{i,j,n} + \sum_{i \in J_i, n' < n} wv_{i,j,n'}, \quad \forall j \in J^h, n \in N \tag{20a}$$

$$\alpha_{j,n} \ge \sum_{i \in J_i} wv_{i,j,n} - \sum_{i' \in J_{i'}, n' < n} wv_{i',j,n'}, \quad \forall j \in J^h, n \in N \tag{20b}$$

Changeovers constraints

Changeover constraints proposed by Shaik et al.Shaik et al., (2009) are used in this work. Changeovers variables are 0-1 continuous variables and constraints (21-22) are used to define the changeover variables. Changeovers between modes of operations cause disturbances and additional costs, thus, few changeovers are desired.

$$\chi_{i',i,j,n} \le wv_{i',j,n}, \quad \forall j \in J, i \in I_j, i' \in I_j, i' \ne i, n < N \tag{21a}$$

$$\chi_{i',i,j,n} \le 1 + wv_{i,j,n'} - \sum_{i'' \in I_j} wv_{i'',j,n'} + \sum_{i'' \in I_j, n'' \in n < n'' < n'} wv_{i'',j,n''}, \quad \forall j \in J, i \in I_j, i' \in I_j, i' \ne i, n < N, n < n' \le N \tag{21b}$$

$$\chi_{i',i,j,n} \ge wv_{i',j,n} + wv_{i,j,n'} - 1 - \sum_{i'' \in I_j, n < n'' < n'} wv_{i'',j,n''}, \quad \forall j \in J, i \in I_j, i' \in I_j, i' \ne i,, n < N, n < n' \le N \tag{21c}$$

$$\eta_{s',s,k,n} \le y_{s',k,n}, \forall k \in K, s \in S_k, s' \in S_k, s' \ne s, n < N \tag{22a}$$

$$\eta_{s',s,k,n} \le 1 + y_{s,k,n'} - \sum_{s'' \in S_k} y_{s'',k,n'} + \sum_{s'' \in S_k, n'' \in n < n'' < n'} y_{s'',k,n''}, \quad \forall k \in K, s \in S_k, s' \in S_k, s' \ne s, n < N, n < n' \le N \tag{22c}$$

$$\eta_{s',s,k,n} \ge y_{s',k,n} + y_{s,k,n'} - 1 - \sum_{s'' \in S_k, n < n'' < n'} y_{s'',k,n''}, \quad \forall k \in K, s \in S_k, s' \in S_k, s' \ne s,, n < N, n < n' \le N \tag{22d}$$

Sequence constraints for production units

Finishing time of any task must be greater than the starting time of that task, as represented by constraint (23). Constraints (24-25) express that a task cannot start at next event point $n+1$ until current task is completed at event point n. Equation (25) enforces the time sequence constraint for different tasks happening in the same unit. The unit needs to go through clean-up maintenance when changeover of service is happening between different operating modes.

$$Tf_{i,j,n} \ge Ts_{i,j,n}, \quad \forall i \in I, j \in J_i, n \in N \tag{23}$$

$$Ts_{i,j,n+1} \ge Tf_{i,j,n}, \quad \forall i \in I, j \in J_i, n \in N, n < N \tag{24}$$

$$Ts_{i,j,n+1} \ge Ts_{i',j,n} + tclean_{i',i}\left(2\chi_{i',i,j,n} - 1\right), \quad \forall j \in J, i \in I_j, i' \in I_j, i \ne i', n < N \tag{25}$$

Constraints (26a-26d) represent that the two consecutive productions with no storage in between happen at the same time because production units operate as continuous processes.

