OPTIMAL PROCESS PLANNING UNDER UNCERTAINTY WITH RISK MANAGEMENT

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Abstract

We propose a computationally-tractable optimization-based framework for risk management in midterm process planning under uncertainty. We employ stochastic programming to account for the uncertainty in which a scenario-based approach is used to represent the underlying probability distribution of the uncertain parameters. The problem is formulated as a two-stage stochastic program with recourse that is extended to incorporate the statistically-significant risk measure of Conditional Value-at-Risk (CVaR). However, since a large number of scenarios are often required to capture the uncertainty of the problem, the model suffers from the curse of dimensionality. To circumvent this problem, we propose a solution strategy with relatively low computational burden that involves a combination of simulation, scenario-based stochastic programming with recourse appended with risk management, and statistical-based scenario reduction technique. First, we solve an approximation of the risk-inclined multiscenario model for a number of randomly generated scenarios with an objective of minimizing the standard deviation of the Monte Carlo sampling estimator, which results in a convex stochastic quadratic program. The advantages of solving the approximation problem are two-fold: First, it only requires the use of a small number of samples. Second, we may utilize the standard deviation value of the Monte Carlo estimator (i.e., the objective value) within a scenario reduction procedure to determine the minimum number of scenarios that is theoretically required to obtain an optimal solution. Subsequently, the VaR parameters of the model are simulated for incorporation in a mean–CVaR stochastic linear programming approximation of the all-encompassing risk-averse planning model. The proposed strategy is implemented on a petroleum refinery planning case study with satisfactory results that illustrate how solutions with relatively affordable computational expense can be attained in a risk-conscious modeling framework in the face of uncertainty.

Keywords

Two-stage stochastic programming, Process planning, Conditional Value-at-Risk.

Introduction

The process systems engineering (PSE) community has been instrumental in carrying out a key role in extending the systems engineering boundaries from a sole focus on process systems to the incorporation of important business issues. The latter involves the inevitable consideration of uncertainty in decision-making that gives rise to a need for risk management in enhancing the robustness of process planning activities under numerous possible operating scenarios. Various approaches have been devised to optimize planning problems under uncertainty in the PSE domain, mainly involving scenario-based two-stage and multi-stage stochastic programming with recourse, chance-constrained optimization, fuzzy programming, flexibility analysis, and robust optimization (Sahinidis, 2004). A closely-related research strand considers the notion of risk in handling process planning under uncertainty. An early work in PSE by Bok et al. (1998), which drew inspiration from the Nobel Prize winning work of Markowitz's mean–variance model (1952, 1959) and the robust stochastic programming approach of Mulvey (1995), involves risk management using variance and has been applied to capacity expansion planning of chemical processing networks. More recent work have applied the Conditional Value-at-Risk measure in research and development activities pipeline management (Colvin and Maravelias, 2011); operational planning of large-scale multipurpose multiproduct industrial batch plants (Verderame and Floudas, 2010); and strategic planning of the petroleum industry supply chain (Carneiro et al., 2010). CVaR has also been employed as a post-optimality measure in the capacity investment planning of multiple vaccines (Tsang et al., 2007). This work seeks to propose a stepwise sequential optimization-based framework with relatively low computational burden for addressing risk management using CVaR in process planning problems under uncertainty.

Conditional Value-at-Risk Risk Measure

CVaR is a risk measure originally intended to be employed for reducing the probability that an investment portfolio will incur high losses. It is closely related to the risk measure Value-at-Risk (VaR) that measures the maximum expected loss in the value of a risky entity at a certain confidence interval over a given period under normal market conditions. CVaR is the expected loss given that the actual loss exceeds some VaR threshold at the same confidence level (Rockafellar and Uryasev, 2002; Szego, 2002). For instance, at a one week 95% confidence interval, VaR reports a single value with 95% certainty that that is the value of the maximum expected loss. CVaR measures the expectation that the value is greater than VaR. Within a production planning setting, for instance, if VaR for a commercial product is \$1 million at a one-month 99% confidence interval, this implies that there is a 1% probability that the value of the product will drop more than \$1 million over any given month. CVaR is the expected loss in the product value that is greater than \$1 million over the same duration associated with the same confidence interval.

