ASSESSING THE BENEFITS OF PRODUCTION-DISTRIBUTION COORDINATION IN AN INDUSTRIAL GASES SUPPLY CHAIN

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Abstract

In this paper, we propose a multi-period MILP model for optimal enterprise-level planning of industrial gas operations with an objective to minimize the total cost of production and distribution. As compared to the fully sequential approach for optimizing production and then optimizing distribution, the proposed fully coordinated model simultaneously considers various trade-offs between production and distribution activities and yields optimal operational decisions for the supply chain as a whole. Through the continuum between fully sequential and fully coordinated approaches, different levels of coordination are investigated. The computational results for small instances show potential savings if a fully coordinated approach is implemented in place of a fully sequential one. This can be attributed to the improved coordination of the production schedule with the product deliveries.

Keywords

Supply chain, Industrial Gases, MILP, Multi-period Optimization, Operational Planning.

Introduction

An industrial gas supply chain consists of multiple plants, products and storage facilities. These serve different types of customers (over-the-fence, pick-up and delivery) through various delivery modes and routes. Oxygen, Nitrogen and Argon are produced at a plant using air as a raw material which is freely available, while electricity is the major cost associated with the production of these gases (Ierapetritou et al. (2002), Karwan and Keblis (2007)). At each plant, there are various modes of production with their respective efficiencies and energy requirements. The inventory of the products is maintained at the production sites in liquid form for distribution to pick-up and delivery (generally via truck or rail) customers while providing back-up supply for over-the-fence gaseous product customers. The minimum inventories of the products are maintained at the production sites and customer locations to ensure that contractual requirements are met. The maximum inventory is limited by the tank capacities.

In this paper, we focus on the optimal operational planning of an industrial gas supply chain through coordinated production and distribution activities at a multi-plant level. In particular, we propose a multi-period MILP model for optimal operational planning of an industrial gas supply chain with the objective of minimizing the total cost of production and distribution. The decisions include production schedules for various plants with their respective operating modes, inventory levels of individual products at each plant site and customer location, product withdrawal schedules from inventories at each plant, product delivery schedules for each customer, and the delivery routes to execute such schedules. The decisions must respect the operating and capacity restrictions over the given planning horizon. In

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contrast to the fully sequential approach for production and then distribution based on the given production schedule, the proposed fully coordinated model simultaneously considers various trade-offs involved between production and distribution.

The production-distribution coordination problem has been studied in the recent years for several applications (Chandra and Fisher (1994), Glankwamdeea et al. (2009)). However, the problem considered here is unique in the context of the industrial gas industry in the following sense:

1. Multiple plants, products and customers are considered within one model that allows all possible interactions and trade-offs among these elements.
2. It involves a variety of customers in the supply chain, for example over-the-fence, delivery and pick-up customers. Moreover, the model supports customer receipt of the products from multiple sources.
3. Production is constrained by specific production modes to meet the product demand requirements. Any loss of product due to these constraints is penalized.
4. The product inventory is managed both at plant sites and customer locations (including vendor managed inventory) and the cost of keeping the inventory in the tanks is negligible.
5. The distribution cost is calculated based on the routing distances with limited truck capacities, You et al. (2011), instead of the per unit volume of the product delivered.

This paper first introduces the various levels of production-distribution coordination of an industrial gas supply chain operation. Then, we present a generic multi-period MILP model for this supply chain considering multiple plants, multiple products and shared customers. The basic differences between the proposed fully coordinated and the fully sequential models are outlined. The effectiveness of the proposed model and the advantage of fully coordinated approach are demonstrated through computational results for a realistic (but not necessarily representative) industrial gas supply chain example as compared to the various levels of production-distribution coordination.

Notice that the proposed model assumes that each production plant is co-located with a distribution depot, where each plant/depot site may be considered as a "source" with a fleet of trucks available for product delivery.

Production-Distribution Coordination Levels

Sequential scheduling refers to the approach of first generating a production schedule based on the historical data or other sources of information, and then generating an optimal distribution schedule based on the production schedule and associated customer demands. In contrast, simultaneous scheduling produces production and distribution schedules through a simultaneous approach. Both scheduling approaches may be applied across multiple sources or limited to a single source.

