ROBUST SCHEDULING UNDER TIME-SENSITIVE ELECTRICITY PRICES FOR CONTINUOUS POWER-INTENSIVE PROCESSES

Sumit Mitra[*a], Ignacio E. Grossmann[a], Jose M. Pinto[b] and Nikhil Arora[c]
[a] Carnegie Mellon University, Pittsburgh, PA 15232
[b] Praxair Inc., Danbury, CT
[c] Praxair Inc., Tonawanda, NY

Abstract

Optimization models for the production scheduling of power-intensive processes such as air separation can help realizing significant economical savings. However, if the electricity is procured in the day-ahead or real-time market, the forecast for the hourly electricity prices contains a significant amount of uncertainty. In this work, we apply robust optimization to the uncertain electricity prices using an uncertainty set that features multiple ranges and can account for correlated data. We describe how the robust counterpart is constructed and provide a case study that shows the differences in the solutions for the deterministic and the robust schedule.

Keywords

Power-intensive processes, air separation plant, time-varying electricity prices, demand response, robust optimization, multiple ranges, correlated data.

Introduction

The profitability of power-intensive processes, such as air separation plants, is largely dependent on the ability to quickly adapt to changes in the electricity prices. With increasing volatility in the electric power market due to deregulation and an increasing share of renewable energies, the need for a smart energy management is even more pressing. In recent work (Mitra et al., 2011), we describe an optimization model for continuous power-intensive processes that facilitates reducing operating expenses due to electricity. Applying the methodology for weekly liquid production planning of different under-utilized air separation plants shows savings of the order of 10 % and more when compared to a simple set point heuristic. In the case study, the electricity prices are assumed to be known with certainty, which is accurate for time-of-use (TOU) pricing where on-peak, off-peak and mid-peak prices are usually negotiated for an entire season of the year. However, for more volatile pricing schemes, such as day-ahead or real-time pricing, the electricity forecast contains uncertainty, which we need to address.

Background

One common approach to deal with parameter uncertainty is the framework of stochastic programming, which has been applied for a medium-size air separation unit by Ierapetritou et al. (2002). However, stochastic programming problems tend to become large with an increasing number of scenarios and are hard to solve. Furthermore, schedules are optimized for the expected total cost and usually do not explicitly "protect" against various realizations of the uncertain parameters.

In contrast to that, the framework of robust optimization focuses on a computationally tractable

[*] To whom all correspondence should be addressed

description of an uncertainty set, against which the solution is "robustified". The robustness of the schedule for air separation plant requires special attention when slow plant dynamics in terms of switching behavior are considered. In the case of a weekly schedule for air separation plants, operational constraints such as minimum downtime (in the order of 24 hours for larger plants) and minimum uptime (48-60 hours) can considerably reduce the degrees of freedom once a shutdown is performed. Furthermore, operating personnel favor robust schedules with respect to anticipated swings in electricity prices.

Li and Ierapetritou (2008) study the impact of three different uncertainty sets proposed in the robust optimization literature for process scheduling under bounded parameter uncertainty: Soyster's worst case approach (1973), Ben-Tal and Nemirovski (1999) as well as Bertsimas and Sim (2003). While Soyster's formulation is considered to be too conservative, Ben-Tal and Nemirovski's (1999) approach introduces nonlinearities. Therefore, they identify the polyhedral uncertainty set of Bertsimas and Sim (2003) to be most promising because it provides a computationally tractable linear formulation, which also allows controlling the degree of conservatism. Bertsimas and Sim's (2003) approach assumes that each uncertain parameter will be either at its nominal or worst-case value and that the total number of parameters that take their worst-case values is restricted.

In the context of uncertain electricity prices, Conejo et al. (2010) use Bertsimas and Sim's polyhedral uncertainty set to optimize the demand response for electricity consumers using a rolling-horizon algorithm. Unfortunately, the polyhedral uncertainty set has two disadvantages when applied to electricity prices. First, the prices will either take their nominal or their worst-case values, which is not the case for electricity prices in reality. Second, the number of parameters, which will take their respective worst-case value (also referred to as "budget of uncertainty"), is hard to choose correctly. The budget of uncertainty is a modeling parameter and describes the degree of conservatism. However, for electricity prices its appropriate value cannot be easily derived from historical data.

