DISSIPATIVITY-BASED NONLINEAR DECENTRALIZED CONTROL DESIGN

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Abstract

This contribution summarizes a methodology, based on dissipative systems theory, for the analysis and control of interconnected nonlinear processes. The global objective of the proposed approach is to design decentralized feedback controllers in such a way that plantwide stability and performance objectives are met, for a known constant interconnection structure. At a plantwide level, extensions of classical results on the stability of large-scale interconnected systems lead to input-output constraints for each subsystems, encoded as supply rates from input ports to output ports. Then for each subsystem, a (possibly parameterized) feedback controller is designed using nonlinear dissipative inequalities to ensure that the aforementioned input-output constraint is met in closed-loop. After a review of dissipativity theory for the analysis of interconnected networks, this paper focuses on the design of nonlinear feedback controllers ensuring that each subsystems meet the interconnection constraints. In particular, this paper presents new results related to the construction of storage functions for control affine systems, as a generalization of some physically-based approaches to dissipative systems theory that appeared recently in the literature. Potential extensions of the proposed approach and areas for future research are also discussed.

Keywords

Interconnected Process Systems, Dissipative Systems Theory, Decentralized Nonlinear Feedback Control.

Introduction

This paper considers the problem of designing decentralized nonlinear feedback controllers in the context of plantwide process control. In the proposed approach, the plant is viewed as an interconnected network of physical process units coupled with a communication network between the controllers (Ydstie, 2002). The problem of large-scale network analysis in the context of control is not new, see for example the stability result given in (Moylan and Hill, 1978). The problem of distributed control design has received some attention in recent years from a graph-theoretic perspective (Langbort et al., 2004). However, for chemical processes described by nonlinear dynamics, design and coordination of decentralized controllers is still an open problem. In particular, as shown in a study by Kumar and Daoutidis (2002), mass and energy recycle flows lead to challenging problems in the analysis of the global behavior of a plant. In the context of both chemical process control and nonlinear systems, model predictive control strategies have been studied to address the problem of distributed controllers design, see for example the contribution (Liu et al., 2009) and the review presented by Rawlings and Stewart (2008). In essence, the general problem of distributed control design for nonlinear process systems leads to the problem of stabilizing local subsystems while enforcing global plant stability and performance using the knowledge of both physical interconnection and communication networks.

From a theoretical point of view, passive and dissipative systems based approaches have been used extensively in the past to study interconnections of nonlinear subsystems. Dissipative systems theory provides a set of tools to address analysis and control design problems for interconnected nonlinear systems. In particular, coordination between nonlinear systems has been studied recently using passivity-based techniques, see for example the recent book (Bai et al., 2011). In the context of chemical process control, recent contributions

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along those lines were given in (Jillson and Ydstie, 2007) and (Rojas et al., 2009). Analysis and controller design based on passivity and dissipativity, two concepts introduced by Willems (1972) and further developed by Hill and Moylan (1976), is a central approach to nonlinear control theory, as an input-output extension of classical Lyapunov techniques. Those concepts were further developed in a broad range of methods linking state-space and input-output approaches for analysis and feedback control design for general classes of nonlinear control systems (Sepulchre et al., 1997; van der Schaft, 2000; Brogliato et al., 2007). Relations between passivity analysis and geometric approaches to nonlinear control design are also reviewed extensively in the literature (Sepulchre et al., 1997; van der Schaft, 2000). In (Byrnes et al., 1991), equivalence of control-affine nonlinear systems to passive systems was studied using differential geometric techniques. In recent years, analysis and controller design based on dissipativity properties proved to be especially useful for mechanical and electrical systems, and in particular, for applications where a storage function can be related to the concept of a stored energy function (van der Schaft, 2000). Many applications and extensions of passivity and dissipativity approaches were proposed in the context of process control, as reviewed for example in (Hangos et al., 2004) and (Bao and Lee, 2007). In practice however, for example in the context of nonlinear chemical processes, the choice of a suitable dissipative representation (storage function and supply rate) for each interconnected subsystem is still an open question.