The production-distribution coordination problem can be studied at various levels. In this paper, we introduce definitions for the various levels of coordination of an industrial gas supply chain (see Table 1).

Table 1. Production-Distribution Coordination Levels

	Sequential (Production then Distribution)	**Simultaneous (Production and Distribution)**
Single plant/depot (Single Source)	No Coordination b/w plants and production-distribution	Coordination b/w production-distribution but No coordination b/w plants
Multi-plant/depot (Multi-Source)	Coordination b/w plants but No Coordination b/w production-distribution	Coordination b/w production-distribution as well as plants (fully coordinated)

In the following section we present a formal description of the industrial gas supply chain problem that is modeled as a generic "fully coordinated" multi-period MILP model. Notice that the models corresponding to the other levels of production-distribution coordination described in Table 1 can be obtained from the fully coordinated model as a special case.

Problem Statement

Given is a set of production plants $P=\{1,2,....p\}$ and operating modes $M=\{0,1,2,..m\}$ with their production capacities, daily electricity prices (off-peak and on peak), potential assigned customers c to each plant, their demand profiles and min/max storage capacities of the plants and customers inventories. Figure 1 represents a typical industrial gas supply chain including production, distribution and inventory for several sources and customers. Notice that some of the customers (e.g. C4) can be sourced from multiple plants in each time period.

Source and customer locations along with truck availability at each source are given parameters. Each time period t is of half day of duration and corresponds to the periods of off-peak and on peak electricity prices on a given day. The time horizons can be the week-ahead or longer. Initial product inventories at production sites and customer locations are also known.

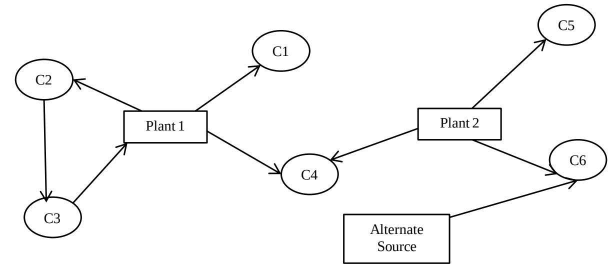

Figure 1. A typical industrial gases supply chain

Operating decisions for each time period t include the following: production rate of the final products at each plant, the mode of operation of each plant, the amount of inventory maintained for each product at each source and customer location, and the amount of each product to be delivered to each customer through a specific route. The objective function is to minimize the total cost of production and distribution.

The savings through full production-distribution coordination is quantified here using an approximate model of sufficient details to be realistic. The proposed "fully coordinated" MILP model for the multi-source simultaneous case described in the following section considers operational decisions concerning the existing supply chain, not design decisions concerning investments in new installations or expansions.

Fully Coordinated Model

We present here a generic MILP model for optimal production and distribution planning of industrial gases at the operational level involving multiple plants, products and shared customers among plants. The objective function (1) is to minimize the total cost that involves (Eq. (2)) sum of the production and distribution costs over each time period t.

$$Min \quad TotalCost \tag{1}$$

$$TotalCost = \sum_t (PCost_t + DCost_t) \tag{2}$$

Constraints (3)-(11) correspond to the production side of the supply chain. In particular, the total production cost in time period t is calculated as the sum of start-up costs and variable costs of production for each plant p operating in one of the modes m and producing products i in that mode as given in eq. (3). Note that the variable production cost, $VO_{p,i,m,t}$, corresponds to the unit energy cost for each plant p operating in mode m to produce product i in time period t. $W_{p,i,m,t}$ is the production rate of product i in time period t at plant p in mode m.

$$PCost_t = \sum_p \left[F_{p,t}^{start} b_{p,t}^{start} + \sum_m \sum_i VO_{p,i,m,t} W_{p,i,m,t} \right] \qquad \forall t \tag{3}$$

The maximum production rate of a product i under each operating mode m should be less than its production capacity limits for that plant in that mode; see constraint (4). Moreover, the plant can only produce in a time period t in mode m if mode m is on for that plant in time period t (i.e. $b_{p,m,t}^{on}$ equals 1). Note that $m=0$ corresponds to the shut down mode for each plant while other modes are $m=1,2,3,..$etc. in which production rates are non-zero.