$$Ts_{i,j,n} \le Ts_{i',j',n} + UH\left(2 - wv_{i,j,n} - wv_{i',j',n}\right), \quad \forall j' \in J, j \in J_{j'}^{seq}, i \in I_j, i' \in I_{j'}, n \in N \tag{26a}$$

$$Ts_{i,j,n} \ge Ts_{i',j',n} - UH\left(2 - wv_{i,j,n} - wv_{i',j',n}\right), \quad \forall j' \in J, j \in J_{j'}^{seq}, i \in I_j, i' \in I_{j'}, n \in N \tag{26b}$$

$$Tf_{i,j,n} \le Tf_{i',j',n} + UH\left(2 - wv_{i,j,n} - wv_{i',j',n}\right), \quad \forall j' \in J, j \in J_{j'}^{seq}, i \in I_j, i' \in I_{j'}, n \in N \tag{26c}$$

$$Tf_{i,j,n} \ge Tf_{i',j',n} - UH\left(2 - wv_{i,j,n} - wv_{i',j',n}\right), \quad \forall j' \in J, j \in J_{j'}^{seq}, i \in I_j, i' \in I_{j'}, n \in N \tag{26d}$$

Sequence constraints for storage tanks:

Constraints (27-29) enforce the sequence time requirement for material movement transfer task from one event point to next event point.

$$Tsf_{j,k,n} \ge Tss_{j,k,n}, \quad \forall k \in K, j \in J_k^{pk}, n \in N \tag{27a}$$

$$Tss_{j,k,n+1} \ge Tsf_{j,k,n}, \quad \forall k \in K, j \in J_k^{pk}, n < N \tag{27b}$$

$$Tsf_{k,j,n} \ge Tss_{k,j,n}, \quad \forall k \in K, j \in J_k^{kp}, n \in N \tag{28a}$$

$$Tss_{k,j,n+1} \ge Tsf_{k,j,n}, \quad \forall k \in K, j \in J_k^{kp}, n < N \tag{28b}$$

$$Tof_{k,o,n} \ge Tos_{k,o,n}, \quad \forall k \in K_P, o \in O_{s \in S_k}, n \in N \tag{29a}$$

$$Tos_{k,o,n+1} \ge Tof_{k,o,n}, \quad \forall k \in K_p, o \in O_{s \in S_k}, n < N \tag{29b}$$

Start time sequence constraints for tanks receiving or sending material from/to multiple destinations are given by constraints (30a-30c).

$$Tss_{j',k,n+1} \ge Tsf_{j,k,n}, \forall k \in K, j \in J_k^{pk}, j' \in J_k^{pk}, j' \ne j, n < N \tag{30a}$$

$$Tss_{k,j',n+1} \ge Tsf_{k,j,n}, \forall k \in K, j \in J_k^{kp}, j' \in J_k^{kp}, j' \ne j, n < N \tag{30b}$$

$$Tos_{k,o',n+1} \ge Tof_{k,o,n}, \quad \forall k \in K_p, o \in O_{s \in S_k}, o' \in O_{s \in S_k}, o' \ne o, n < N \tag{30c}$$

Sequence constraints for material transfer in and out of tanks happening at the same event point is enforced by equations (31-32). The start and finish time of the material transfer must align as to not violate the material balance requirement.

$$Tss_{j,k,n} \ge Tss_{k,j',n} - UH\left(2 - out_{s,k,j',n} - in_{s,j,k,n}\right), \quad \forall k \in K, s \in K_s, j \in J_k^{pk}, j' \in J_k^{kp}, n \in N \tag{31a}$$

$$Tss_{j,k,n} \le Tss_{k,j',n} + UH\left(2 - out_{s,k,j',n} - in_{s,j,k,n}\right), \quad \forall k \in K, s \in K_s, j \in J_k^{pk}, j' \in J_k^{kp}, n \in N \tag{31b}$$

$$Tsf_{j,k,n} \ge Tsf_{k,j',n} - UH\left(2 - out_{s,k,j',n} - in_{s,j,k,n}\right), \quad \forall k \in K, s \in K_s, j \in J_k^{pk}, j' \in J_k^{kp}, n \in N \tag{31c}$$

$$Tsf_{j,k,n} \le Tsf_{k,j',n} + UH\left(2 - out_{s,k,j',n} - in_{s,j,k,n}\right), \quad \forall k \in K, s \in K_s, j \in J_k^{pk}, j' \in J_k^{kp}, n \in N \tag{31d}$$

$$Tss_{j,k,n} \ge Tos_{k,o,n} - UH\left(2 - l_{k,o,n} - in_{s,j,k,n}\right), \quad \forall k \in K_P, s \in K_s, o \in O_s, j \in J_k^{pk}, n \in N \tag{32a}$$