Problem Statement

The midterm production planning problem addressed in this work can be stated as follows. We are given the following information:

- available process units and their yields and capacities;
- costs of crude oil and refined saleable products;
- capital and operating costs of process units; and
- market demands by customers for products.

Our goal is to determine the amount of materials that are processed in each process stream of each process unit by considering the following uncertain parameters:

- market demands, i.e., production amounts of desired products;
- prices of crude oil and saleable products; and
- product yields of crude oil from chemical reactions in the crude distillation unit

It is assumed that:

- the uncertain parameters of prices, costs, and demands are externally imposed, that is, they are exogenous uncertainties;
- the uncertain parameters are independent random variables that exhibit the behavior and properties of discrete probability distribution functions; and
- the physical resources of the plant are fixed.

Model Formulation and Proposed Solution Strategy

The problem is formulated as a recourse-based two-stage stochastic program with a multiobjective weighted mean–risk objective function as given by the following (Ruszczyński and Shapiro, 2009; Ahmed, 2006):

$$
\begin{aligned}
\max_{x\in X} \quad & c^{\mathrm{T}} x - \mathrm{E}_{\xi\in\Xi}\left[Q\left(x,\xi\right)\right] - \Delta\left[Q\left(x,\xi\right)\right] \\
\text{s.t.} \quad & Ax = b \\
& x \geq 0 \\
\text{with} \quad & \qquad\qquad\qquad\qquad\qquad\qquad (1) \\
Q\left(x,\xi\right) = \min_{y\in Y(x)} \quad & q(\xi)^{\mathrm{T}} y(\xi) \\
\text{s.t.} \quad & W(\xi) y = h(\xi) - T(\xi) x \\
& y \geq 0
\end{aligned}
$$

where $Q(x,\xi)$ is the second-stage recourse costs function and $\Delta(x,\xi)$ is the dispersion statistic adopted as a proxy to represent the risk function. Provided that the vector of random parameters ξ has a small number of possible realizations (or scenarios), it is computationally intractable to solve problem (SP) exactly with current state of computing power and solution algorithms. The two major approaches for approximately solving SP are by performing: (1) numerical integration over the random continuous probability space Ξ (e.g., Pistikopoulos and Ierapetritou, 1995); and (2) discretization of the underlying probability measures or distributions of the continuous space Ξ by using a finite number of scenarios. In this work, we consider the latter approach in formulating the following approximating problem to (1) by utilizing discrete scenarios:

$$
\begin{aligned}
\max_{x\in X} \quad & c^{\mathrm{T}} x - \mathrm{E}_{\xi\in\Xi}\left[Q\left(x,\xi\right)\right] - \Delta\left[Q\left(x,\xi\right)\right] \\
\text{s.t.} \quad & Ax = b \\
& W(\xi_s) y_s = h(\xi_s) - T(\xi_s) x, \quad s = 1,\cdots \qquad (2) \\
& x \geq 0 \\
& y_s \geq 0, \quad s = 1,\cdots
\end{aligned}
$$

To solve problem (2) to optimality, we propose a sequential stepwise optimization-based solution strategy, as shown in Figure 1, that involves a combination of simulation, optimization of scenario-based two-stage stochastic programs with recourse appended with risk management, and statistical-based scenario reduction technique.

