FROM MULTI-PARAMETRIC PROGRAMMING THEORY TO MPC-ON-A-CHIP MULTI-SCALE SYSTEMS APPLICATIONS

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Abstract

An overview of multi-parametric programming and control is presented with emphasis on historical milestones, novel developments in the theory of multi-parametric programming and explicit MPC as well as their application to the design of advanced controller for complex multi-scale systems.

Keywords

Multi-parametric programming; explicit/multi-parametric MPC; MPC-on-a-chip; fast MPC; multi-scale applications

Introduction

Multi-parametric programming is an important optimization tool for systematically analyzing the effect of varying parameters and/or uncertainties in mathematical programming problems. Its importance is widely recognized and many significant advances have emerged in the last ten years both in the theory and practice of multi-parametric programming, especially in engineering (Pistikopoulos, 2009). A significant milestone is its adoption in model-based control and specifically Model Predictive Control, which has created a new field of research in control theory and applications - the field of explicit/multi-parametric MPC or explicit MPC control.

In an optimization framework (Figure 1a) with an objective function to minimize, a set of constraints to satisfy and a number of bounded parameters affecting the solution, multi-parametric programming obtains:

- The objective function and the optimization variables as functions of the parameters (Figure 1c), and
- The space of parameters (known as critical regions) where these functions are valid (Figure 1b).

i.e. the exact mapping of the optimal solution profile in the space of parameters (Figure 2). The optimization can then be replaced by its optimal solution mapping and the optimal solution for a given value of the parameters can be computed efficiently by performing simple function evaluations, without the need to solve the optimization. The advantage to replace optimization by simple and efficient computations has given multi-parametric programming wide spread recognition and has triggered significant advances in its theory and applications. In the context of model predictive control (MPC – online optimization) multi-parametric programming can be used to obtain the optimal control inputs as explicit functions of the system state variables/process data. This is the notion of explicit/multi-parametric MPC or explicit MPC – also known as “MPC-on-a-chip” technology. Key advances in the various areas of multi-parametric programming are summarized in Table 1 while for a complete list of references and a full overview of multi-parametric programming see Pistikopoulos, 2009 and the forthcoming review paper by Pistikopoulos et al. 2012. The advances of multi-parametric programming and its applications in

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advanced model-based control are the subjects of a two volume book (Pistikopoulos et al., 2007a, 2007b).

In this paper, we overview multi-parametric programming, explicit/multi-parametric MPC and the MPC-on-a-chip concept and we briefly present recent advances in the theory and applications of multi-parametric programming and explicit MPC. First, a comprehensive framework for multi-parametric programming and control for real-time control of process systems is presented. Then some recent theoretical advances in multi-parametric programming and control are presented. Finally, we describe a number of recent applications of explicit MPC in the area of fuel cell, pressure swing adsorption and other applications.

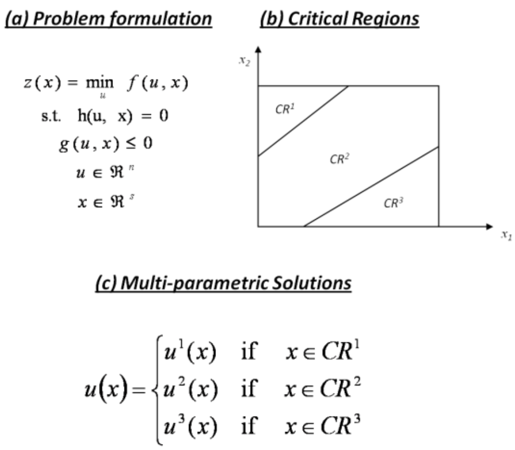

Figure 1. Multi-parametric programming: (a) problem formulation, (b) critical regions, (c) explicit solution

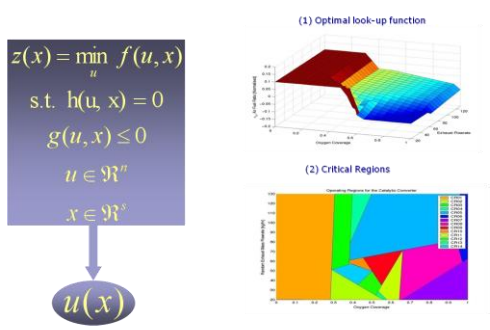

Figure 2. Optimal solution mapping of multi-parametric programming

Towards fast MPC

Traditional Model Predictive Control (MPC) aims to provide a sequence of control actions/inputs over a future time horizon, which seeks to optimize the controller performance based upon the predicted states of the system (Rawlings and Mayne, 2009, Lee, 2011, Morari, 2011). This is achieved by repetitively solving an on-line optimization problem, which describes the (past, present, future) behavior of the system. MPC problems are typically formulated as the following optimization problem:

$$
\begin{aligned}
&\min_{u(0),\ldots,u(N)} J = \sum_{k=0}^{N_y} y'(k)Qy(k) + \sum_{k=0}^{N_u} u'(k)Ru(k) \\
&\text{s.t.} \quad x(k+1) = \ell(x(k),u(k)) \\
&\qquad y(k) = \psi(x(k),u(k)) \\
&\qquad x_{\min} \le x(k+1) \le x_{\max}, \; k = 0,1,\ldots,N_y \\
&\qquad u_{\min} \le u(k) \le u_{\max}, \; k = 0,1,\ldots,N_u
\end{aligned}
\tag{1}
$$

where x and u are the vectors of state and control/inputs variables, respectively; ℓ and ψ are the vectors of state and output equations respectively; N_y, N_u are the prediction/output and control/input horizons, respectively; Q and R are weights on deviations of the state and control variables, and k denotes a time interval. The basic idea of MPC is illustrated in Figure 3, where at the current time interval k, the optimization problem is solved to minimize the state and control deviations from the set point, by implementing the optimal values of the control/input variables. Note that only the first control element is implemented and this sequence is repeated at the next time interval, for the new state measurements or estimates, until the desired or set point values are obtained.