Recently, Duzgun and Thiele (2010) use multiple ranges for a more detailed modeling of the realizations of the uncertain parameters in the context of R&D project selection. They introduce bins with different parameter ranges and assume that each parameter will fall in one bin, attaining its respective worst case within the bin. The number of parameters for each bin is restricted and a modeling decision, e.g. observed from historical data.

In this work, we utilize the uncertainty set by Duzgun and Thiele (2010) to build a computationally tractable uncertainty set for electricity prices based on historic price data. We modify their model to also account for correlations in the observed data. The resulting uncertainty set is integrated in the operational planning model for air separation plants that is described in Mitra et al. (2011) and a case study is solved to show the impact.

Problem Statement

Given is a power-intensive process that produces a set of products $g \in G$, which can be partitioned into *Storable* and *Nonstorable* products. The plant has different operating modes $m \in M$ depending on the equipment that is running and is subject to hourly-varying electricity prices e^h (hour $h \in H$). A price forecast is given for the horizon of a week. Furthermore, historical data for each individual hour of the week is available. The problem is to find a weekly production plan for the power-intensive process, such that operational costs due to power consumption are minimized while satisfying various operational constraints of the plant. At the same time, the probabilistic data should be used to control the financial risk associated with the obtained weekly production profile.

Process Representation

We represent the plant with a set of modes $m \in M$, each one having a distinct feasible space of operation. In every hour, the plant operates in exactly one mode. Furthermore, the dynamics of the plant require additional logic constraints that e.g. ensure minimum stay relationships (minimum up-/downtime) or forbid certain transitions between modes. Additionally, mass balances for the *Storable* products have to be enforced to account for inventory levels. The power consumption PW^h in hour h is approximated by a linear correlation between the operational decision variables $x_{m,g}^h$ of the plant using correlation coefficients $\phi_{m,g}$ as shown in Eq. (1b). Note that the operational decision variables can be continuous (e.g. production levels) as well as discrete (e.g. state of the plant in terms of modes). The exact derivation of all equations can be found in Mitra et al. (2011). For the purpose of this paper, we state the deterministic problem of finding the schedule with minimum costs that satisfies a pre-defined demand for a given forecast of hourly electricity prices e^h the following way:

$$\min_{PW^h,\, x_{m,g}^h} \sum_h e^h PW^h \tag{1a}$$

$$\text{s.t. } PW^h = \sum_{m,g} \phi_{m,g} x_{m,g}^h \tag{1b}$$

$$x_{m,g}^h \in X \tag{1c}$$

Note that the operational constraints of the problem are summarized in the set of constraints X as described in Eq. (1c).

Robust Optimization

The general idea of robust optimization is to robustify the solution of an optimization problem with respect to uncertainty in the problem data that can be in cost coefficients, in the constraint matrix or the right-hand side

(Bertsimas and Sim, 2004). In our case, we consider uncertainty in the cost coefficients e^h. We assume that the uncertainty for e^h is bounded in the interval $[\mathrm{E}(e^h)-\hat{e}^h, \mathrm{E}(e^h)+\hat{e}^h]$, where $\mathrm{E}(e^h)$ is the nominal (expected) value for the electricity price at hour h and $\hat{e}^h$ the expected maximal deviation from the nominal value. All e^h are part of the uncertainty set E, whose exact structure is a degree of freedom for the modeler. The robust optimization problem of the deterministic problem in (1) with uncertain cost coefficient can be written as:

$$\min_{PW^h,\, x_{m,g}^h} \max_{e^h \in E} \sum_h e^h PW^h \tag{2a}$$

$$\text{s.t. Constraints (1b)-(1c)} \tag{2b}$$

Uncertainty Set

The appropriate selection of the uncertainty set E is a non-trivial task due to concerns regarding tractability and conservatism. As stated earlier, we study the influence of the uncertainty set described by Duzgun and Thiele (2010) that features multiple ranges for a more detailed modeling of the realizations of the uncertain parameters. It has the advantage that it allows to tune the degree of conservatism according to historical data while providing a formulation that is tractable. The formulation is the following:

$$\min_{PW^h,\, x_{m,g}^h} \max_{e^h,\, w_k^h} \sum_h e^h PW^h \tag{3a}$$

$$\text{s.t. Constraints (1b)-(1c)} \tag{3b}$$

$$\underline{e}_k^h w_k^h \le e_k^h \le \overline{e}_k^h w_k^h,\ \forall h \in H, k \in K \tag{3c}$$