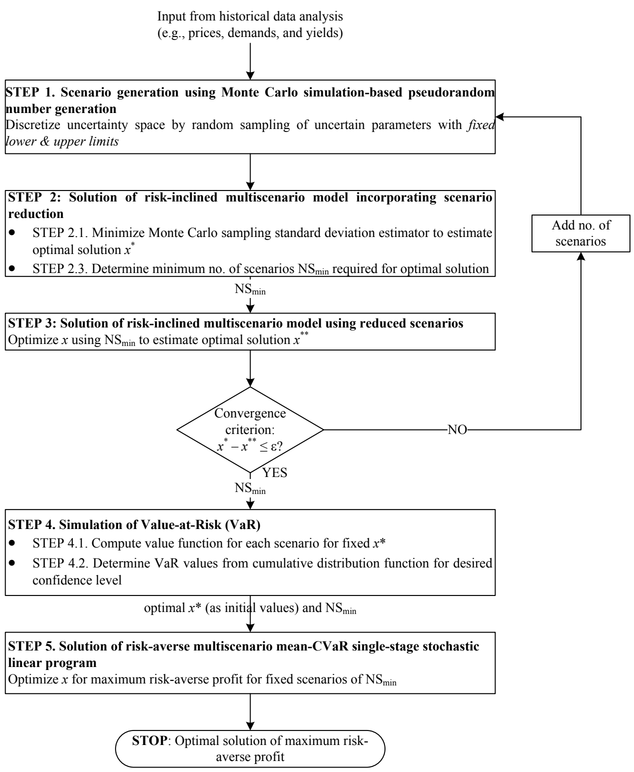

Figure 1. Proposed solution strategy

Step 1. Scenario Generation Using Monte Carlo Simulation-Based Sampling

We first employ a Monte Carlo approach using pseudorandom number generation to generate scenarios that approximate the original full space of the probability distribution, which underlies the uncertain parameters.

Step 2. Formulation and Solution of Risk-Inclined Multiscenario Model Incorporating Scenario Reduction Procedure

Using the generated random samples of scenarios in Step 1, we estimate the expected value function in program (2) by employing the Monte Carlo-simulation based sampling estimator $\bar{z}$ that is given as (Liu and Sahinidis, 1996; Hammersley and Handscomb, 1964):

$$\bar{z}\left(\xi_s\right) = \frac{1}{\mathrm{NS}} \sum_{s=1}^{\mathrm{NS}} p_s q(\xi_s)^{\mathrm{T}} y_s \qquad (3)$$

Using $\bar{z}$, we compute an approximate solution for program (2) by considering an objective function of minimizing the standard deviation $\hat{\sigma}$ of $\bar{z}$ that is given by (You et al., 2009; Mak et al., 1999; Shapiro and Homem-de-Mello, 1998):

$$\max_{\substack{x \in X;\\ y_s \in Y(x)\\ s=1,\cdots}} \quad c^T x - \hat{\sigma} = c^T x - \sqrt{\frac{1}{\mathrm{NS}-1} \sum_{s=1}^{\mathrm{NS}} \left(\mathrm{E}_s - \bar{z}\left(\xi_s\right) \right)^2} \qquad (4)$$

s.t. constraints

where

$$\hat{\sigma} = \frac{1}{\sqrt{\mathrm{NS}-1}} \sum_{s=1}^{\mathrm{NS}} \left(p_s q(\xi_s)^{\mathrm{T}} y_s - \frac{1}{\mathrm{NS}} \sum_{s=1}^{\mathrm{NS}} p_s q(\xi_s)^{\mathrm{T}} y_s \right) \qquad (5)$$

Thus program (4) yields a stochastic linear program that entails affordable computational load.

Step 3. Formulation and Solution of Risk-Inclined Model with Reduced Scenarios

Subsequently, we utilize the value of $\hat{\sigma}$ in a scenario reduction procedure that allows us to determine the minimum number of scenarios, $\mathrm{NS}_{\min}$ that is theoretically required to obtain the same optimal solution as given by program (4) for a desired level of accuracy within a specified confidence interval H (You et al., 2009; Shapiro, 2000; Liu and Sahinidis, 1996). The formula for computing $\mathrm{NS}_{\min}$ is given by:

$$\mathrm{NS}_{\min} = \left[\frac{z_{\alpha/2} \hat{\sigma}}{H} \right]^2 \qquad (6)$$

where $H = \dfrac{2 z_{\alpha/2} \hat{\sigma}}{\sqrt{\mathrm{NS}}}$. We set a convergence criterion for the difference between the optimal objective value of Steps 2 and 3 to a small number ε (typically 0.005). If this criterion is not satisfied, we elect to backtrack to Step 1 to consider a larger number of scenarios that is more representative of the problem.