$$W_{p,i,m,t} \le b_{p,m,t}^{on} U_{p,i,m} \qquad \forall p,i,m,t \tag{4}$$

The minimum production rate of product i for each plant p operating in mode m and producing product i in time t is defined by a turndown ratio, α_p (e.g. 70%), applied to the maximum capacity in this mode, $U_{p,i,m}$. Let us note that the turndown ratio is for the plant only and not depending on the mode. This condition is represented by the constraint (5). Notice that we assume that plants are flexible enough to operate between their maximum and minimum capacity levels in each mode.

$$b_{p,m,t}^{on} \alpha_p U_{p,i,m,t} \le W_{p,i,m,t} \qquad \forall p,i,m,t \tag{5}$$

The restriction that each plant can operate in a single mode during time period t is given by the logic constraint (6). Moreover, we assume here that switching times and costs from one mode to another are negligible but the model can easily be extended to include these factors.

$$\sum_m b_{p,m,t}^{on} = 1 \qquad \forall p,t \tag{6}$$

The logic constraints (7)-(8) state that the binary variable $b_{p,t}^{start}$ that represents the start-up of the plant from a shut-down mode ($m=0$) to any other mode ($m=1,2,...$etc.) will be 1 if and only if plant p was shut-down in the previous time period t-1, and is turned on in the current time period t. In particular, constraint (7) represents this logic for time period 1, while constraint (8) corresponds to the rest of the time periods. Note that these constraints are required to capture the start-up cost for each plant p in constraint (3). Constraints (7) and (8) can further be reformulated as the mixed-integer liner constraints.

$$b_{p,1}^{start} \Leftrightarrow \bigvee_{m=1}^{m} b_{p,m,1}^{on} \qquad \forall p \tag{7}$$

$$b_{p,t}^{start} \Leftrightarrow b_{p,m=0,t-1}^{on} \wedge \left(\bigvee_{m=1}^{m} b_{p,m,t}^{on} \right) \qquad \forall p, t \ge 2 \tag{8}$$

Each plant is associated with a storage facility for each product i. The level of product i in inventory at plant p in time t, $I_{p,i,t}^{pr}$, must lie between the minimum desired level (redline) and the maximum storage capacity of the facility for that product i, i.e. inequality (9). The minimum inventory level ensures that the demand of over-the-fence/on-site customers can be met using this inventory as a back-up source. Moreover, this redline is a given parameter which can have variations over the planning horizon (not constant) based on the over-the-fence customer demand profile, while the maximum limit is related to the capacity of the storage facility (a constant value) for product i.

$$Q_{p,i,t}^{\min,pr} \le I_{p,i,t}^{pr} \le Q_{p,i}^{\max,pr} \qquad \forall p,i,t \tag{9}$$

Constraint (10) represents the material balance constraint for each product i in inventory at plant p for each time period t. In particular, the amount of product in storage at plant is equal to the inventory of the product at the previous time period, plus the production over time period t less the total product distributed to the pickup customers, $D_{p,i,c,t}^{pickup}$, and total deliveries for that product by trucks from the plant, $D_{p,i,t}^{truck}$.

$$I_{p,i,t}^{pr} = I_{p,i,t-1}^{pr} + \sum_m W_{p,i,m,t} - \sum_{c \in PickUp} D_{p,i,c,t}^{pcikup} - D_{p,i,t}^{truck} \qquad \forall p,i,t \tag{10}$$

The amount of product i distributed to the pick-up customers from each plant must meet their demands, $S_{p,i,c,t}^{pickup}$, in each time period t, eq. (11).

$$D_{p,i,c,t}^{pickup} = S_{p,i,c,t}^{pickup} \qquad \forall p,i,c \in PickUp,t \quad (11)$$