$$Tss_{j,k,n} \le Tos_{k,o,n} + UH\left(2 - l_{k,o,n} - in_{s,j,k,n}\right), \quad \forall k \in K_P, s \in K_s, o \in O_s, j \in J_k^{pk}, n \in N \tag{32b}$$

$$Tsf_{j,k,n} \ge Tof_{k,o,n} - UH\left(2 - l_{k,o,n} - in_{s,j,k,n}\right), \quad \forall k \in K_P, s \in K_s, o \in O_s, j \in J_k^{pk}, n \in N \tag{32c}$$

$$Tsf_{j,k,n} \le Tof_{k,o,n} + UH\left(2 - l_{k,o,n} - in_{s,j,k,n}\right), \quad \forall k \in K_P, s \in K_s, o \in O_s, j \in J_k^{pk}, n \in N \tag{32d}$$

Constraints (33-34) connect material transfer in/out of a tank from one event point to next event point.

$$Tss_{j,k,n+1} \ge Tss_{k,j',n}, \forall k \in K, j \in J_k^{pk}, j' \in J_k^{kp}, n < N \tag{33a}$$

$$Tss_{k,j',n+1} \ge Tss_{j,k,n}, \quad \forall k \in K, j \in J_k^{pk}, j' \in J_k^{kp}, n \in N \tag{33b}$$

$$Tos_{k,o,n+1} \ge Tsf_{j,k,n}, \quad \forall k \in K_P, j \in J_k^{pk}, o \in O_{s \in S_k}, n < N \tag{34a}$$

$$Tss_{j,k,n+1} \ge Tof_{k,o,n}, \quad \forall k \in K_P, j \in J_k^{pk}, o \in O_{s \in S_k}, n < N \tag{34b}$$

Sequence constraints for production units and tanks:

Sequence constraints for production and material flow into/out of storage tank happening at the same time event point is given by constraints (35-36).

$$Ts_{i,j,n} \le Tss_{j,k,n} + UH\left(2 - wv_{i,j,n} - in_{s,j,k,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{pk}, i \in I_j, n \in N \tag{35a}$$

$$Ts_{i,j,n} \ge Tss_{j,k,n} - UH\left(2 - wv_{i,j,n} - in_{s,j,k,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{pk}, i \in I_j, n \in N \tag{35b}$$

$$Tf_{i,j,n} \le Tsf_{j,k,n} + UH\left(2 - wv_{i,j,n} - in_{s,j,k,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{pk}, i \in I_j, n \in N \tag{35c}$$

$$Tf_{i,j,n} \ge Tsf_{j,k,n} - UH\left(2 - wv_{i,j,n} - in_{s,j,k,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{pk}, i \in I_j, n \in N \tag{35d}$$

$$Ts_{i,j,n} \le Tss_{k,j,n} + UH\left(2 - wv_{i,j,n} - out_{s,k,j,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{kp}, i \in I_j, n \in N \tag{36a}$$

$$Ts_{i,j,n} \ge Tss_{k,j,n} - UH\left(2 - wv_{i,j,n} - out_{s,k,j,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{kp}, i \in I_j, n \in N \tag{36b}$$

$$Tf_{i,j,n} \le Tsf_{k,j,n} + UH\left(2 - wv_{i,j,n} - out_{s,k,j,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{kp}, i \in I_j, n \in N \tag{36c}$$

$$Tf_{i,j,n} \ge Tsf_{k,j,n} - UH\left(2 - wv_{i,j,n} - out_{s,k,j,n}\right), \quad \forall s \in S, k \in K_s, j \in J_k^{kp}, i \in I_j, n \in N \tag{36d}$$

Intermediate due dates:

Intermediate due dates requirements for group A products, which are stored in product tanks, are given by constraints (37a-37b). To consider demurrage, slack variables are utilized and later penalized in the objective function.