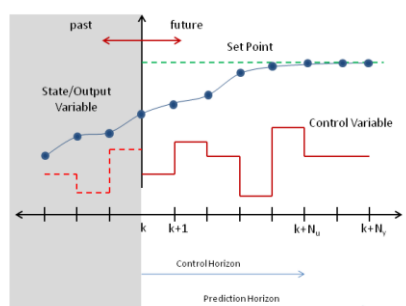

Figure 3. Model Predictive Control implementation.

The key advantage of MPC is that it is model-based and it can take into account the constraints on the state and control variables. However, a key limitation is the (potentially high) computational effort involved due to the repetitive solution of the underlying optimization problem of the MPC, which may limit its implementation to processes with "slow" dynamics (Pistikopoulos, 2009, Lee, 2011). Adding to this, MPC is an implicit control method i.e. the optimal control values are only determined numerically at the current state values without any a priori knowledge of the governing control laws.

Table 1.Milestones in multi-parametric programming

Field	Contributors
Multi-Parametric Linear Programming (mp-LP)	Gass and Saaty (1955); Gal and Nedoma (1972); Gal (1995); Acevedo and Pistikopoulos (1997a); Dua et al. (2002)
Multi-Parametric Quadratic Programming (mp-QP)	Townsley and Candler (1972); Propoi and Yadykin (1978); Best and Ding (1995); Dua et al. (2002); Pistikopoulos et al. (2002)
Multi-Parametric Nonlinear Programming (mp-NLP)	Fiacco (1976); Kojima (1979); Bank et al. (1983); Fiacco and Kyparisis (1986); Acevedo and Pistikopoulos (1996); Dua and Pistikopoulos (1998)
Multi-Parametric Dynamic Optimization (mp-DO)	Sakizlis et al. (2001b)
Multi–Parametric Global Optimization (mp–GO)	Fiacco (1990); Dua et al. (1999, 2004a)
Multi–Parametric Mixed Integer Linear Programming (mp–MILP)	Marsten and Morin (1975); Geoffrion and Nauss (1977); Acevedo and Pistikopoulos (1997b); Dua and Pistikopoulos (2000)
Multi–Parametric Mixed Integer Quadratic and non-Linear Programming (mp–MINLP)	McBride and Yorkmark (1980); Dua and Pistikopoulos (1999); Dua et al. (2002)

Explicit/multi-parametric MPC (also explicit MPC or mp-MPC), on the other hand, is an advanced control method that uses multi-parametric programming methods to solve the online optimization problem of MPC and obtain the exact mapping of the optimal control variables as functions of the state variables (see Figure 1c). Thereby, the online optimization problem involved in MPC is replaced with simple function evaluations that require a smaller online computational effort. This advantage has made it possible for MPC to be implemented on the simplest of computational hardware such as microchips thus making mp-MPC suitable for fast control applications such as portable devices and embedded control systems. The concept of replacing the online optimization via the exact mapping of its optimal solution, has become known as "online optimization via off-line optimization" while the ability of multi-parametric MPC to be implemented on the simplest possible hardware has become known as the "MPC-on-a-Chip" technology. These concepts as well as the framework for the design and implementation of explicit MPC are illustrated in Figure 4. A number of key developments in explicit/multi-parametric MPC is shown in Table 2 while a thorough review and listing of publications is given Pistikopoulos, 2009 and in the forthcoming Pistikopoulos et al., 2011.

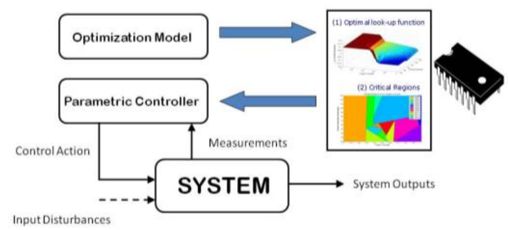

Figure 4. MPC-on-a-chip concept.

The key advantages of MPC-on-chip implementation are that (i) it is computationally efficient since it requires simple function evaluations, (ii) it does not require any on-line optimization software, (iii) its explicit form makes mp-MPC ideal for safety critical applications, and (iv) allows for advanced model-based controllers to be implemented in portable and/or embedded devices. This has paved the way for many advanced control applications in chemical, energy, automotive, aeronautical, biomedical systems, amongst others.