$$e^h = \sum_{k \in K} e_k^h,\ \forall h \in H \tag{3d}$$

$$\sum_{k \in K} w_k^h = 1,\ \forall h \in H \tag{3e}$$

$$\sum_h w_k^h \le \Gamma_k,\ \forall k \in K \tag{3f}$$

$$w_k^h \in \{0,1\},\ \forall h \in H, k \in K \tag{3g}$$

It is assumed that the uncertainty space can be split up into multiple ranges $k \in K$. The ranges represent relative deviations from the mean value for each individual parameter e^h. Each e^h falls in exactly one bin *k* (see Eq. (3c)-(3e)). The number of parameters per bin is limited by the parameter Γ_k as stated in Eq. (3f). The binary variable w_k^h describes whether e^h falls into bin *k* (Eq. (3g)). The lower and upper bounds $\underline{e}_k^h$ and $\overline{e}_k^h$ are constructed with respect to the relative deviations of the hourly prices from their corresponding mean values. Based on the analysis of historical data for a given season of the year, we generate the bins for the relative deviations of the hourly prices for a whole week and define the allowed observations per bin accordingly.

In formulation (3), we do not account for correlations in the random variables across different hours. However, in reality these correlations exist, i.e. if the electricity price is already high in the morning of a summer day it is likely to increase throughout the day. Therefore, similarly to Bertsimas and Sim (2004), we introduce the uncertain variables c_i, with $i \in I$ being the set of independent random variables. The electricity prices can be rewritten as $e^h = \sum_i c_i \zeta_i^h$, with the parameter ζ_i^h describing the influence of c_i on the hourly price e^h. We formulate the robust optimization problem over the uncertainty set of the independent random variables C, which is analogously described to E with the following set of equations:

$$\min_{PW^h,\, x_{m,g}^h} \max_{c_i, z_{i,k}} \sum_h \sum_i c_i \zeta_i^h PW^h \tag{4a}$$

$$\text{s.t. Constraints (1b)-(1c)} \tag{4b}$$

$$\underline{c}_{i,k} z_{i,k} \le c_{i,k} \le \overline{c}_{i,k} z_{i,k},\ \forall i \in I, k \in K \tag{4c}$$

$$c_i = \sum_{k \in K} c_{i,k},\ \forall i \in I \tag{4d}$$

$$\sum_{k \in K} z_{i,k} = 1,\ \forall i \in I \tag{4e}$$

$$\sum_{i \in I} z_{i,k} \le \Gamma_k,\ \forall k \in K \tag{4f}$$

$$z_{i,k} \in \{0,1\},\ \forall i \in I, k \in K \tag{4e}$$

Similarly to w_k^h, the binary variables $z_{i,k}$ indicate whether c_i falls into bin *k* or not. In the LP relaxation of formulation (9)-(14) $z_{i,k}$ is relaxed as

$$0 \le z_{i,k} \le 1,\ \forall i \in I, k \in K \tag{4g}$$

Robust Counterpart

In order to derive the robust counterpart of formulation (4), the linear relaxation of the inner maximization is investigated. As shown by Duzgun and Thiele (2010), the integer variables $z_{i,k}$ will take integer variables in the solution of the linear relaxation. The key idea of the proof is the assignment polytope structure that can be found in Eqns. (4e)-(4f). Furthermore, $z_{i,k}$ will take its maximum value in its range since the power consumption PW^h is nonnegative. Therefore, it follows that $c_i = \sum_{k \in K} \overline{c}_{i,k} z_{i,k}$ for all *i* and the robust counterpart can be written as:

$$\min \sum_{i \in I} \nu_i + \sum_k \eta_k \Gamma_k + \sum_{i,k} \xi_{i,k} \tag{5a}$$

$$\text{s.t. Constraints (1b)-(1c)} \tag{5b}$$

$$\nu_i + \eta_k + \xi_{i,k} \ge \sum_{h \in H} \overline{c}_{i,k} \zeta_i^h PW^h,\ \forall i \in I, k \in K \tag{5c}$$

$$\eta_k,\ \xi_{i,k} \ge 0,\ \forall i \in I, k \in K \tag{5d}$$

In this formulation (5), ν_i is the dual multiplier for Eq. (4e), η_k is the dual multiplier for Eq. (4f) and $\xi_{i,k}$ stems from the upper bound constraint in the LP relaxation of Eq. (4g).