$$Tos_{k,o,n} + UH\left(1 - l_{k,o,n}\right) \ge times_o - Tearly_o, \quad \forall o \in O, k \in K_p, n \in N \tag{37a}$$

$$Tof_{k,o,n} + UH\left(1 - l_{k,o,n}\right) \le timef_o + Tlate_o, \quad \forall o \in O, k \in K_p, n \in N \tag{37b}$$

All tasks should start and finish before the end of the scheduling time horizon as stated in (38a-38d). The scheduling horizon is bounded as $H \le UH$.

$$\sum_n \left(Tf_{i,j,n} - Ts_{i,j,n}\right) \le H, \qquad \forall i \in I, j \in J_i \tag{38a}$$

$$\sum_n \left(Tsf_{j,k,n} - Tss_{j,k,n}\right) \le H, \quad \forall j \in J, k \in K_j^{pk} \tag{38b}$$

$$\sum_n \left(Tsf_{k,j,n} - Tss_{k,j,n}\right) \le H, \quad \forall j \in J, k \in K_j^{kp} \tag{38c}$$

$$\sum_n \left(Tof_{k,o,n} - Tos_{k,o,n}\right) \le H, \quad \forall k \in K_p, o \in O_{s \in S_k} \tag{38d}$$

Objective function

The objective function (39) is used to maximize the performance and revenue of total production. The performance is represented by the minimization of utilization of units and tanks, start up set-ups, changeovers, demurrage, and under and over production and revenue term is calculated from the sales of the final products. The penalty weights are assigned arbitrary to each term depending on its importance in schedule and note that the different penalty parameters have significant effect on the computational time required to obtain an optimal solution.

$$\begin{aligned} z = & \sum_{i,j \in J_i,n} c_{i,j}^1 wv_{i,j,n} + \sum_{s,k \in K_s,n} c_k^2 y_{s,k,n} + \sum_{s,k \in K_s,n} c_k^5 st_{s,k,n} \\ & + \sum_{s,k \in K_s \cap K^h, j \in J_k^{pk},n} c_k^3 in_{s,j,k,n} + \sum_{s,k \in K_s \cap K^h, j \in J_k^{kp},n} c_k^4 out_{s,k,j,n} \\ & - \sum_{s \in S_f, j_s^p, n} c_s^6 Uof_{s,j,n} - \sum_{k \in K_P, o \in O_{s \in K_s}, n} c_s^7 Lf_{k.o,n} + \sum_{j \in J^h,n} c_j^8 \alpha_{j,n} \\ & + \sum_{k \in K^h,n} c_k^9 \beta_{k,n} + \sum_{j \in J, i \in I_j, i' \in I_j, i' \ne i, n} c_{i,i'}^{10} \chi_{i,i',j,n} + \sum_{s \in S^f} c_s^{12} rg_s^u \\ & + \sum_{k \in K, s \in S_k, s' \in S_k, s \ne s', n} c_{s,s'}^{11} \eta_{s,s',k,n} + \sum_{s \in S^f} c_s^{13} rg_s^l + \sum_{o \in O} c_o^{14} dg_o^u \\ & + \sum_{o \in O} c_o^{15} dg_o^l + \sum_{o \in O} c_o^{16} Tearly_o + \sum_{o \in O} c_o^{17} Tlate_o \end{aligned} \tag{39}$$

Valid Inequalities

Valid inequalities are added in the formulation to improve the computational efficiency of the proposed model. Constraints (40a-40c) enforce that if there is a material flow into/out of the tank at event point n , then the binary variable $y_{s,k,n}$ is 1.

$$\sum_{j \in J_k^{pk}} in_{s,j,k,n} \le \sum_{j \in J_k^{pk}} y_{s,k,n}, \quad \forall k \in K, s \in S_k, n \in N \tag{40a}$$

$$\sum_{j \in J_k^{kp}} out_{s,j,k,n} \le \sum_{j \in J_k^{kp}} y_{s,k,n}, \quad \forall k \in K, s \in S_k, n \in N \tag{40b}$$

$$\sum_{o \in O_s} l_{k,o,n} \le y_{s,k,n}, \quad \forall k \in K_p, s \in S_k, n \in N \tag{40c}$$

Constraints (41a-41b) require material s flow in/out of tanks to be active at event point n if the unit j is processing the material at that event point.