The main contribution of the present paper is the construction of a storage function for processes described by control affine nonlinear models. In particular, a decomposition approach is proposed to study the problem of closed-loop dissipativity assignment and feedback stabilization, exploiting an idea originally presented in (Sira-Ramírez and Angulo-Núñez, 1997) based on the decomposition of the drift vector field in terms of dissipative and non-dissipative components and extending previous results on stabilization presented in (Hudon et al., 2008) to an input-output setting. One consequence of the constructive approach presented here is its potential to outline the excess or shortage of passivity of a given controlled subsystem. This type of general construction is of importance when one seeks to use dissipativity theory to study interconnected nonlinear systems (Sepulchre et al., 1997). The main advantage of the proposed approach is that no *a priori* knowledge of a storage function is required to carry the decomposition. Based on the obtained approximate dissipative representation, the present paper proposes a domination-based controller design technique to ensure input-output performance of each subsystem for plantwide analysis.

The paper is organized as follows. First, the problem of analysis and stabilization, through distributed state feedback control, of interconnected control affine subsystems using dissipativity is presented. The central part of the paper presents the construction of a storage function based on a geometric decomposition of the drift vector field. The obtained storage function is then used to design nonlinear feedback controllers that ensure, in closed-loop, desired dissipativity properties. Areas for future research are also briefly discussed.

Problem Formulation and Background

This section summarizes the problem under study. As discussed above, following (Ydstie, 2002), plantwide process control problems can be viewed as interconnected networks of process units, denoted P_i, exchanging mass and energy flows, coupled with an interconnected network of controllers, denoted C_i. This general description of plantwide control problems is depicted in Figure 1.

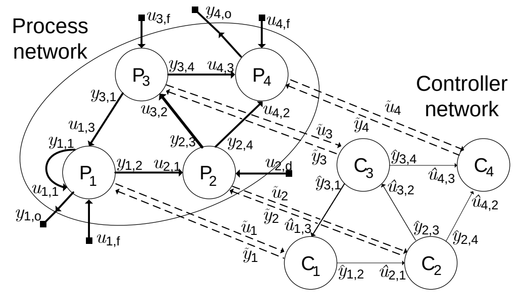

Figure 1. Process and information networks

The general problem associated to such a process and information network is the design of controllers $C_i(\cdot)$, such that the global network is stable and achieve a certain level of performance, for example, as discussed in (Rojas et al., 2009), optimal disturbance attenuation.

Recall, from (Willems, 1972; Sepulchre et al. 1997; van der Schaft, 2000), the following basic definitions about dissipative systems. Consider a control affine system Σ given by

$$\Sigma: \begin{cases} \dot{x} = f(x) + \sum_{j=1}^{m} g_j(x)u_j, & x \in \mathcal{X}, u \in \mathcal{U} \\ y = h(x), & y \in \mathcal{Y}. \end{cases} \tag{1}$$

The system Σ is said to be dissipative with respect to the supply rate $s(u,y) : \mathcal{U} \times \mathcal{Y} \to \mathbb{R}$ if there exists a storage function $V : \mathcal{X} \to \mathbb{R}^+$, and that for all $t_1 \geq t_0$, and all input functions $u(\cdot)$, the following inequality holds

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} s(u(t), y(t))dt, \tag{2}$$

with $x(t_0) = x_0$ and $x(t_1)$ is the state resulting, at time t_1 from the solution of (1) taking x_0 as initial condition and $u(t)$ as control input the function. If V is differentiable with respect to time t for all $x \in \mathcal{X}$ and $u(\cdot)$, inequality (2) is equivalent to

$$\dot{V}(x) \leq s(u(t), y(t)). \tag{3}$$

The system is said to be lossless if inequalities (2) or (3) are equalities. A state space system Σ with $\mathcal{U} = \mathcal{Y} = \mathbb{R}^m$ is said to be passive if it is dissipative with respect to the supply rate $s(u,y) = u^T y$. The system Σ is strictly input passive if there exists $\delta > 0$ such that Σ is dissipative with respect to $s(u,y) = u^T y - \delta\|u\|^2$ and is said to be strictly output passive if there exists $\varepsilon > 0$ such that Σ is dissipative with respect to $s(u,y) = u^T y - \varepsilon\|y\|^2$. Finally, Σ is conservative if it is lossless with respect to the supply rate $s(u,y) = u^T y$.

In the present paper, we consider interconnections of multi-port subsystems of the type presented in Figure 2, where for a each subsystem P_i, we denote the state by $x_i \in \mathbb{R}^{n_i}$ the interconnection input by $\tilde{u}_i \in \mathbb{R}^{m_I}$, the manipulated input by $u_i \in \mathbb{R}^{m_i}$, and the interconnection output by $\tilde{y}_I \in \mathbb{R}^{m_I}$.