Framework for multi-parametric programming & explicit MPC

A comprehensive framework for the systematic design and off-line validation and testing of multi-parametric programming and robust explicit MPC was presented in Pistikopoulos (2009) is shown in Figure 5. This framework features four key steps which are presented next:

Step 1: development of a detailed "high-fidelity" mathematical model of the process,
Step 2: development of a reduced-order/ approximating model suitable for explicit MPC design,
Step 3: design of robust explicit MPC controller
Step 4: implementation and validation/testing of the designed controller on the "high-fidelity" model

Step 1 involves the development of a ""high-fidelity" mathematical model that provides a detailed description of the real system operation - this mathematical model is mainly for performing detailed simulations and optimization studies. Step 2 involves the development of an approximating model for the system at hand, which is derived by using model order-reduction or system identification methods. This step is important in order to arrive at a suitable MPC formulation that can be solved by the available multi-parametric programming and control techniques. Then, in step 3 an explicit/multi-parametric MPC controller is designed by applying the available methods of multi-parametric programming and explicit control based on the approximate model derived in step 2. Finally, step 4 involves the off-line validation of the controller. This is done by incorporating the functional expression of the explicit controller into the high-fidelity model and then performing dynamic simulation studies to test/validate the controller performance. Since the controller is based on an approximate model of the process, deviations from the desired behavior may occur and the steps of the framework have to be repeated until a desired performance is obtained. The controller is then implemented to the system.

All the steps of the framework in Figure 5 can be performed off-line before any real implementation on the system takes place. However, the high-fidelity model in step 1 is derived only once and does not use any new information created from the operation of the process that could possibly improve the model further and hence the controller performance. Thus the framework of Figure 5 is an open-loop procedure for the design of explicit controllers.

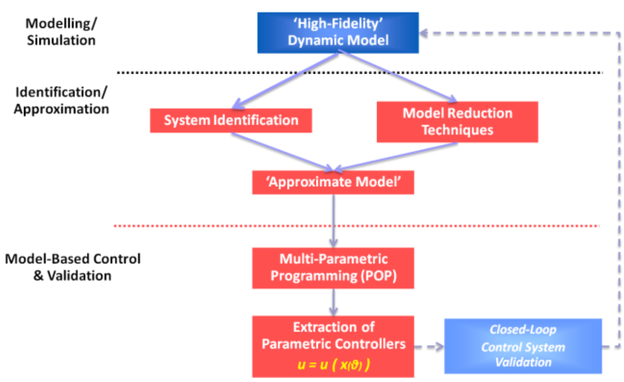

Figure 5. A framework from multi-parametric programming & explicit MPC

A closed-loop framework for the design, validation and on-line implementation of explicit controllers is presented here and shown in Figure 6. In this framework, the open-loop framework (in Figure 5) for multi-parametric programming and explicit MPC is used off-line to derive and validate the explicit controller. However, due to the bidirectional link between the open-loop framework and the real process, real-time date available at any time from the real process can be used to further improve the model and hence the explicit control design. Additionally, optimization studies performed off-line on the high-fidelity model can lead to optimal designs and operational profiles which can be directly used to improve the design and operation of the real system. Hence a closed-loop framework for real-time multi-parametric programming and explicit MPC is established.

Table 2. Milestones in multi-parametric Model Predictive Control.

Area of mp-MPC	Contributors
Multi–Parametric Model Predictive Control	Pistikopoulos (1997, 2000); Bemporad et al. (2000b); Pistikopoulos et al. (2002); Bemporad et al. (2002); Johansen and Grancharova (2003); Sakizlis et al. (2003)
Multi–Parametric Continuous Time Model Predictive Control	Sakizlis et al. (2002b); Kojima and Morari (2004); Sakizlis et al. (2005)
Hybrid Multi–Parametric Model Predictive Control	Bemporad et al. (2000a); Sakizlis et al. (2001a)
Robust Multi–Parametric Model Predictive Control	Kakalis (2001); Bemporad et al. (2001); Sakizlis et al. (2002c, 2004a); Faísca et al. (2008)
Multi–Parametric Dynamic Programming	de la Peña et al. (2004); Faísca et al. (2008)
Multi-Parametric Non-linear Model Predictive control	Johansen (2002); Bemporad (2003a); Sakizlis et al. (2007)

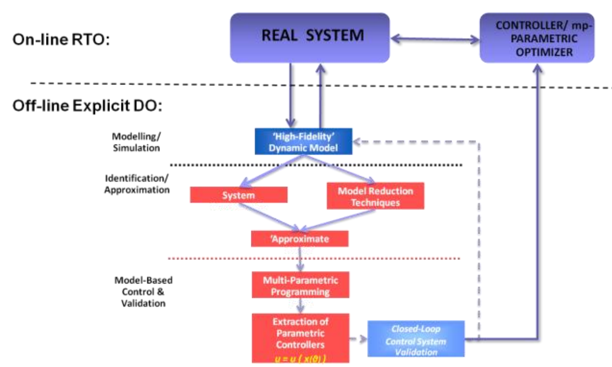

Figure 6. Closed-loop framework for multi-parametric programming and explicit MPC

Recent developments in multi-parametric programming & mp-MPC

The recent developments in multi-parametric programming and explicit/multi-parametric control include: i) model-order reduction and explicit MPC, ii) multi-parametric Nonlinear Programming (mp-NLP) and explicit Nonlinear MPC, iii) multi-parametric Mixed-Integer Nonlinear Programming, iv) robust explicit MPC and v) constrained moving horizon estimation and explicit

MPC. In the following sections we present an overview of the recent developments in each of these areas.