$$\sum_{k\in K_j^{pk}\cap K_s} in_{s,j,k,n} \ge \sum_{i\in I_j\cap I_s^p} wv_{i,j,n},\ \forall j\in J^{pk}, s\in S_j^p, n\in N \tag{41a}$$

$$\sum_{k\in K_j^{kp}\cap K_s} out_{s,k,j,n} \ge \sum_{i\in I_j\cap I_s^c} wv_{i,j,n},\ \forall j\in J^{kp}, s\in S_j^c, n\in N \tag{41b}$$

Constraint (42) enforces the material balance constraint in addition to the constraint presented in equation (3).

$$\sum_{k\in K_j^{kp}\cap K_s} Kof_{s,k,j,n} + \sum_{j'\in J_j^{seq}\cap J_s^p} JJf_{s,j',j,n} + Uif_{s,j,n} = \tag{42}$$

$$\sum_{k\in K_j^{pk}\cap K_s} Kif_{s,j,k,n} + \sum_{j'\in J_j^{seq}\cap J_s^c} JJf_{s,j,j',n} + Uof_{s,j,n}, \forall j\in J, n\in N$$

If two units are consecutive without any storage tank between them, then constraint (43a) imposes the simultaneous operation of these units due to the continuous operation mode. However, this constraint is not imposed on parallel production units that can produce the same type of products. For units that follow or are followed by parallel units, valid inequalities 43b and 43c are included.

$$\sum_{i'\in I_{j'}} wv_{i',j',n} = \sum_{i\in I_j} wv_{i,j,n}, \tag{43a}$$

$$\forall j\in J, j'\in J, j'\in J_j^{seq}, j\notin J^h, j'\notin J^h, n\in N$$

$$\sum_{j'\in J^h\cap j\in J_{j'}^{seq}} \sum_{i'\in I_{j'}} wv_{i',j',n} \ge \sum_{i\in I_j} wv_{i,j,n},\ \forall j\in J\cap J^{seq}_{j'\in J^h}, n\in N \tag{43b}$$

$$\sum_{j'\in J_j^{seq}\cap J^h} \sum_{i'\in I_{j'}\cap I_i^{seq}} wv_{i',j',n} \ge \sum_{i\in I_j} wv_{i,j,n}, \tag{43c}$$

$$\forall j\in J, \exists j'\subset J_j^{seq}\cap J^h, n\in N$$

The demand order set O is arranged according to the ascending due date start time. That is, $times_o \le times_{o'},\quad \forall o, o'\ne o, o'>o$. Inequalities 44a are applied to require the demand order o to be satisfied by product tank k earlier than order $o'>o$. Furthermore, if the initial inventory of the products is less than the required total minimum demand orders, then the production should take place before the demand is fulfilled. This requirement is captured by constraint (44b).

Here, $pr_s = \sum_{o\in O_s} D^-_{o,s} - \sum_{k\in K_s} sto_{s,k}$.

$$l_{k,o',n} \le \sum_{k'\in K_s, n'\le n} l_{k',o,n}, \tag{44a}$$

$$\forall s\in S_b, k\in K_s, o\in O_s, o'\in O_s, o'>o, n\in N$$

$$l_{k,o,n} \le \sum_{i\in I_s^p, j\in J_i, n'\le n} wv_{i,j,n'}, \tag{44b}$$

$$\forall s\in S_b, k\in K_s, o\in O_s, pr_s>0, n\in N$$

Case Study and Results

We apply our model to a case study that consists of 14 multipurpose production units that processes 4 different raw materials to produce 18 final products. Of the 14 production units, two are parallel units (R1 and R2). Units (L1-L5) have cleaning downtime impose between certain task mode changeovers. The intermediates that are consumed by unit (M1) are stored in five dedicated tanks and other seven intermediates share three different storage tanks but with restricted allocation to these tanks. Final products (mp1, mp2, and mp3) are stored in three dedicated product. Demand of all eighteen final products is bounded by the maximum and minimum limits. In our problem we consider five demand orders and scheduling time-horizon of 240 hours. The state-task network of the problem is given in Figure 1.