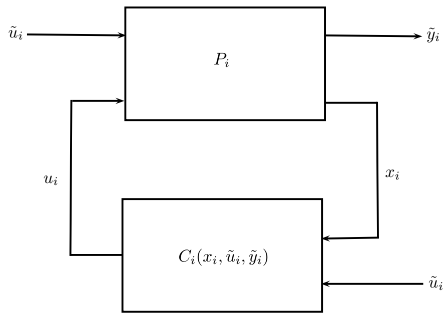

Figure 2. Multi-port system

Each system P_i is assumed to be affine in the control inputs $u_{ij}(\cdot)$ and the interconnection inputs $\tilde{u}_{ij}(\cdot)$, *i.e.*,

$$P_i: \quad \begin{cases} \dot{x}_i = f_i(x_i) + \sum_{j=1}^{m_i} g_{ij}(x_i)u_{ij} + \sum_{j=1}^{m_I} \tilde{g}_{ij}(x_i)\tilde{u}_{ij} \\ \tilde{y} = \tilde{h}(x_i). \end{cases} \tag{4}$$

Consider the interconnection of N subsystems, interconnected through a constant interconnection of the form

$$\begin{bmatrix} \tilde{u}_1 \\ \vdots \\ \tilde{u}_N \end{bmatrix} = \begin{bmatrix} H_{11} & \dots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \dots & H_{NN} \end{bmatrix} \begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_N \end{bmatrix}. \tag{5}$$

Following (Langbort et al., 2004), it is assumed that the information network is the same as the process network, *i.e.*, the controller C_i has access to all information required to achieve an objective given by the supply rate from $\tilde{u}_i$ to $\tilde{y}_i$, denoted by $s_i(\tilde{u}_i, \tilde{h}_i(x))$. In (Rojas et al., 2009), it was demonstrated that for a large class of plantwide problems, this interconnection structure can be used to computed decentralized control objectives. Following the original work from Moylan and Hill (1978) and using the classical notation from (Hill and Moylan, 1976), it was shown that for a fixed known interconnection structure H and global (Q,S,R) objectives, the global stability problem can be translated into a collection of decentralized (Q_i,S_i,R_i) objectives for each (P_i,C_i) pair. In particular, it was shown in (Rojas et al., 2009) that asymptotic stability of the entire process network is ensure if

$$Q - SH - H^T S + HRH < 0. \tag{6}$$

In practice, and depending on the nonlinearity of a given subsystem P_i, it could be difficult to compute a controller C_i that delivers exactly a desired (Q_i,S_i,R_i). In the present paper, we propose an approximate approach to the problem, using parameterized domination-based feedback controllers to ensure that distributed objectives are met. More precisely, the crucial problem to be addressed in the sequel is the following: Given a decentralized dissipative objective encoded as the supply rate $s_i(\tilde{u}_i, \tilde{y}_i)$, design a feedback controller $C_i(\cdot)$ such that for the pair (P_i,C_i) in closed-loop, the dissipative inequality

$$\dot{V}_i(x) \leq s_i(\tilde{u}_i(t), \tilde{y}_i(t)) \tag{7}$$

holds. One of the major difficulties in such dissipativity-based plantwide control is the determination of the dissipatvity of subsystems. As mentioned previously, the problem to be emphasize in the present paper is the construction of a storage function $V_i(x)$ such that the last inequality holds, for $i = 1,\dots,N$. In the present paper, we propose an approximate geometric approach to construct storage functions, and discuss an approach to feedback controllers design using the obtained storage function such the above inequalities hold, *i.e.*, ensuring that the objectives for subsystems $1,\dots,N$, encoded as supply rates, hold such that a desired global objective for the interconnected network is met. The next section discusses the proposed constructive approach for storage functions.

