# Evolution of an Industrial Nonlinear Model Predictive Controller

Robert E. Young, R. Donald Bartusiak and Robert W. Fontaine Engineering and Manufacturing Services Department ExxonMobil Chemical Company Baytown, Texas 77520

#### Abstract

Motivated by a specific manufacturing problem in 1990, Exxon Chemical Company embarked on the development of a nonlinear multivariable model-based predictive controller. The controller's evolution has included collaboration among academic researchers, engineers from industry, and process control software vendors. The resulting control algorithm was patented by Exxon Chemical Company and commercialized by Dynamic Optimization Technology Products, Inc. At the same time, several other academic interactions produced results supporting the implemention of these controllers in our manufacturing facilities. This paper chronicles the evolution of the controller development, and presents the details of the control algorithm. The control algorithm features are discussed, and where applicable, compared to other model predictive control (MPC) algorithms. Finally, two industrial examples are presented to illustrate the methodology.

#### Keywords

Model-based control, Predictive control, Nonlinear control, Industrial control

### Introduction

For the past 10 years, Exxon Chemical Company has pursued development of a methodology to address industrial process control challenges characterized by nonlinear process responses. Motivated by problems in manufacturing plants, the evolution of this nonlinear control methodology has drawn on the expertise and experience of practitioners (both from manufacturing sites and technology organizations), of academic researchers, and of process control vendors.

Over these same 10 years, many changes have occurred within our company, the process control industry, and to the process control systems technology. Academic research has significantly increased our understanding of MPC, especially stability of linear MPC algorithms. Computer science and optimization technologies have improved vendor's packages, making industrial implementation easier and more effective. Through all of these changes, the motivation and development of this algorithm have persisted. While by no means complete, the evolution of this control algorithm is an interesting story about how diversely motivated groups of people can interact to produce a tool capable of solving commercially relevant and intellectually challenging process control problems.

# **Motivating Problem**

In 1990, Exxon Chemical Company started up a new polymerization plant using a new catalyst system. While the details of the process are proprietary, the process involved a single reactor vessel with a simple monomer recovery/compression recycle. The product is a copolymer composed of two monomers. The control objective is to control particular polymer resin properties, specifically polymer melt viscosity and polymer density. In

this case, these variables are controlled by manipulating reactor feed temperature and feed composition. The reactor pressure, feed flow rate and feed temperature are measured disturbance variables. None of the in-reactor compositions are measured. When compared to similar plants, the control of this unit was unable to achieve the expected prime or "right-first-time" production.

Polymerization processes have been considered challenging process control problems for many years (Ray, 1986). In these processes, the goal is to control polymer product properties, such as polymer melt viscosity and comonomer incorporation, as well as manufacturing targets such as production rate and slurry concentration. Reaction temperature has very significant effects on reaction rates, and hence, both the polymer properties and the process operability. Typically, these variables are controlled to targets by manipulating the feed rate and composition, catalyst feed rates, and reactor cooling. Often, regulatory control of polymerization reactor is achieved with a combination of PID feedback/feedforward and ratio controllers (Congalidis et al., 1989). These control schemes are often adequate for regulatory control because the process is *linear* enough near the operating point that more sophistication is not warranted. This observation continues to be true for many industrial polymer processes operating today.

However, the apparent gains and time constants between the control variables and manipulated variables often exhibit significant nonlinear behavior when a polymer plant makes different grades of polymer. Often, the simple regulatory control schemes must be tuned at each operating condition to achieve good control over the entire operating window of given plant. To maximize prime production, manufacturing planning attempts set schedules with every adjacent grade having overlapping specifications with the previous grade. Also, the process nonlinearity must not be severe enough to cause significant overshoot. Often, planning can not achieve either of these objectives, resulting in off-prime polymer production.

The transition control problem has been examined by both academic researchers and industrial practitioners (McAuley and MacGregor, 1992; Debling et al., 1994). Gain scheduling or multiple-model controller designs have been suggested approaches to solving this problem. However, these approaches require a model or data for each grade and, need an algorithm to decide when to switch. The downside to either approach is the significant added cost of controller maintenance. Industrially, a nonlinear predictive controller designed to execute polypropylene reactor transitions was successfully implemented (Hillestad and Andersen, 1994). This controller design is characteristic of linear MPC applications with the exceptions that it employs a nonlinear model and includes a state estimator.

Finally, polymerization processes are very susceptible to changes in unmeasured disturbances such as very small concentrations of polymerization poisons in the feed or catalyst activation changes for any variety of reasons. A recent study of a polymerization process demonstrates that linear MPC can not achieve acceptable controller performance when faced with typical industrial disturbance signals (Bindlish and Rawlings, 2000).