It is noteworthy that the value of Steps 2 and 3 lies in the minimization of uncertainty, albeit not so much of risk for the reason that variance is a symmetric metric that penalizes both the downside risk, which is desirable, as well as the upside risk (which is not desirable) as pointed out by Samsatli et al. (1998). The idea is to relegate an explicit handling of the minimization of risk to a later step (Step 5), which offers the advantage of using the optimal solution from Step 2 as initial values for the solution of a risk-averse model that only needs to consider a reduced number of scenarios ($\mathrm{NS}_{\min}$) and is hence, computationally tractable.

Step 4. VaR Simulation

In this step, we formulate appropriate value functions for the uncertain parameters to simulate the values of VaR using $\mathrm{NS}_{\min}$ and the optimal solution from Step 3 by estimating their associated cumulative distribution function (CDF). The values are incorporated in the next step to solve the ultimately desirable risk-averse optimization program.

Step 5: Formulation and Solution of Mean–CVaR Stochastic Program

The risk-averse model admits a mean–risk structure in its objective function using the risk measure CVaR. We adopt the computationally-attractive linear programming approximation of CVaR proposed by Rockafellar and Uryasev (2000):

$$\mathrm{CVaR}_\alpha = \mathrm{VaR} + \frac{1}{1-\alpha}\sum_{s=1}^{\mathrm{NS}} p(\xi_s)\left(f\left(x, y_s\right) - \mathrm{VaR}\right) \tag{7}$$

Formulation of the CVaR-based objective function that yields a convex optimization problem is given by:

$$\begin{aligned}
\max_{x, y_s} \quad & \left(\begin{array}{c} c^T x - \sum_{s=1}^{\mathrm{NS_{min}}} p(\xi_s)\left(q(\xi_s)^{\mathrm{T}} y_s + r(\xi_s)^{\mathrm{T}} x\right) \\ -\sum_{n=1}^{\mathrm{NP}} \theta_n \cdot \mathrm{CVaR}_{\alpha,n}\left(x, \xi_s, y_s\right) \end{array} \right) \\
\text{s.t.} \quad & Ax = b \\
& W(\xi_s) y_s = h(\xi_s) - T(\xi_s) x, \quad s = 1, \cdots \\
& x \geq 0 \\
& y_s \geq 0, \quad s = 1, \cdots
\end{aligned} \tag{8}$$

Numerical Example

We illustrate the proposed solution strategy on a case study involving midterm petroleum refinery planning taken from Khor et al. (2008). Uncertainty in prices, demands, and yields are considered, initially using 100 scenarios (Step 1). Formulation of the risk-inclined multiscenario refinery planning model is as follows (Step 2):

$$\max_x \ \mathrm{Profit1} - \sum_{s=1}^{\mathrm{NS}} p_s \left(\begin{array}{l} \mathrm{Profit2}_s + \mathrm{PenaltyDemand}_s \\ + \mathrm{PenaltyYield}_s \end{array} \right) \tag{9}$$