Construction of a Storage Function

The problem of constructing storage functions for nonlinear systems of the form (1) has been studied extensively in recent years. In particular, several physically-based techniques were proposed, most notably energy- and power- shaping methods (Jeltsema and Scherpen, 2009). In the context of chemical engineering, Ydstie and Alonso (1997) proposed to use a thermodynamically-defined quantity, the availability, as a storage function for dissipative analysis of process systems. This quantity, in the context of the discussion from (Jeltsema and Scherpen, 2009) is an example of a mixed-potential, *i.e.*, a potential that combines a conserved quantity (an energy-like component) and a metric quantity (an entropy-like component). The present approach to construct a storage function can be viewed from that perspective. The general idea on which the proposed storage function construction procedure is based is to study the evolution of a volume element in the phase space under the action of the drift vector field $f(x)$. In order to do so, the proposed approach relies on differential geometry techniques, in particular, elements of exterior calculus, briefly discussed in the sequel. A complete review of exterior calculus on $\mathbb{R}^n$ can be found in (Edelen, 2005). We denote a smooth vector field in $\Gamma(\mathbb{R}^n)$ as $X(x) = \sum_{i=1}^n v^i(x)\frac{\partial}{\partial x_i}$ and a smooth differential one-form in $\Lambda^1(\mathbb{R}^n)$ as $\omega(x) = \sum_{i=1}^n \omega_i(x)dx_i$, where $v^i(x)$ and $\omega_i(x)$ are smooth functions on $\mathbb{R}^n$. The standard basis for vectors in $\Gamma(\mathbb{R}^n)$ and one-forms in $\Lambda^1(\mathbb{R}^n)$ are denoted by $\frac{\partial}{\partial x_i}$ and dx_i, respectively. The wedge product is denoted by $\wedge$ and the interior product of a differential form ω with respect to a vector field X is denoted by $X \lrcorner \omega$. The exterior derivative can be viewed as a generalization of the differential of a function in the direction of a vector field. In this context, a function $h(x)$ can be viewed as a zero form, *i.e.*, $h \in \Lambda^0(\mathbb{R}^n)$. In particular, the exterior derivative of a function $dh(x)$ has the interpretation of a gradient in coordinates. The interior product is an inner multiplication that can be understood as a contraction of indices in tensor calculus. For a vector field X and a k-form α, the $(k-1)$-form obtained by taking the interior product $X \lrcorner \alpha$ can be viewed as a covariant tensor. The exterior derivative and the interior product can be used together to determines the Lie derivative of a k-form α along a vector field X by using the Cartan's formula:

$$\mathcal{L}_X \alpha = X \lrcorner d\alpha + d(X \lrcorner \alpha). \tag{8}$$

In particular, for a zero-form $h(x)$, since $X \lrcorner h = 0$, the Lie derivative can be computed simply as $\mathcal{L}_X h = X \lrcorner df$.

The procedure used in the present paper relies on the canonical Riemannian metric in $\mathbb{R}^n$, given as $g = dx_1 \otimes dx_1 + \ldots + dx_n \otimes dx_n$ with associated volume form in $\Lambda^n(\mathbb{R}^n)$, expressed as $\mu = dx_1 \wedge dx_2 \wedge \ldots \wedge dx_n$. For a given drift vector field $f(x) = \sum_{i=1}^n f_i(x) \frac{\partial}{\partial x_i}$, we seek to construct a dissipative representation with an associated natural storage function. The central element to be exploited in the sequel is the divergence of the drift vector field, *i.e.*,

$$\text{div } f = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f_i(x). \tag{9}$$

From a physical point of view, by virtue of the Liouville theorem, it is known that a system is conservative if div $f = 0$. To study dissipativity (and in particular Lyapunov stability), we consider the problem of shaping the closed-loop vector field such that the divergence of the resulting vector field is negative definite, with a rate prescribed by the desired supply rate.

In the language of exterior calculus, the divergence of a vector field, can be encoded as follows. A $(n-1)$ differential form j is first obtained by taking the interior product of the volume μ with respect to the drift vector field $f(x)$, *i.e.*,

$$j = \left(\sum_{i=1}^{n} f_i(x) \frac{\partial}{\partial x_i} \right) \lrcorner \mu \tag{10}$$

$$j = \sum_{i=1}^{n} f_i(x) dx_1 \wedge \ldots \wedge \widehat{dx_i} \wedge \ldots \wedge dx_n, \tag{11}$$

where $\widehat{dx_i}$ denotes a removed element such that j is a $(n-1)$ form. Taking the exterior derivative of j, and by the property of the wedge product that $dx_i \wedge dx_i = 0$, we obtain,

$$dj = \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i}(x) dx_1 \wedge \ldots \wedge dx_n = \text{div } f(x)\mu. \tag{12}$$