Model order- reduction/approximation and explicit MPC

The main difficulty in the off-line design and on-line implementation of explicit MPC controllers is the capability to handle the expensive computations involved either in the off-line optimization or the online calculations of the controller especially in the case of large-scale processes. Therefore, model order-reduction methods are used to provide approximating reduced order models (with reduced number of state variables) for the large scale processes. The reasons for using model order-reduction techniques for multi-parametric programming include:

1. the insufficient availably memory for solving the explicit MPC problem off-line,
2. the desire to reduce the computational time at which the explicit controller is derived, and
3. the need to reduce the size of the explicit solution (by deriving a smaller number of parameters and critical regions) in order to speed up the online calculations

In addition, since most high-fidelity mathematical models are too complex and cannot be used with the available multi-parametric programming and control methods, model approximations are necessary to create a model that is suitable for use with the existing explicit control design methods (Johansen, 2003 and Pistikopoulos, 2009). The key issue is that since reduced-order models are only approximations of the real process, the optimality and feasibility of the reduced explicit MPC controller is not guaranteed. In Narciso et al. (2008) a systematic method was developed that combines balanced truncation model reduction with explicit MPC techniques for linear dynamic systems, which ensures the optimality and feasibility of the explicit MPC design – this is the first reported work which adopted the combined model reduction and explicit MPC design approach for solving the above issues.

Recently Lambert and Pistikopoulos (2011) proposed an approach for the combined model order-reduction and explicit MPC of nonlinear dynamic systems

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= g(x,u) \end{aligned} \tag{2}$$

with constraints on the inputs U and outputs Y. The proposed methodology uses dynamic integration methods (such as Monte-Carlo integration) and "meta-model" approaches (such as Legendre polynomials) to derive a linear discrete-time approximation of the predictions of the output as a function of the initial state x_t and the input prediction u_j

$$y_j = \Psi_j\left(x_t, u_1, u_2, \ldots, u_{N-1}\right), \quad j = 1, \ldots, N \tag{3}$$

over the prediction horizon N. Thus the nonlinear dynamic system (1) can be replaced by a set of algebraic expressions which only depend on the initial condition and the future sequence of control variables. An explicit MPC formulation can then be derived for the system, in which the future control sequences are the optimization variables and the initial condition is the parameter. It was shown that this problem can be solved as a convex mp-QP problem (Lambert and Pistikopoulos, 2011).

Multi-parametric Nonlinear Programming

The general mp-NLP problem can be stated as follows:

$$\begin{aligned} z(\theta) = &\min_x f(x,\theta) \\ \text{s.t.} \quad & g(x,\theta) \le 0 \\ & x \in \mathbb{R}^n,\ \theta \in \mathbb{R}^p \end{aligned} \tag{4}$$

Where x is the vector of optimization variables, θ is the vector of parameters, f the objective function, $z(\theta)$ the value function and g the vector of linear inequalities. Developments in mp-NLP have not followed the rapid progress in the developments of multi-parametric linear programming (mp-LP) and multi-parametric mixed-integer linear programming (mp-MILP). Most of the work on mp-NLP has focused on convex problems (Pistikopoulos, 2009, Dominguez and Pistikopoulos, 2010). Previous work on mp-NLP was focused on the development of outer mp-LP approximation with a prescribed error of the underlying mp-NLP problem (Dua and Pistikopoulos, 1998) while in Narciso, 2009 a geometric, vertex search-based (GVS) method was presented for partitioning the parameter spaces to obtain piecewise linear approximation of the nonlinear solution.

In the recent work, novel multi-parametric Quadratic Approximation (mp-QA) algorithms, initially proposed by Johansen (2002) were developed by Dominguez and Pistikopoulos (2009) for the mp-NLP problem. The main idea, as in the previous outer approximation algorithms, is to alternate between a primal NLP and master mp-QP problem to derive the global (or an approximation with a specified tolerance ε) solution. However, the difference with outer approximation algorithms is that, where a first approximation of the nonlinear objective is used, the mp-QA method uses second order (quadratic) approximations of the mp-NLP problem as follows:

$$\begin{aligned} z_q(\theta) = &\min_u f\left(u_c^*\right) + \nabla_u f\left(u_c^*\right)\left(u-u_c^*\right) \\ &\quad + \frac{1}{2}\left(u-u_c^*\right)^T \nabla_u^2 f\left(u_c^*\right)\left(u-u_c^*\right) \\ \text{s.t.} \quad & g\left(u_c^*\right) + \nabla_u g\left(u_c^*\right)\left(u-u_c^*\right) \le 0 \\ & g\left(u^*\right) + \nabla_u g\left(u^*\right)\left(u-u^*\right) \le 0 \\ & u^* \in U_{\omega,k}^*, \quad \forall\ \omega = 1,\ldots,\Omega_k,\ k = 1,\ldots,K \\ & u_c^* \in U \subseteq \mathbb{R}^{n+p} \end{aligned} \tag{5}$$

Where the point of approximation u_c^* is determined by computing the centre point of a critical region, $\nabla_u f$ and $\nabla_u^2 f$ are the Jacobian and Hessian matrices respectively, and $U_{\omega,k}^*$ is the set of feasible points found at iteration k for every infeasible vertex, θ_ω. In Dominguez and

Pistikopoulos (2010) it was shown that mp-QA methods can obtain significant improvements in terms of quality of the optimal solution approximations and performance (number of NLP and mp-LPs/mp-QPs required to solve mp-NLP). Ongoing research in mp-NLP is currently focusing on the development of multi-parametric programming approximation algorithms for the general non-convex case.