Constraints (12)-(21) correspond to the distribution side of the supply chain. In particular, equation (12) calculates the total distribution cost in time period t. The first term in the expression is the sum of the delivery costs of all the products i over routes r originating from each plant p through the use of all possible trucks $k(i)$ available for delivery of that product i . The binary variable $y_{p,i,k,r,t}$ equals to 1 if truck k is used at plant p for delivery through route r in time period t for product i. Notice that we assume that the set of trucks $k(i)$ can only be assigned to deliver the product type i (fleet assignment). In principle, the set of routes should include all possible routes that originate from all the plants/depots and involves single (direct shipment) and/or multiple customers visit in each route. However, due to the exponential increase in the number of these routes with the number of plants and customers, we include only those routes r within the model that are determined based on the pre-defined clustering of the customers for each plant. Parameter $dis_{p,r}$ is the total distance of the route r originating from plant p while $c_{p,k}$ is the delivery cost per unit distance traveled of the truck k available at plant p.

The other terms in the distribution cost, eq. (12), include the delivery cost from alternate sources to customers, plus purchase costs of product i for any unsatisfied customer demand in time period t that is served by an alternate source alt. The binary variable $y_{c,alt,p,i,k,t}^{purch}$ equals 1 if truck $k(i)$ of product i from plant p is used to deliver the product to customer c from alternate source alt in time period t.

$$\begin{aligned} DCost_t &= \sum_p \sum_i \sum_{k(i)} \sum_r \left[dis_{p,r} \cdot c_{p,k} \cdot y_{p,i,k,r,t} \right] \\ &+ \sum_{alt} \sum_c \sum_i \left[\sum_p \sum_{k(i)} del_{c,alt,p,t}\, y_{c,alt,p,i,k,t}^{purch} \right] \qquad \forall t \quad (12) \\ &+ \sum_{alt} \sum_c \sum_i \left[C_{alt,i,t}^{purch} D_{c,alt,i,t}^{purch} \right] \end{aligned}$$

Notice that the above distribution cost is independent of the partial/full truck loading as it is only based on the routing distance instead of volume delivered.

The total amount of product i retrieved from an alternate source alt during time period t, must be less than the maximum available amount of the product in the tank at alternate source during that time period t as given by constraint (13).

$$\sum_c D_{c,alt,i,t}^{purch} \le Q_{alt,i,t}^{\max,purch} \qquad \forall alt,i,t \quad (13)$$

The delivery of product i from alternate source alt to customer c using truck $k(i)$ from plant p, has to satisfy the maximum capacity of the truck, i.e. constraint (14).

$$D_{c,alt,i,t}^{purch} \le \sum_p \sum_{k(i)} y_{c,alt,p,i,k,t}^{purch} Q_k^{truck} \qquad \forall c,alt,i,t \quad (14)$$

If a delivery of product i is made by trucks $k(i)$ through a route r, then it has to satisfy a minimum fraction f of the total truckload need to be delivered and the maximum total capacity of the trucks as given by constraint (15), where C_r is the set of customers that can receive deliveries through route r and $k(i)$ is the set of trucks that can deliver product type i.

$$f \sum_{k(i)} y_{p,i,k,r,t} Q_k^{truck} \le \sum_{c \in C_r} D_{c,i,r,t}^{route} \le \sum_{k(i)} y_{p,i,k,r,t} Q_k^{truck} \qquad \forall p,i,r,t \quad (15)$$

Constraint (16) represents that the total amount of product i delivered to customer c during time period t, i.e. the summation of the deliveries through all possible routes involving this customer and amount delivered by the alternate sources.

$$D_{c,i,t} = \sum_{r \in R_c} D_{c,i,r,t}^{route} + \sum_{alt} D_{c,alt,i,t}^{purch} \qquad \forall c,i,t \quad (16)$$

The amount of product i distributed from plant p using trucks equals the sum of the product deliveries made to all the routes originating from that plant, i.e. R_p , as given in equation (17).

$$D_{p,i,t}^{truck} = \sum_{r \in R_p} D_{p,i,r,t}^{proute} \qquad \forall p,i,t \quad (17)$$

Constraint (18) ensures that the total amount of product i delivered from all the plants p to a route r equals to the total amount of the product distributed to the customers in that route. Notice that we do not consider the case where some portion of the product in a truck can return back to the plant after deliveries.

$$\sum_{p \in P_r} D_{p,i,r,t}^{proute} = \sum_{c \in C_r} D_{c,i,r,t}^{route} \qquad \forall i,r,t \quad (18)$$