The first step in the proposed construction consists in constructing a differential one-form $\omega \in \Lambda^1(\mathbb{R}^n)$ that encodes the divergence of the drift vector field $f(x)$. Such a one-form is obtained by using the Hodge star operator $\star$ of the $(n-1)$ form j, *i.e.*,

$$\omega = \star j = \star(f(x) \lrcorner \mu). \tag{13}$$

If the one-form ω is closed, *i.e.*, if $d\omega = 0$, it can be shown that it is also locally exact, by virtue of the Poincaré Lemma, and the system is conservative (in particular, the dynamics is generated by the gradient of a potential function). However, if the one-form is not closed, ω can be expressed as the sum of an exact component and an anti-exact component. Such decomposition can be carried locally using a homotopy operator $\mathbb{H}$, such that

$$\omega = d(\mathbb{H}\omega) + \mathbb{H}d\omega. \tag{14}$$

Hence, the one-form ω is decomposed in terms of an exact component and an anti-exact components, denoted by $\omega_e = d(\mathbb{H}\omega)$ and $\omega_a = \mathbb{H}d\omega$, respectively. In coordinates, the homotopy operator is defined as follows. For a differential form ω of degree k on a star-shaped region S centered at an equilibrium x^*, the homotopy operator is given as

$$(\mathbb{H}\omega) = \int_0^1 \mathfrak{X}(x^* + \lambda(x - x^*)) \lrcorner \omega(x^* + \lambda(x - x^*))\lambda^{k-1} d\lambda, \tag{15}$$

where $\omega(x^* + \lambda(x - x^*))$ denotes the differential form evaluated on the star-shaped domain in the local coordinates defined above. By the properties of exterior derivative $d \circ d = 0$, hence the exact part ω_e is closed and exact (*i.e.*, ω_e is the exterior derivative of a 0-form, the function $\mathbb{H}\omega$).

Following the approach in (Hudon et al., 2008), we consider the approximate dissipative potential function $\psi(x) = \mathbb{H}\omega$ to construct a storage function for the system. In general, this function is not necessarily positive definite. First note that the exact one-form $\omega_e(x)$ is given, in terms of the obtained potential, as

$$\omega_e(x) = \sum_{i=1}^{n} \frac{\partial \psi(x)}{\partial x_i} dx_i. \tag{16}$$

To define a positive storage function based on the obtained dissipative potential, we consider the following change of coordinates, based on the obtained potential to a target dissipative system. Consider a gradient system

$$\dot{z} = -\nabla P(z) \tag{17}$$

with the positive semi-definite potential centered at the origin $P(z) = \frac{1}{2} \sum_{i=1}^n z_i^2$ which from the discussion above is an exact system, with an exact one-form given by

$$\bar{\omega}_e(z) = \sum_{i=1}^{n} z_i dz_i. \tag{18}$$

The coordinate change considered in the present note consists in taking the new coordinates z_i to be the gradient directions of the locally-defined approximate potential $\psi(x)$ in the original coordinates, *i.e.*,

$$z_i = \frac{\partial \psi}{\partial x_i}. \tag{19}$$

The central problem here is to find an expression that relates the one-form in the original coordinates $\omega_e(x)$ to the one-form in the new coordinates (18). In essence, the idea amounts to reshape the dissipative potential $\psi(x)$ such that its derivative with respect to the dynamics is positive definite. This is achieved by computing a transformation that preserves the exact one-form $\bar{\omega}_e(z)$. Taking the exterior derivative on both sides of (19), one obtains the following expression relating both systems of coordinates

$$dz_i = \sum_{j=1}^{n} \frac{\partial \psi^2}{\partial x_i \partial x_j} dx_j, \tag{20}$$

or, if the differential coordinates are expressed in vector form $dz = [dz_1, \ldots, dz_n]^T$ and $dx = [dx_1, \ldots, dx_n]^T$

$$dz = (D^2\psi(x))dx, \tag{21}$$

where $D^2\psi(x)$ denotes the Hessian of the potential $\psi(x)$. For the particular choice of target dynamics chosen here, the target one-form in the x-coordinates $\bar{\omega}_e(x)$ is given as

$$\bar{\omega}_e(x) = (\Delta\psi(x))\omega_e(x) + \frac{\partial\psi}{\partial x}(D^2\psi(x) - \Delta\psi(x))dx, \qquad (22)$$

where $\Delta\psi(x)$ is the Laplacian of the potential $\psi(x)$. The main task of the control design presented in the next section amounts to the domination of the deviation from the original system to the targeted system, as encode in the term

$$\bar{\omega}_a = \frac{\partial\psi}{\partial x}(D^2\psi(x) - \Delta\psi(x))dx. \qquad (23)$$