Case Study Setup

Air separation plant

The air separation plant that we consider has two liquefiers that produce the storable products (liquid oxygen, nitrogen and argon) and does not have a pipeline customer. If the second liquefier ramps up production, the plant is 4 hours in a transitional state in which almost no product is produced. Furthermore, once the plant is shut down, it has to stay offline for at least 24 hours. After a start-up, which takes 4 hours, the plant has to run for at least 48 hours. We use industrial data provided by Praxair to conduct the case study.

We assume that the plant's liquid product inventory is at 40% at the beginning of the week, and the same value is used for the target inventory at the end of the week. Every 6 hours a constant pre-defined amount of product is removed from the storage tank. The demand is equivalent to 72% capacity utilization.

Electricity Data

In this case study we analyze electricity data on an hourly basis for the PJM day-ahead market (Ott, 2003). In Figure 1, we can see the distribution of the relative deviations from their hourly seasonal average for the spring seasons 2005-2007. The graph was generated considering 13 weeks of the spring season for each year.

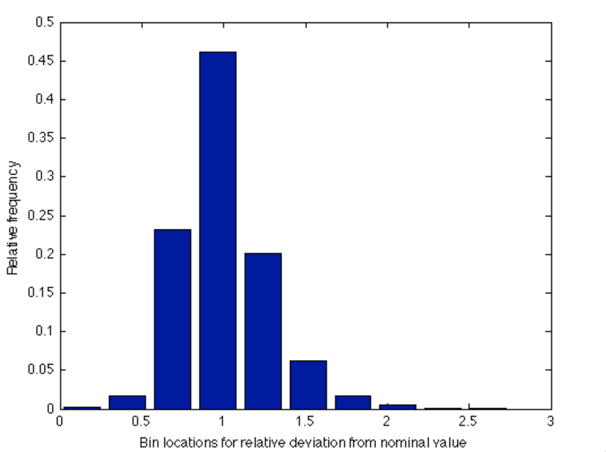

Figure 1. Distribution of relative deviations from nominal values ("1") in electricity prices for a typical week in spring.

To account for correlations in electricity prices, we have to use a dimension-reduction technique. We choose Nonnegative Matrix Factorization (NMF, Berry et al., 2007), which performs a low-rank approximation of the data matrix A in the following way:

$$A \approx U\ V \qquad (6)$$

The columns of U represent transformations of the variables in A. V describes the relative contributions of the original variables to the transformed variables. Each row of V is scaled to unit length. All entries in U and V are nonnegative, which is of particular importance when the dual of the inner maximization in (4) is taken. Since the data matrix A represents the relative deviation from the nominal (forecasted) value $\mathrm{E}(e^h)$, we can construct our coefficients ζ_i^h as follows:

$$\zeta_i^h = v_{i,h}\mathrm{E}(e^h) \qquad (7)$$

For our case study, we reduce the 168 hours of a week to 10 independent random variables. The distribution in the transformed variables is shown in Figure 2.

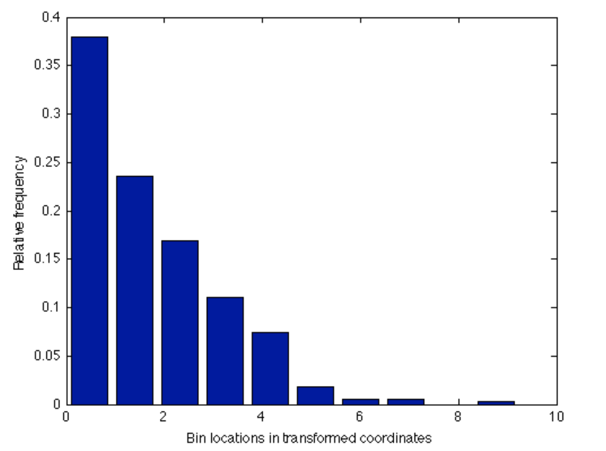

Figure 2. Distribution of deviations in the transformed variables

The frequencies for each bin (scaled by the number of independent random variables, here 10) are taken as the input for the Γ_k parameters in our uncertainty set model.