The level of product i inventory at each customer c in time t, $I_{c,t}^{cu}$, must lie between minimum desired level (safety stock) and maximum storage capacity of the tank, i.e. inequality (19). Notice that the safety stock is given as a parameter with variations over the planning horizon based on the demand profile of that particular customer.

$$Q_{c,i,t}^{\min,cu} \le I_{c,i,t}^{cu} \le Q_{c,i}^{\max,cu} \qquad \forall c,i,t \quad (19)$$

Constraint (20) represents the material balance constraint for product i inventory at each customer location. In particular, the amount of product i in the customer storage tank in time period t, is equal to the inventory of that product at the previous time period plus the amount of the product delivered, $D_{c,i,t}$, to that customer in time period t less the amount of product consumed by the customer, $S_{c,i,t}$, in that time period t.

$$I_{c,i,t}^{cu} = I_{c,i,t-1}^{cu} + D_{c,i,t} - S_{c,i,t} \qquad \forall c,i,t \qquad (20)$$

Each truck $k(i)$ for product i at plant p can only be assigned to either a single route r associated to that plant in each time period t, or it can be used to deliver product from an alternate source *alt* to customer c as given in constraint (21). We assume here for simplicity that no multiple trips for a single truck from a plant in each time period t are allowed. However, multiple trucks can be assigned to the same route in a time period t to meet the customer demands in the route. The model can easily be extended to include multiple trips for each truck in time period t.

$$\sum_{r \in R_p} y_{p,i,k,r,t} + \sum_{alt} \sum_{c} y_{c,alt,p,i,k,t}^{purch} \le 1 \qquad \forall p,i,k(i),t \quad (21)$$

We introduce here the proposed fully coordinated MILP model **(P1)** that involves constraints (1)-(21). The differences for the sequential cases are described next.

Sequential vs. Simultaneous Model

The sequential models are derived from the fully coordinated model **(P1)** by decomposing the production and the distribution optimization into two separate programs that will be connected through a sequence of decisions involving the two programs. We introduce first the production optimization program **(P2)** generating the production schedule that minimizes the production cost based on the constraints on the production side (3)-(11). Second, we introduce the distribution optimization program **(P3)** generating the distribution schedule that minimizes the total cost of delivery with respect to the distribution side constraints (12)-(21). We will study in this paper the sequence **(P2)→(P3)** to set the production decisions first and then observing the consequences on the availability of the product, before solving the distribution model.

Notice that in the production model **(P2)**, the truck withdrawal amount in each time period t is given by the parameters in eq. (10). Two options have been considered to set the production targets: either the truck withdrawals are forecasted directly based on historical frequencies **(P2a)** or by aggregating the customer demand forecasts itself available on the distribution side to evaluate the truck withdrawal forecasts **(P2b)**. The distribution model **(P3)** generates the delivery routes, based on the future plant inventory levels fixed by the solution of the production model. Moreover, the product purchase cost from a particular plant is based on the average production cost at the plant. This purchase cost is included in the distribution costs, eq. (12), to consider attractiveness in the objective function for deliveries among multiple sources in each time period t.

In the next section, we present the results of the fully coordinated model for a realistic example, and compare the savings in contrast to different levels of coordination.

Example

In this example, we consider 3 plants with 2 main products (LIN i.e. liquid nitrogen and LOX i.e. liquid oxygen). The plants can be operated in 2 modes (High LIN and High LOX) with specific capacity limits or they can be shut down. There are 17 customers (10 LIN and 7 LOX) that need to be served by trucks, 42 pick-up customers that pick up the product from the plants on specific days. There are 20 trucks that are product specific (10 for LIN, 10 for LOX). The time horizon of 1 week is discretized into 14 time periods t each with half day of duration and corresponds to off-peak and on peak electricity prices on a day.

The models (P1), (P2) and (P3) are implemented in GAMS 23.6.3 and solved with the solver CPLEX 12.2. The multi-plant simultaneous model (P1) has been solved first. Figure 2 represents the optimum production profile vs. electricity prices for one plant, which shows the sensitivity to energy costs with less production during peak hours as compared to the off-peak hours.