$$\begin{aligned}
\text{where} \quad & \mathrm{Profit1} = \sum_{i \in I} c_i x_i \\
& \mathrm{Profit2}_s = \sum_{i \in I} c'_{i,s} x'_{i,s} \\
& \mathrm{PenaltyDemand}_s = \sum_{i \in I_{\mathrm{D}}} \sum_{k \in K} q_{i,k} z_{i,s,k} \\
& \mathrm{PenaltyYield}_s = \sum_{i \in I_{\mathrm{Y}}} \sum_{m \in M} r_{i,m} y_{i,s,m}
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & x_i \leq d_i, \quad \forall i \in I_{\mathrm{D}} \\
& x'_{i,s} + z_{i,s,k_1} - z_{i,s,k_2} = d'_{i,s}, \forall i \in I_{\mathrm{D}}, \forall s \in S \\
& \sum_{i \in I} a_{i,j} x_i = 0, \quad \forall j \in J \\
& -a'_{s,i} x'_{i,s} + x'_{i'} + y_{i,s,m_1} - y_{i,s,m_2} = 0, \forall \left(i, i'\right) \in I'_{\mathrm{Y}}, \forall s \in S \\
& x'_{i,s} = x'_{i,s'}, \quad \forall i \in I, \forall s \in S \\
& \underline{x}_i \leq x_i \leq \overline{x}_i \\
& \underline{x}'_{i,s} \leq x'_{i,s} \leq \overline{x}'_{i,s}, \quad \forall i \in I, \forall s \in S \\
& x_i, x'_{i,s}, z_{i,s,k_1}, z_{i,s,k_2}, y_{i,s,m_1}, y_{i,s,m_2} \geq 0
\end{aligned} \tag{10}$$

The objective function for the approximating problem to (9) that minimizes the standard deviation of the Monte Carlo estimator is given by:

$$\max_{x, x', z, y} \ c^T x - \frac{1}{\sqrt{\mathrm{NS}-1}} \sum_{s=1}^{\mathrm{NS}} \left(\begin{array}{l} \sum_{i \in I} c'_{i,s} x'_{i,s} + \sum_{\substack{i \in I_{\mathrm{D}} \\ k \in K}} q_{i,k} z_{i,s,k} + \sum_{\substack{i \in I_{\mathrm{Y}} \\ m \in M}} r_{i,m} y_{i,s,m} \\ - \frac{1}{\mathrm{NS}} \sum_{s=1}^{\mathrm{NS}} \left(\begin{array}{l} \sum_{i \in I} c'_{i,s} x'_{i,s} + \sum_{\substack{i \in I_{\mathrm{D}} \\ k \in K}} q_{i,k} z_{i,s,k} \\ + \sum_{\substack{i \in I_{\mathrm{Y}} \\ m \in M}} r_{i,m} y_{i,s,m} \end{array} \right) \end{array} \right) \tag{11}$$

Optimal solution of the risk-inclined program is indicated on the refinery flow diagram in Figure 2.

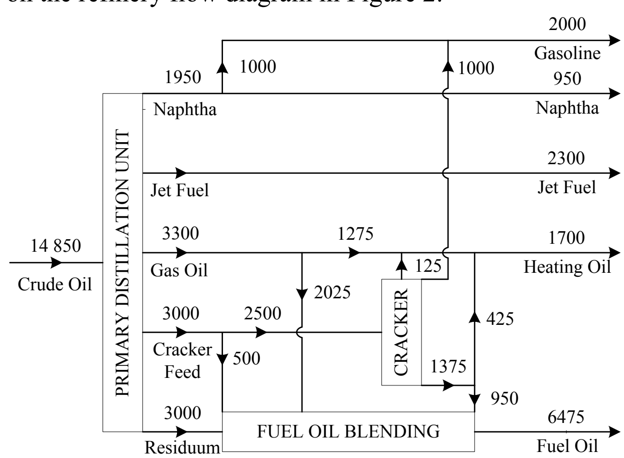

Figure 1. Optimal solution of the risk-inclined program (Step 2)

The value function for price uncertainty is formulated as:

$$f_s^{\text{profit}} = \sum_{i \in I_{\mathrm{P}}} c'_{i,s} x_i, \quad \forall s \in S \tag{12}$$

while the value function for aggregated uncertainty in demands and yields is given by:

$$f_s^{\text{penalty}} = \sum_{i \in I_{\mathrm{D}}} \sum_{k \in K} q_{i,k} z_{i,s,k} + \sum_{i \in I_{\mathrm{Y}}} \sum_{m \in M} r_{i,m} y_{i,s,m}, \quad \forall s \in S \tag{13}$$