Multi-parametric Mixed-Integer Linear Programming

Multi-parametric programming problems with discrete variables arise naturally in many engineering problems (Dua et al., 1999, Floudas, 1995). The general formulation of multi-parametric mixed-integer Nonlinear Programming (mp-MINLP) problems is written as follows

$$
\begin{aligned}
z(\theta) = \min_{x,y} \; & f(x,y,\theta) \\
\text{s.t.} \quad & h(x,y,\theta) = 0 \\
& g(x,y,\theta) \le 0 \qquad (6) \\
& x \in X \subseteq \mathbb{R}^n,\; y \in \{0,1\}^q \\
& \theta \in \Theta \subseteq \mathbb{R}^p
\end{aligned}
$$

where x is the vector of continuous variables, y is the vector of binary variables and θ the vector of parameters. The general mp-MINLP problem has not yet been treated due to its complex nature. However, an important case of the above problem is the multi-parametric mixed-integer Linear Programming (mp-MILP) problem with parameters/uncertainties in both the objective functions and constraints, which is given as follows:

$$
\begin{aligned}
z(\theta) = \min_{x,y} \; & (c+H\theta)^T x + (d+L\theta)^T y \\
\text{s.t.} \quad & A(\theta)x + E(\theta)y \le b + F\theta \\
& x \in \mathbb{R}^n,\; y \in \{0,1\}^q,\; \theta \in \Theta \subseteq \mathbb{R}^p \qquad (7) \\
& A(\theta) = A^N + \sum_{j=1}^{q} \theta_j A^j \\
& E(\theta) = E^N + \sum_{j=1}^{q} \theta_j E^j
\end{aligned}
$$

The mp-MILP problem with uncertainties in both the objective function and the left-hand (LHS) and right-hand side (RHS) of the inequalities, arise in many applications such as planning/scheduling problems, hybrid control and process synthesis (Faisca et al, 2009). The presence of uncertainties in both the objective and the constraints significantly increases the complexity and computational effort of deriving explicit solutions to (4).

The problem with no LHS uncertainties (i.e. in $A(\theta)$) was first studied in Faisca et al. 2009 and an algorithm was proposed which alternates between solving a Master MINLP problem (to obtain the binary variables) and a Slave mp-NLP problem (to obtain the explicit solution). Although the Master problem is solved to global optimality as a binary variable has to be calculated, it was shown that global optimization of the slave problem can be avoided and it can be solved as a simple mp-LP problem. Only recently a method for solving the general mp-MILP problem (4) was presented in Wittmann and Pistikopoulos (2011), which relies on a two-stage method described as follows. In the first stage of the method, following robust optimization methods (Lin et al. 2004), the general mp-MILP problem is transformed into its Robust Counterpart (RC) problem by considering (4) as a robust mp-MILP problem where θ is the uncertainty. Then, by replacing the linear inequalities in (4) with the following

$$
\begin{aligned}
& [a_i^N]^T x + [e_i^N]^T y + \sum_{j=1}^{q} \left(\theta_j^N \left([a_i^j]^T x + [e_i^j]^T y \right) + r_j u_i^j \right) \\
& \qquad \le b_i + [f_i]^T \theta \qquad (8) \\
& -u_i^j \le [a_i^j]^T x + [e_i^j]^T y \le u_i^j
\end{aligned}
$$

where $r_j = (\theta_j^{max} - \theta_j^{min})/2$ and θ_j^N are the range and nominal value of θ_j thus removing the parameters from the LHS of the inequalities and transforming (4) into an mp-MILP problem with uncertainties only in the objective and the RHS of the constraints. Then, in the second stage the method of Faisca et al. (2009) is used to obtain the explicit solution. The advantages of the proposed method is that by taking the RC of the mp-MILP problem and removing the LHS uncertainties no additional global optimization procedures are required, which is computationally less expensive than the original problem.

Robust Explicit/multi-parametric MPC

There is an undisputable need for robust explicit MPC controllers for dynamic systems with bounded disturbances and model uncertainties. Explicit MPC controllers designed with nominal dynamic models (see equation 1 in section 1) cannot guarantee feasibility, in terms of constraint satisfaction, performance (in terms of optimality) and stability when disturbances and/or model uncertainties are present. The challenge here is the development of methods and algorithmic tools for the design of robust explicit/multi-parametric controllers which can guarantee constraint satisfaction and system stability for any value of the disturbances/uncertainties. Previous research efforts have mainly focused on the design of robust explicit MPC controllers for linear discrete-time dynamic systems with additive disturbances in the state equation of the linear model and uncertainties in the system matrices.

The formulation of robust explicit MPC with model uncertainties is given as follows:

$$
\begin{aligned}
V^*(x) = \min_{\mathrm{U}} \; & \sum_{k=0}^{N-1} \left\{ x_k^T Q x_k + u_k^T R u_k \right\} + x_N^T P x_N \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + Ed,\; x_0 = x,\; A = A_0 + \Delta A,\; B = B_0 + \Delta B \qquad (9) \\
& -\varepsilon_a |A_0| \le \Delta A \le \varepsilon_a |A_0|,\; -\varepsilon_\beta |B_0| \le \Delta B \le \varepsilon_\beta |B_0| \\
& Cx_k \le d,\; k = 0,\ldots,N \\
& Mu_k \le \mu,\; k = 0,\ldots,N-1
\end{aligned}
$$

where the system matrices A, B are uncertain in that they are given as the sum of a nominal value A_0 and an uncertain value ΔA. The objective is to satisfy the constraints in (9) for all values of ΔA. A framework for the design of robust explicit MPC controllers was presented recently in Pistikopoulos et al., 2009 and in Kouramas et al., 2012. This framework features three key steps: i) a dynamic programming step, in which the MPC optimization (9) is reformulated in a multi-stage optimization setting, ii) a robust reformulation step in which the constraints of each stage (of the multi-stage optimization) are reformulated to account for the worst-effect of the uncertainty and iii) a multi-parametric programming step, in which each stage is solved as a multi-parametric Quadratic Programming problem to derive the control variables as explicit functions of the states.