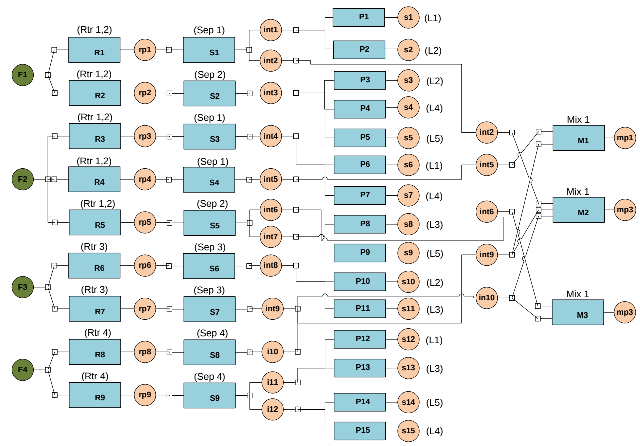

Figure 1. STN representation of the plant

The problem is solved on a Dell Precision (IntelR XeonTM with CPU 3.20 GHz, 3.19 GHz, and 2 GB memory) running on Windows XP using CPLEX 12.2.0/GAMS 23.6.3. The proposed model requires 4 event points in order to satisfy group A and group B products demand. The objective function corresponds to minimization of costs and maximization of revenue. The optimal objective function is found to be 1982.87 and statistics of model with and without valid inequalities are reported in Table 1.

Table 1. Model statistics and results

	Without Valid-inequalities	With Valid inequalities
Events	4	4
Binary Variables	384	384
Continuous Var.	2677	2677
Constraints	5857	6217
Nonzeros	24546	25684
Objective	1982.87	1982.87
% gap	48.75	0.00
CPU times	27147.72	1783.06
nodes	412021	82084
Iterations	709225859	5680953

When the valid inequalities are included in the model, the number of variables remains the same, but the number of constraints and nonzero elements increases. Valid inequalities have no effect on quality of the optimal solution; rather their effect is concentrated in significantly reducing the computational effort needed to find the optimal solution. The CPU time required to reach optimal solution is improved significantly when the valid inequalities are present versus when they are not included in the model. In the problem studied, it is not possible to find optimal solution within 7 hours; however the model with valid-inequalities obtains the optimal solution in 1783 seconds an improvement of 1 order of magnitude. The optimal objective schedule has minimum changeovers and for units L1-L5, the changeovers are favored between those tasks that have minimum clean-up downtime imposed.

Conclusions

In this paper, we present a scheduling model for large-scale continuous processes using unit-specific event points and continuous-time representation. A set of valid inequalities are proposed that reduces the CPU resolution time by a significant factor for large scale problems. Propose valid inequalities perform best when the problem consists of parallel and sequential units, product tanks, multipurpose units and tanks, and intermediate demand due-dates requirements.

Nomenclature

Indices: i :Tasks, n :Event point, j :Units, o : Orders, s :Tasks

Sets: I_j :Tasks performed in unit j, I_s^P / I_s^C :Tasks produce/consume material s, J :Production units, J_s^P / J_s^C :Units produce/consume material s, J_i :Units which are suitable for performing task i, J^h :Parallel production units, J_k^{pk} / J_k^{kp} :Units that produce/consume material stored in tank k, J_j^{seq} :Units that follow unit j (no storage in between), K :Storage tanks, K^h :Parallel tanks, K_j^{pk} / K_j^{kp} :Tanks that store material produce/ consumed by unit j, K_p :Tanks that can store final products , K_s :Tanks that can store material s , N :Event points, O :Demand Orders for Group A products, S :States, S_b / S_f :Group A/Group B products, S_k :Materials that can be stored in tank k, S_i^p / S_i^c :Materials produced/consumed by task i, S_j^p / S_j^c :Materials produced/consumed by unit j