Results

We solve the deterministic and the robust optimization problem, and can observe that both solutions behave similarly with a few exceptions that we will explain in the following. Figure 3 shows the nominal values for the electricity prices of one week as well as minimum and maximum values. Here the nominal values are the means for each hour of the spring of 2008. As we can see in Fig. 4, the power consumption is countercyclical to the electricity prices in both solutions. During the night, when electricity is cheap, the second liquefier is activated (above 0.7 relative power consumption). During the first 48 hours, there are only slight differences between the two solutions with respect to the timing of the start-up of the second liquefier. For the following three days, the main difference is the shutdown of the second liquefier in the robust solution to reduce the exposure to the market volatility on Wednesday morning (hour 55). Over the weekend, we can observe that the robust solution delays the start-up of the second liquefier on Friday night (hour 117), and keeps it running during Saturday while exploiting the lower volatility during hours 128-143.

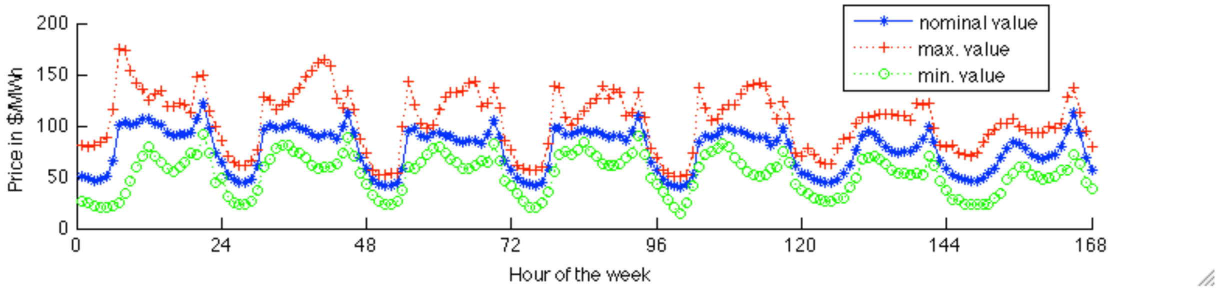

Figure 3. Electricity prices for a typical week in spring 2008 (mean, max and min values)

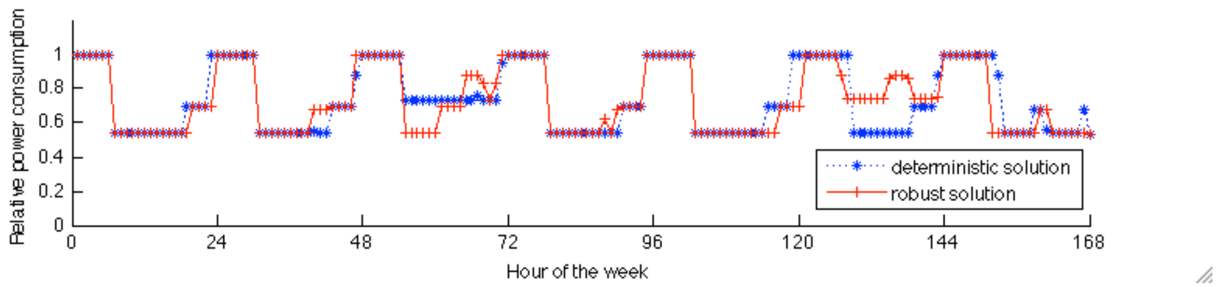

Figure 4. Power consumption profiles for deterministic and robust solution

From the profiles it is not immediately obvious which solution should be implemented. Hence, we compare the two solutions with respect to 2000 electricity profiles that were randomly generated based on the relative deviations, which were observed in 2005-2008 data. In Figure 5, we can see that both solutions perform similarly with the robust solution having slightly smaller tails.

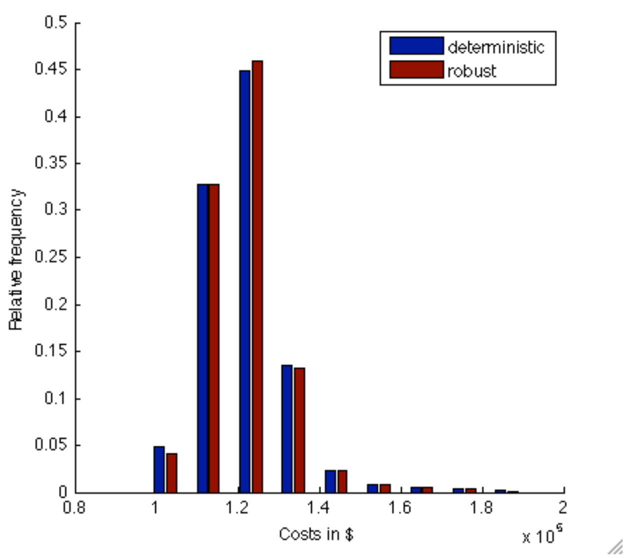

Figure 5. Comparison of cost distributions for randomly generated electricity profiles.