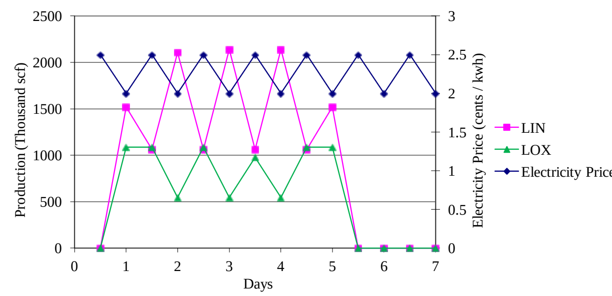

Figure 2. Production rate vs. electricity prices for plant 1

Figure 3 compares the inventory profiles at one production site in the cases where either the simultaneous model (P1) is solved or the sequential model (P2b)→(P3) based on customer demand forecasts is solved. Notice that we assume here that the plant inventories at the end of planning horizon must be at-least as large as the initial inventories. In the simultaneous model, the inventory level ends at the same level as the initial level. With the sequential model relying on forecasts, we can observe an extra product inventory at the end of the planning horizon. Other plant inventories also show similar trends in the solution.

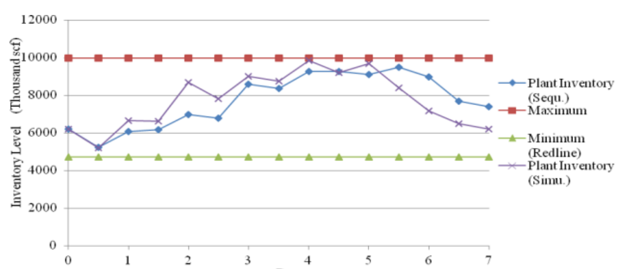

Figure 3. Sequential vs. Simultaneous case LIN inventory level for plant 1

Table 2 compares the total production and distribution costs from fully coordinated model for this example with other levels of coordination. We can observe that the multi-source simultaneous case gives the least cost, while the single plant sequential case that relies on truck forecast is the most expensive (~20% cost penalty).

Table 2. Cost penalties (%) for various levels of Production-Distribution coordination versus multi-plant simultaneous model (reference case)

	Sequential (based on truck withdrawal forecast)	Sequential (based on customer demand forecast)	Simultaneous
Single Plant (Single Source)	25%	15%	11%
Multi-plant (Multi-source)	22%	8%	0%

Figure 4 shows the impact of the different levels of coordination on the breakdown between production and distribution costs separately (without actual costs mentioned for confidentiality purpose). We can note first that production and distribution costs are in the same order of magnitude. Second, it is worth noticing that the both production and distribution sides benefit from a better coordination ("win/win" solutions) even if savings are slightly more important on the production side. Third, the cost reduction by switching from a pure "blind" truck forecasts approach to a "myopic" customer demand forecasts approach is very significant. The gain from the customer demand forecast approach to the fully coordinated approach is more limited. In summary, we can say that interactions among various plants, customers and alternate sources in one model results in the minimum total cost, better schedules and optimal inventory profiles at production sites and customer locations.

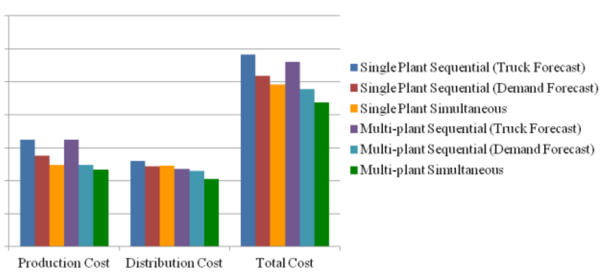

Figure 4. Impact on production and distribution costs for each level of coordination

Conclusions

In this paper, we have proposed simultaneous and sequential MILP models for optimal production and distribution planning of an industrial gas supply chain that involves multi-plant, multi-product, a variety of customers and routing decisions. Numerical results on one small test case show potential savings due to switching sourcing/routing strategies, electricity price differences and efficient inventory management, resulting in a better control over the production and distribution schedules. The proposed models will further be extended to include other details on both sides to be more realistic. This will allow further verifications of the promising results obtained until now.

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