For the purpose of prototyping, we employ manual intervention to estimate the CDF for a value function by plotting the value function for each of the NS_{min} against its corresponding cumulative probability. The plots are used to determine VaR_{profit} and $VaR_{penalty}$ at the a priori desired confidence level of 95% as shown in Figures 3 and 4.

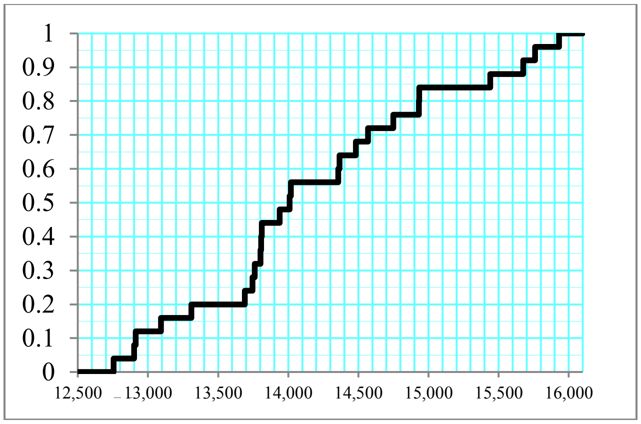

Figure 3. Cumulative distribution function for determining VaR_{profit} *(Step 4)*

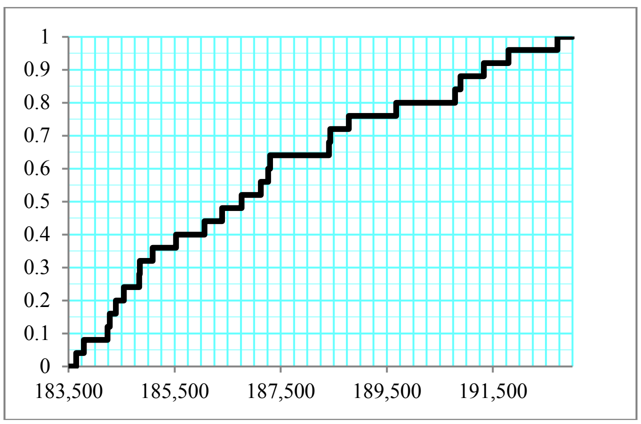

Figure 4. Cumulative distribution function for determining $VaR_{penalty}$ *(Step 4)*

Finally, the linear value functions give rise to a stochastic linear programming approximation of the mean–CVaR risk-averse program:

$$\max \left(\begin{array}{l} \text{Profit1} - \sum_{s=1}^{\text{NS}} p_s \left(\begin{array}{l} \text{Profit2}_s + \text{PenaltyDemand}_s \\ + \text{PenaltyYield}_s \end{array} \right) \\ -\theta_1 \cdot \text{CVaR}_{\text{profit}} - \theta_2 \cdot \text{CVaR}_{\text{penalty}} \end{array} \right) \quad (14)$$

s.t. constraints in (10)

Table 1 summarizes the important optimal results while Table 2 provides the model size and computational statistics for the risk-averse refinery planning model.