Simultaneous design of moving horizon estimation and explicit MPC

In explicit/multi-parametric MPC and in general in any case of feedback controllers, the implementation of the controller relies on the assumption that the state and disturbance values are readily available from the system measurements. However, in many real applications the state values cannot be measured directly from the system and need to be obtained from the system output measurements with state estimation techniques (Rawlings and Mayne, 2009). Traditionally, in the case that no system constraints were considered in the problem, the controller and the estimator can be designed separately and then implemented together, following the separation principle (see Figure 7). However, this is not the case when constraints are considered in the problem, since the estimation error introduced in the system can significantly degrade system stability and result in constraint violations. As it is shown in Figure 7, if $e_x = x - \hat{x}$ is the estimation error, x is the real state and $\hat{x}$ is the state estimate, then the system state constraint $D(\hat{x}+e_x) \le d_x$ might get violated when error variations occur.

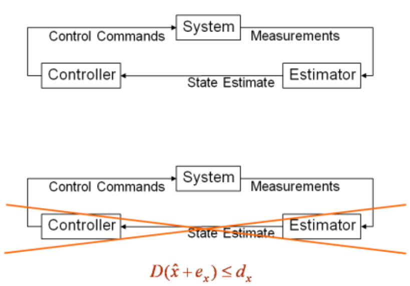

Figure 7. State estimation and explicit/multi-parametric MPC

In order to avoid this issues simultaneous methods for the design of estimation and MPC have been developed, based on robust output-feedback tube MPC methods (see Mayne et al., 2006, Sui et al. 2008, Rawlings and Mayne 2009). The main idea in this approach is to obtain the dynamic model equations and the bounding set of the estimation error as well as the state estimate dynamics, and use them to 'robustify' the systems constraints in the MPC formulation to avoid any constraint violations. This approach has been successfully applied only on MPC problems with unconstrained estimators such as Luenberger observers and unconstrained Moving Horizon Estimators (MHE). Nevertheless, the general and more difficult case of constrained Moving Horizon Estimation (MHE) was not fully treated until only recently. In the work of Voelker et al., 2010 and Voelker et al., 2011 a method was presented for the simultaneous design of the general case of constrained MHE and explicit MPC based on multi-parametric programming. In this method the optimization problem involved in the constrained MHE problem is solved using multi-parametric programming techniques to derive the estimation error dynamics and bounds for the constrained MHE. A robust output-feedback tube MPC methods was then established, by incorporating the estimation error dynamics and bounds, that ensures that the system constraints are not violated and stability is preserved.

Explicit nonlinear MPC

In explicit nonlinear MPC (or mp-NMPC) the optimization problem of the underlying MPC formulation is a mp-NLP problem. For this reason the developments on mp-NMPC controllers has always relied on the developments in mp-NLP. the work in mp-NMPC mainly focuses on the development of algorithms for the approximation of the solution of the mp-NLP problem involved in the MPC formulation. Johansen (2004) presented an algorithm for obtaining an mp-QP - based approximation for obtaining the control variables as piecewise affine functions of the state. Sakizlis et al. (2005) presented a method for reformulating and solving the mp-NMPC optimization problem with multi-parametric global optimization techniques. Additionally Sakizlis et al. (2005) presented a multi-parametric dynamic optimization approach for continuous-time mp-NMPC, which is based on deriving the exact solution of the continuous-time control variables as functions of the time and state variables. Recently, Dominguez and Pistikopoulos (2011) presented an algorithm for mp-NMPC based on their algorithm for multi-parametric quadratic approximations (see previous section). In this method the state-space is partitioned into polyhedral sets (hyper-cubes) the union of which represents and approximation of the feasible set of states. In each hypercube an approximating mp-QP problem is solved to obtain a local approximating explicit solution of the inputs as linear piecewise functions of the state.

MPC-on-a-chip multi-scale applications

The significant advances in multi-parametric programming and explicit/multi-parametric MPC have been followed by a number of important applications. Many of these applications are ideal for the use of explicit/multi-parametric MPC controllers and the MPC-on-a-chip concept, since they involve real, complex processes with limited available control hardware for advanced control applications. A detailed report of these applications is given in Table 3. In this section we overview two important applications of explicit MPC design on fuel cells and Pressure Swing Adsorption (PSA) systems for hydrogen separation.

Fuel cell systems

An important application for explicit/multi-parametric MPC is for the control of fuel cells. Fuel cells such as the Proton Exchange Membrane (PEM) fuel cells are popular alternatives for electrical power generation and especially for portable and automotive applications (Arce et al., 2011, Panos et al., 2011) where the control hardware cannot accommodate demanding computations. Hence, PEM fuel cell systems are a suitable application to which explicit MPC and the MPC-on-a-chip concept can bring distinct benefits. The control of PEM fuel cell systems have in general attracted a lot of attention in the relevant literature first due to their importance for mobile applications but also due to the challenging nonlinear dynamics which are mainly due to the complex electrochemical reactions and membrane characteristics.