Parameters: $D_{os}^+ / D_{os}^-, r_s^+ / r_s^-$: Demand requirements bounds, $R_{ij}^{\max} / R_{ij}^{\max}$:minimum or maximum rate of production, $U_k^{\max} / U_k^{\max}$:Minimum or maximum rate of unloading, $sto_{s,k}$:Initial amount of state s in tank k, UH :Available time horizon, $V_k^{\max}$:Maximum available storage capacity of storage tank k, $\rho_{si}^{\max} / \rho_{si}^{\max}$:Proportion of state s produced/consumed by task i. *Binary Variables*: wv_{ijn} :Assignment of task i in unit j at event point n, $in_{sjkn} \backslash out_{skjn}$:flow of s into/out of tank k from unit j at point n, l_{kon} :flow out of tank k to order o at event point n, y_{skn} :material s is stored in tank k at event point n. *Positive Variables*: bp_{sijn} / bc_{sijn} :material s produced/consumed by task i in unit j at point n, $dg_o^l / dg_o^u, rg_s^l / rg_s^u$:Over and under production, H :Total time horizon, $JJf_{sjj'n}$:Flow from unit j to consecutive unit j' at point n, $Kif_{sjkn} \backslash Kof_{skjn}$:Flow of material s into/out of tank, $Lf_{okn} \backslash Uof_{sjn}$:Flow of product from tank\unit, $Rif_{skn} \backslash Uif_{sjn}$:Flow of raw material to tank\unit, st_{skn} :Amount of material present in tank k, $Tearly_o \backslash Tlate_o$:Due date violations for order o, $Tf_{ijn} \backslash Ts_{ijn}$:Finish/start time for task i in unit j, $Tof_{kon} \backslash Tos_{kon}$:Finish/start time of flow from tank k for order o at event point n, $Tsf_{jkn} \backslash Tss_{jkn}$:Finish/start time of flow from unit to tank, $Tsf_{kjn} \backslash Tss_{kjn}$:Finish/start time of flow from tank to unit, $\alpha_{jn} \backslash \beta_{kn}$:1 if the unit\tank becomes active for very first time at event point n, $\eta_{ss'kn}$:1 if service changeover in tank k from s at event point n to s' at later event point, $\chi_{ii'jn}$:1 if task at unit j changes from i at event point n to i'at later event point..

Acknowledgments

The authors gratefully acknowledge financial support from the National Science Foundation under Grant CBET 0966861 and GAANN.

References

Floudas, C. A. and Lin, X. (2004). Continuous-Time Versus Discrete-Time Approaches For Scheduling of Chemical Processes: a Review. *Comput. Chem. Eng., 28(11)*, 2109.

Giannelos, N. F. and Georgiadis, M. C. (2002). A Novel Event-Driven Formulation for Short-Term Scheduling of Multipurpose Continuous Processes. *Ind. Eng. Chem. Res., 41(10)*, 2431.

Ierapetritou, M. G., Hene, T. S. and Floudas, C. A. (1999). Effective Continuous-Time Formulation for Short - Term Scheduling: 3. Multiple Intermediate Due Dates. *Ind. Eng. Chem. Res., 38(9)*, 3446.

Kondili, E., Pantelides, C. C. and Sargent, R. W. H. (1993). A General Algorithm for Short-Term Scheduling of Batch Operations--I. MILP Formulation. *Comput. Chem. Eng., 17(2)*, 211.

Mendez, C. A. and Cerd, J. (2002). An Efficient MILP Continuous-Time Formulation for Short-Term Scheduling of Multiproduct Continuous Facilities. *Comput. Chem. Eng., 26(4-5)*, 687.

Mouret, S., Grossmann, I. E. and Pestiaux, P. (2010). Time Representations and Mathematical Models for Process Scheduling Problems. *Comput. Chem. Eng., 35(6)*, 1038.

Shaik, M. A. and Floudas, C. A. (2007). Improved Unit-Specific Event-Based Continuous-Time Model for Short-Term Scheduling of Continuous Processes: Rigorous Treatment of Storage Requirements. *Ind. Eng. Chem. Res., 46(6)*, 1764.

Shaik, M. A., Floudas, C. A., Kallrath, J. and Pitz, H.-J. (2009). Production Scheduling of a Large-Scale Industrial Continuous Plant: Short-Term and Medium-Term Scheduling. *Comput. Chem. Eng., 33(3)*, 670.