Table 1. Computational results for the risk-averse refinery planning model

Parameter	**Value**
NS_{min} (Step 3)	25
VaR_{profit} (Step 4)	\$12,900/day
$VaR_{penalty}$ (Step 4)	\$183,800/day
Confidence level β (Step 5)	0.95
Optimal risk-averse profit (Step 5)	\$25,760/day

Table 1. Model size and computational statistics for the risk-averse model

Solver	GAMS/CPLEX
Number of continuous variables	521
Number of constraints	265
CPU time/resource usage	0.136 s
Number of iterations	180

Concluding Remarks

In this work, we have proposed an optimization-based framework with relatively low computational expense for handling risk management in process planning under uncertainty. We incorporate the use of the risk measure CVaR within the proposed framework that affords a computationally-attractive linear programming formulation. The framework also features the application of a Monte Carlo-based scenario reduction scheme to determine the minimum number of scenarios required to obtain an optimal solution in computationally-tractable fashion. An immediate future work is to extend the approach to a multiobjective optimization formulation, which yields a Pareto curve of optimal solutions, that is capable of examining the tradeoffs between the typically conflicting objectives of expected profit and risk.

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Notation

Sets and Indices

I	set of materials i
I_D	set of i under demand uncertainty, $I_D \subseteq I$
I_Y	set of i under yield uncertainty, $I_Y \subseteq I$
J	set of processes j
K	set of conditions k under demand uncertainty = $\{k_1, k_2\}$ with k_1: shortfall due to underproduction; k_2: surplus due to overproduction
M	set of conditions m under yield uncertainty = $\{m_1, m_2\}$ with m_1 represents shortage in yield and m_2 represents excess in yield
N	set of uncertain parameters $n = \{1, \cdots, \text{NP}\}$
S	set of scenarios $s = \{1, \cdots, \text{NS}\}$
Ξ	set of possible occurrences of ξ

Deterministic Parameters

$a_{i,j}$	yield coefficient of material i in process j
c_i	deterministic unit sales price of product type i
d_i	market demand for product type i
$\underline{x}_i$, $\overline{x}_i$	lower and upper bounds on flowrate of material i

Stochastic Parameters

$a'_{s,i}$	yield coefficient of material i per realization of scenario s
$d_{i,s}$	demand for product i per realization of scenario s
p_s	probability of scenario s
V_n	Monte Carlo sampling variance estimator
β	confidence level to compute VaR and CVaR
θ_1, θ_2	risk factors (weights)
ξ	random variables vector of uncertain parameters

Recourse Parameters

$c'_{i,s}$	stochastic unit sales price of product i per realization of scenario s
$h(\xi)$	second-stage right-hand side vector that is a function of ξ
$Q(x, \xi)$	second-stage recourse costs function
$q(\xi)$,$r(\xi)$	second-stage vector of recourse penalty costs that is a function of ξ
q_{i,k_1}	fixed penalty cost per unit demand $d_{i,s}$ of underproduction k_1 of product i per realization of scenario s (also the cost of lost demand)
q_{i,k_2}	fixed penalty cost per unit demand $d_{i,s}$ of overproduction k_2 of product i per realization of scenario s (also the cost of inventory to store production surplus)
r_{i,m_1}	fixed unit penalty cost for shortage in yield m_1 from material i for product k
r_{i,m_2}	fixed unit penalty cost for excess in yield m_2 from material i for product k
$T(\xi)$	second-stage technology matrix that is a function of ξ
$W(\xi)$	second-stage recourse matrix that is a function of ξ

First-Stage Deterministic Decision Variables

x	vector of first-stage decision variables
x_i	flowrate of material i

Second-Stage Stochastic Recourse Decision Variables

$\text{VaR}_{\text{profit}}$	acceptable loss variable of Value-at-Risk (VaR) under price uncertainty
$\text{VaR}_{\text{penalty}}$	acceptable loss variable of Value-at-Risk (VaR) under aggregated demand and yield uncertainty
$x'_{i,s}$	flowrate of material i per realization of scenario s
y	vector of second-stage decision variables
y_{i,s,m_1}	amount of shortage in yield m_1 from material i per realization of scenario s
y_{i,s,m_2}	amount of excess in yield m_2 from material i per realization of scenario s
z_{i,s,k_1}	amount of unsatisfied demand for product i due to underproduction k_1 per realization of scenario s
z_{i,s,k_2}	amount of excess product i due to overproduction k_2 per realization of scenario s