A typical PEM fuel cell system is shown in Figure 8 and consists of the fuel cell stack, a compressor for the recirculation of hydrogen to the fuel cell and a cooling system for controlling the temperature of the stack. One objective here is to control the electrical power or voltage produced by the fuel cell by manipulating the fuel (hydrogen) and oxygen flowrates in the stack. An additional but very important objective is to maintain the stack temperature in a certain range of values in order to ensure efficient operation of the fuel cell and avoid damaging the stack. Despite being a widespread application for implementing and testing control algorithms, the explicit/multi-parametric MPC of PEM fuel cells has only recently attracted some attention.

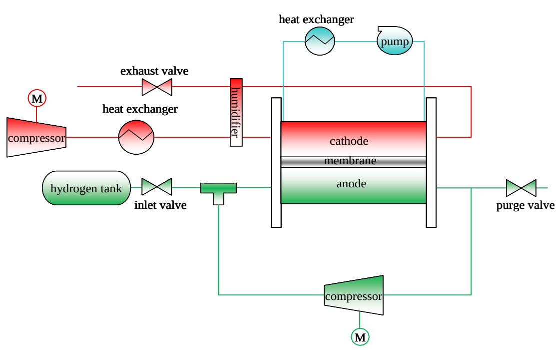

Figure 8. PEM Fuel Cell System

The application of multi-parametric programming and explicit MPC for fuel cells was performed by Arce et al., 2009, where an explicit MPC controller was designed for the control of the excess oxygen ratio in a real fuel cell: the objective here is to maintain the excess oxygen ratio above a critical level in order to maintain a safe and efficient operation. This work was further extended in Arce et al., 2010 and an explicit controller was designed for controlling the temperature of the PEM fuel cell stack by manipulating the voltage of the fun of the cooling system, in the presence of current disturbances. In Panos et al., 2010 and Panos et al., 2011, an explicit MPC controller was designed for the simultaneous control of the fuel cell voltage and temperature. In this work instead of designing two different controllers to control the voltage and temperature separately, a single multi-variable controller was designed. Finally, in Ziogou et al., 2011 an explicit MPC controller was designed for a PEM fuel cell system for the power, excess oxygen and hydrogen ratio and temperature control of the fuel cell stack.

All the above applications involved real PEM fuel cell systems which are based in University of Seville, Spain (Arce et al., 2009, Arce et al., 2010), Imperial College London, UK (Panos et al, 2011) and the Chemical Process Engineering Research Institute (CPERI), Greece (Ziogou et al., 2011). Figure 9 shows the process instrumentation diagram of the experimental PEM fuel cell system in Imperial College London. In conclusion, the results in the above applications showed that explicit/multi-parametric programming is a promising control technology for this type of small applications.

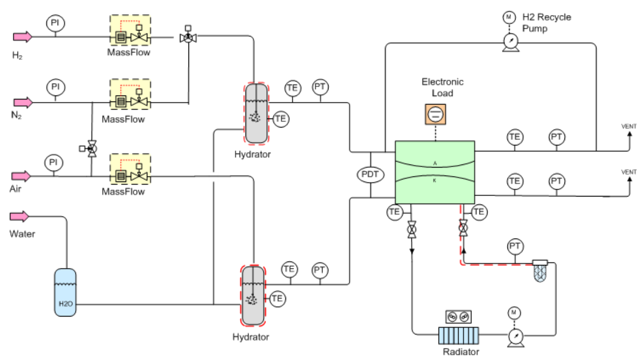

Figure 9. Process Instrumentation Diagram for PEM Fuel Cell System

PSA systems

Another application of explicit/multi-parametric MPC is in the control of Pressure Swing Adsorption (PSA) systems. Pressure swing adsorption (PSA) is a flexible, albeit complex gas separation system. PSA operation is not only highly nonlinear and dynamic but also poses extra challenges due to its periodic nature; directly attributed to the network of bed interconnecting valves whose active status keeps changing over time (Figure 10). The timing of these valves in turn control the duration of process steps that each PSA bed undergoes in one cycle. In the past, only a few studies have appeared in the open literature on PSA control even though there is an increasing interest to improve their operability Due to its inherent nonlinear nature and discontinuous operation, the design of a model based PSA controller, especially with varying operating conditions, is a challenging task and only a few studies have appeared in the literature on PSA control even though there is an increasing interest to improve their operability (Khajuria and Pistikopoulos, 2011). In Khajuria and Pistikopoulos (2011) an explicit multi-parametric MPC controller was designed for the first time, for the control of hydrogen purity produced in a PSA unit. In this section we overview this application and we present its key findings.