It should be noted that the proposed coordinate transformation is valid if the Hessian of the computed potential function is non-singular, which is ensured if the potential is convex (or concave), as assumed in (Jillson and Ydstie, 2007). A similar argument was used recently in (Favache et al., 2011) in the context of power-shaping. Note also that, by construction of the homotopy operator, the equilibrium of the x-system x^* is mapped to the origin of the target system.

The dissipative design problem is derived as follows. For the target system in z-coordinates, define the storage function

$$V(z) = \frac{1}{2}\sum_{i=1}^{n} z_i^2 \geq 0, \;\; V(0) = 0, \qquad (24)$$

for which

$$\dot{V} = -\sum_{i=1}^{n} z_i^2 \leq 0. \qquad (25)$$

Hence, $V(z)$ is an admissible storage function. Observe that in the z-coordinates, the time derivative of the storage can be re-expressed as an interior product,

$$\dot{V}(z) = \mathcal{L}_{-\nabla P(z)}V(z) = -\nabla P(z) \lrcorner dV(z) = -\nabla P(z) \lrcorner \bar{\omega}_e(z), \qquad (26)$$

and it can be easily computed that $\dot{V}(z)$ is negative semidefinite, since

$$-\nabla P(z) \lrcorner \bar{\omega}_e(z) = \left(\sum_{i=1}^{n} -z_i \frac{\partial}{\partial z_i}\right) \lrcorner \left(\sum_{i=1}^{n} z_i dx_i\right) = -\sum_{i=1}^{n} z_i^2 \leq 0. \qquad (27)$$

In the original coordinates, the Lyapunov stability condition with respect to the function $V(x)$, which can be obtained using (19), is thus given by

$$\dot{V}(x) = f(x) \lrcorner ((\Delta\psi(x))\omega_e(x) + \bar{\omega}_a(x)) \leq 0. \qquad (28)$$

To relate this condition to the divergence, which is encoded in the anti-exact part ω_a, re-write the above inequality in terms of ω and ω_a, by using the fact that $\omega = \omega_e + \omega_a$. In terms of the drift vector field, the inequality becomes

$$f(x) \lrcorner ((\Delta\psi(x))(\omega(x) - \omega_a(x)) + \bar{\omega}_a(x)) \leq 0. \qquad (29)$$

This last expression will be used as a basis for the stabilization design in the next section.

Decentralized State Feedback Design

We now consider the construction of state feedback control for the control affine system (4), using the storage function $V_i(x)$ constructed above based on the drift vector field, such that the subsystem P_i is stable and achieve a desired supply rate $s_i(\tilde{u}_i, \tilde{y}_i)$ in closed-loop. The approach presented here can be viewed as an extension of the approach proposed in (Sira-Ramírez and Angulo-Núñez, 1997) based on the decomposition of the drift vector field in terms of dissipative and non-dissipative components. Here, the objective of the controller is to dominate the non-dissipative parts of the dynamics, encoded in ω_a and $\bar{\omega}_a$, and assign a certain supply rate $s_i(\tilde{u}_i, \tilde{y}_i)$ to the subsystem P_i.

In the present note, it is assumed that we have access to the full state x. It is also assumed that the communication network is such that a controller C_i can send an estimation of $\tilde{h}_i$ to any controller C_j, *i.e.*, the controller network is identical to the process network, $H_C = H$ (Langbort et al., 2004). As a result, the value of $\tilde{u}_j$ is known to the controller C_j, and we seek to design feedback controllers $C_j = u_j(x, \tilde{u}, \tilde{h}(x))$ such that the dissipation inequality (29) holds. In general, the right-hand side of (29) is fixed by plantwide stability analysis, as reviewed briefly in the preceding section, following (Rojas et al., 2009). Following the notation from (Byrnes et al., 1991), we are seeking to design controllers of the form $u_j(x, \tilde{u}_i) = \alpha(x) + \beta(x)s_i(\tilde{u}_i, \tilde{h}_i(x))$.