In Table 1, we can observe that the standard deviation decreases by 2%, while the mean of the robust solution increases by 0.1% (i.e. the "price of robustness").

Table 1. Cost comparison for randomly generated electricity profiles.

	Deterministic	Robust	Difference
Mean	*$121,430*	*$121,550*	*+0.10%*
Standard deviation	*$10,098*	*$9,896*	*-2.01%*

Computational Results

The model statistics are shown in Table 2. It is interesting to note that the problem size does not increase significantly. With $|I|=10$ (number of uncertain parameters) and $|K|=10$ (number of bins), we introduce only 100 new constraints and 120 new continuous variables. The number of binary variables does not change. We use CPLEX 12.2.0.1 within the GAMS modeling environment (Brooke et al., 2010) to solve the resulting MILP problems on a MacBook Pro with a 2.53 Ghz Intel Core i5 and 4GB RAM. The CPU time required increases from 6 s (deterministic) to 16 s (robust).

Table 2. Model statistics for case study.

	Deterministic	Robust
# Constraints	*42,163*	*42,263*
# Variables	*29,401*	*29,521*
# Binary	*3,528*	*3,528*
CPU time	*6 s*	*16 s*

Conclusion

We have applied robust optimization to the scheduling of power-intensive processes under uncertain electricity prices. The uncertainty set featured multiple ranges and accounted for correlated data. For an illustrative case study with an air separation plant, we observed that the robust solution behaved similarly to the deterministic solution with a few exceptions. The differences caused the robust solution to have a 2% lower standard deviation in the cost distribution that was calculated for randomly generated electricity profiles. At the same, the mean of the robust solution increased by only 0.1%. While these numbers suggest that the robust solution might be worth it to implement, the results can also be interpreted in a way that the deterministic solution itself is a somewhat “robust” solution. Hence, it would be interesting to study the performance of the deterministic and the robust solution within a rolling horizon scheme as suggested by Conejo et al. (2010).

Acknowledgments

We thank the Center for Advanced Process Decision-making (CAPD) at Carnegie Mellon University and Praxair for financial support.

References

Ben-Tal, A.; Nemirovski, A. (1999). Robust solutions to uncertain programs. Operations Research Letters, 25, 1.

Berry, M. W.; Browne, M.; Langville, A. N.; Pauca, V. P.; Plemmons, R. J. (2007). Algorithms and Applications for Approximate Nonnegative Matrix Factorization. Computational Statistics and Data Analysis, 52, 155.

Bertsimas, D.; Sim, M. (2003). Robust Discrete optimization and Network Flows. Mathematical Programming, 98, 49.

Bertsimas, D.; Sim, M. (2004). The price of robustness. Operations Research, 52(1), 35.

Brooke, A.; Kendrick, D.; Meeraus, A. (2010). GAMS: A users guide, release 23.3. South San Francisco: The Scientific Press.

Conejo, A.J.; Morales, J.M.; Baringo, L. (2010). Real-Time Demand Response Model. IEEE Transactions on Smart Grid, 1, 236.

Duzgun, R.; Thiele, A. (2010). Robust Optimization with Multiple Ranges: Theory and Application to R & D Project Selection. Submitted to European Journal of Operational Research, Available at http://www.optimization-online.org/

Ierapetritou, M.G.; Wu, D.; Vin, J.; Sweeny P.; Chigirinskiy M. (2002). Cost Minimization in an Energy-Intensive Plant Using Mathematical Programming Approaches. Industrial & Engineering Chemistry Research, 41, 5262.

Li, Z.; Ierapetritou, M. G. (2008). Robust Optimization for Process Scheduling Under Uncertainty. Industrial & Engineering Chemistry Research, 47, 4148.

Mitra, S; Grossmann, I. E.; Pinto, J. M.; Arora, N. (2011) Optimal Production Planning under Time-sensitive Electricity Prices for Continuous Power-intensive Processes. Accepted to Computers & Chemical Engineering.

Ott, A. L. (2003). Experience with PJM Market Operation, System Design and Implementation. IEEE Transactions on Power Systems, 18, 528.

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. Operations Research, 21, 1154.