The PSA system considered in this study consists of four beds containing activated carbon as an adsorbent. Each of the four beds undergoes a cyclic operation that comprises nine process steps as shown in Figure 10, separating a mixture of 70% Hydrogen H_2 and 30% CH_4 into high purity H_2. The control objective is to maintain the hydrogen purity (controlled variable) at 99.99% by varying the adsorption time (manipulated variable). The framework for multi-parametric programming and explicit MPC was applied for the PSA problem and its steps are shown next. In step 1 a first-principles mathematical model of the PSA system is obtained, which consists of Partial Differential Algebraic Equations (PDAEs). In step 2 simulations studies are performed to evaluate the performance of the PSA and obtain input-output data for performing system identification to obtain a reduced-order linear state-space system for the explicit MPC design. In step 3 an explicit MPC controller was designed based on multi-parametric Quadratic Programming methods (Pistikopoulos et al., 2007a). The optimal profile and its critical regions for the explicit MPC controller is shown in Figure 11. Finally the controller was evaluated by directly implementing it on the high-fidelity model derived in the first step and performing simulation for a number of operating conditions such as nominal operating conditions and operation under disturbances in the PSA feed. The explicit controller performance is then compared to two PID controllers (Figure 12). It is easy to see that the PID1 violates the constraints on the adsorption time (denoted by the bold black line in Figure 12) despite its rapid steady-state behavior. Further retuning, represented by PID2, improves its behavior but keeps the adsorption time above 55s. The explicit MPC controller provides much superior performance as it achieves almost the same controller response time at much lower values of mean control effort. Fig. 12 further shows that the adsorption time trajectory during the response time is far away from the defined constraints, without requiring any re-tuning efforts. Therefore, it is shown that explicit/multi-parametric MPC is also a promising technology for medium scale processes such as the PSA system.

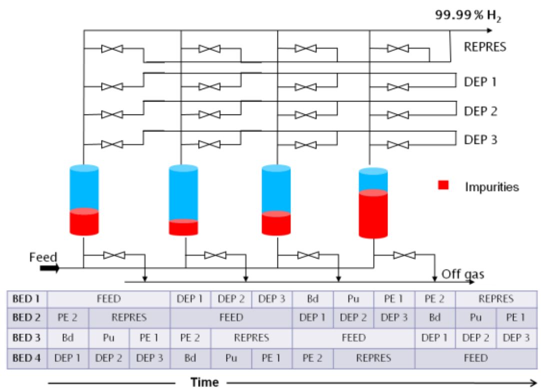

Figure 10. PSA system and operation

Other applications

Other recent applications of explicit MPC include the guidance and control of unmanned air vehicles and control of biomedical systems such as insulin delivery for type 1 diabetes, anesthesia and chemotherapeutical agents. Both applications demonstrate the importance and potential of the MPC-on-a-chip concept. In the case of unmanned air vehicles (UAVs) the objective is to navigate the aerial vehicle through a predetermined flight path in the presence of wind while ensuring that important constraints on fuel , throttle and control surfaces movement are not violated (Voelker et al. 2009). Another challenge in this area is state and disturbance estimation since the measurements of many state variables and of the wind are not always available (Voelker et al., 2011). Biomedical systems and devices is also another area for which the MPC-on-a-chip concept can bring significant benefits. The main objective here is to derive explicit MPC controllers for the delivery of insulin for type 1 diabetes, anesthesia and chemotherapeutical agents. This has been the subject of the MOBILE project which is an ERC Advanced Grant project (Modeling, Control and Optimization of Biomedical Systems, ERC Advanced Grant, 2009-2013). The ability to derive off-line and in an explicit form the control performance of the device (for the delivery of the insulin, anesthesia or the chemotherapeutical agents), based on the simulated patient characteristics, allows for increased safety and understanding of the preclinical and clinical testing in the approval and implementation stages (Pefani et. al 2011, Zavitsanou et. al 2011, Krieger A. et al. 2011).

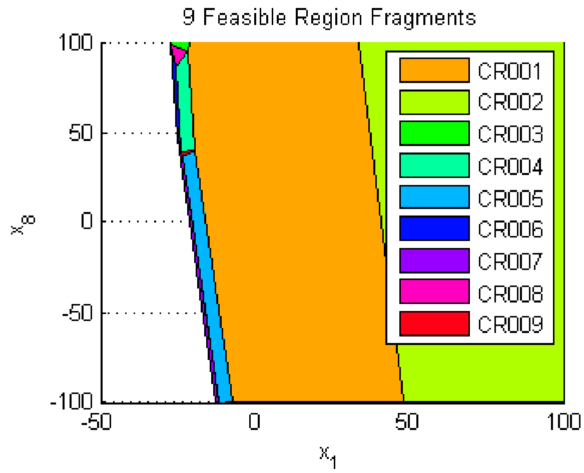

Figure 11. Critical regions of the explicit MPC controller for the PSA system.

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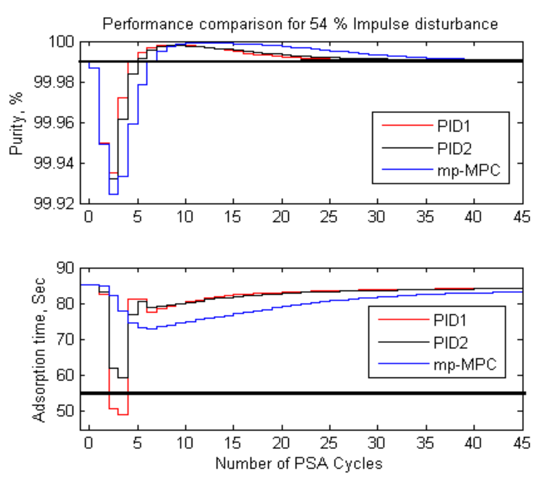

Figure 12. Closed-loop implementation of explicit MPC and comparison with PID for a 54% disturbance in the PSA feed.

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