To alleviate some problems related to the inversion-based design proposed in (Sira-Ramírez and Angulo-Núñez, 1997), and to be able to compensate for the remaining shortage of passivity if the drift is unstable (or use the excess of passivity if the system is open-loop stable), we consider a variation of damping controllers of the Jurdjevic–Quinn type (Byrnes et al., 1991). Those domination-based controllers of the form $u_j = -\mathcal{L}_{X_j}V(x)$ were also considered in (Sepulchre et al., 1997). In this present case, we let the control be of the form

$$u_j = -k_{1,j}(x)g(x) \lrcorner \bar{\omega}_e(x) - k_{2,j}(x, \tilde{u})s_i(\tilde{u}, \tilde{y}). \qquad (30)$$

In closed-loop, the dissipative inequality (3) for the system (4) can be re-written as

$$\left(f(x) + \sum_{j=1}^{m_i} g_j(x)u_j + \sum_{j=1}^{m_I} \tilde{g}_j(x)\tilde{u}_j\right) \lrcorner \bar{\omega}_e \leq s_i(\tilde{u}_i(t), \tilde{y}_i(t)). \qquad (31)$$

Using the decomposition (29) and the controllers (30), one obtains

$$\begin{aligned} &f \lrcorner (\Delta\psi(x))\omega(x) - f \lrcorner ((\Delta\psi(x))\omega_a(x) + \bar{\omega}_a(x)) \\ &\quad - \sum_{j=1}^{m_i} \left(k_{1,j}(\cdot)(g_j \lrcorner \bar{\omega}_e)^2 + k_{2,j}(\cdot)(g_j \lrcorner \bar{\omega}_e)s_i(\tilde{u}, \tilde{y})\right) \\ &\quad + \sum_{j=1}^{m_I} (\tilde{g}_j \lrcorner \bar{\omega}_e)\tilde{u}_j \leq s_i(\tilde{u}_i(t), \tilde{y}_i(t)). \end{aligned} \qquad (32)$$

Controller design is then carried in two steps. First, we design the gain functions $k_{1,j}(\cdot)$ to dominate the approximation and ensure local stability, *i.e.*, ensure that

$$f(x)\lrcorner((\Delta\psi(x))(\omega(x)-\omega_a(x))+\bar{\omega}_a(x)) \\ -\sum_{j=1}^{m_i} k_{1,j}(\cdot)(g_j\lrcorner\bar{\omega}_e)^2 \leq 0. \tag{33}$$

The second task consists in the design of the gain functions $k_{2,j}(\cdot)$ such that the input-output dissipativity objective is met, *i.e.*,

$$-\sum_{j=1}^{m_i} k_{2,j}(\cdot)(g_j\lrcorner\bar{\omega}_e)s_i(\tilde{u},\tilde{y})+\sum_{j=1}^{m_I}(\tilde{g}_j\lrcorner\bar{\omega}_e)\tilde{u}_j \leq s_i(\tilde{u}_i(t),\tilde{y}_i(t)). \tag{34}$$

This particular choice of controller structure is generally more robust than the exact cancelation controller approach proposed in (Sira-Ramírez and Angulo-Núñez, 1997). However, as it is the case for damping controllers of the Jurdjevic–Quinn type, it relies on a controllability-like assumption on the pair of vector fields $(f(x),g(x))$, which in the present case can be written in a perturbed form $(f(x)+\tilde{g}(x)\tilde{u},g(x))$. As a consequence, this type of controller might suffer lost of controllability depending on the structure of $\tilde{g}(x)$. This question would be considered in future research.

Conclusions

This paper presented an approach to the design, in the context of plantwide systems, of nonlinear feedback controllers that achieve a desired input-output performance, determined (or parameterized) *a priori* from the analysis of the interconnection network. The key idea in the proposed approach consists in encoding control objectives as parameterized supply rates to be achieved through feedback control design. The main contribution of the present paper consists in the development of a nonlinear feedback control design strategy ensuring that each subsystems meet those objectives in closed-loop, based on the construction of an approximate storage function for the system. Using geometric theory of dissipative nonlinear systems at a local level, the approach has the advantage of outlining potential structural limitations to the desired global performance. Current areas of research also focus on controller design under limited information exchange between subsystems and robustness to model uncertainties of the proposed construction. Applications of the proposed approach to chemical process control systems will be presented in a forthcoming paper.

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