In the case of the new Exxon Chemical facility, the combination of the process/equipment design and new catalyst chemistry resulted in a highly-interactive nonlinear process. The nonlinear effects of both measured and unmeasured disturbances could not be rejected by the state-of-the art control technology used on similar reactor systems. In short, both the regulatory control and the transition performance was limited by the nonlinear behavior of the process to either servo or load changes.

## Early Controller and Model Development

During this same time frame, multivariable model predictive control based on identified linear process models (Cutler and Ramaker, 1980; Richalet et al., 1978) was being used to solve significant industrial control problems. At the time, nonlinear control was already a strong area of academic research and significant effort had been made to develop nonlinear MPC algorithms (Bequette, 1991). The program at CPC IV (Arkun and Ray, 1991) contained several presentations on both topics indicating that both industry and academia had already recognized the importance of both technologies.

After careful examination of the motivating polymerization control problem, the Exxon Chemical process control technology organization determined that using linear MPC could not yield the desired process performance. Even given the significant academic research, there was no commercially available software to bring nonlinear MPC technology to bear on the polymerization problem.

### Early Academic Collaboration

Specifically focusing on how to model and control the nonlinear behavior of the process, Exxon Chemical Company elected to collaborate with academic researchers. A request for competitive bids was issued and awarded for two specific projects. Both of these projects were contractual agreements with specific milestone dates, deliverables, and non-disclosure agreements.

First, to focus on understanding the process, they contracted the University of Maryland to develop nonlinear models of the polymerization reactor. As a result of this effort, Professor K. Y. Choi and co-workers developed a fundamental model of the reaction process and estimated kinetic parameters for this model from pilot-plant data. This process model is composed of the dynamic mass and energy balances that describe the polymer reaction system. The polymer population balances were condensed through the use of moments (Ray, 1972) after applying the quasi-steady-state assumption to the growing polymer chains. The model is very similar to others found the open literature (McAuley et al., 1990; Zacca and Ray, 1993; Ogunnaike, 1994) The fundamental model was combined with empirical correlations to relate the polymer moments to polymer resin properties.

Also, Georgia Institute of Technology was contracted to investigate and develop a nonlinear state estimator and multivariable predictive controller to be used to control the polymerization reactor. This work was conducted by Professor Yaman Arkun, Professor Joseph Schork and their co-workers. The state estimation work revolved around the implementation of different Kalman filter and Luenberger observer designs. The controller algorithm developed was a quasi-linearized QDMC algorithm (Peterson et al., 1992; Charos and Arkun, 1993; Sriniwas et al., 1995). The controller used the nonlinear model to predict process trajectories and to compute disturbance estimates. During the final stages of this contract work, the observer/NLMPC algorithms were tested using the fundamental model developed at the University of Maryland.

Both of these programs were two year contracts, successfully meeting all of the expectations set out at the beginning. However, as is often the case, these initial investigations were most successful at providing a more detailed specification about how Exxon Chemical wanted to address both the specific polymerization control problem and the general nonlinear control problem.

#### Internal Development Program

In the first quarter of 1993, the Exxon Chemical project team evaluated the results of the two academic contracts and defined an internal development project. This project was focused on both the specific polymerization reactor control problem and the development of a nonlinear MPC (NLC) structure for use within Exxon Chemical Company.

The earliest milestones involved evaluation and on-line implementation of the modeling equations. The predictions made using the academic model did not match the plant data well. After some analysis, the model structure was modified and new parameters estimated from plant data. After these two changes, the model tracking errors were significantly reduced.

During the modeling phase of the development, work was underway formulating the controller. Characteristic of the NLMPC developed at Georgia Institute of Technology, any nonlinear controlled variable (CV) dynamics were retained in the closed-loop response of the controller. In other words, the closed loop CV response was still a function of the operating point. Illustrating this behavior is easiest with an example. Consider the following nonlinear single-input single-output(SISO) process:

$$400x^3\frac{dx}{dt} = -x + 0.7(1-x)e^{\frac{-1}{u}} \tag{1}$$

This contrived example is motivated by a component mass balance, a single irreversible reaction, and a concentration dependent density. The parameters have no special meaning but were selected to provided a simple and illustrative example. Figure 1 is the response of this system of two setpoint changes—from 0.25 to 0.30 and from 0.30 to 0.35. The process is controlled using a linearized MPC controller as implemented in the Mathworks Model Predictive Control Toolbox (Morari and Ricker, 1995). The controller parameters specified for these simulations are the prediction horizon equal to the control horizon set to 25, the output weight (ywt) equal to one and the input weight (uwt) equal to 0.04. The manipulated variable (MV) or input is constrained to be greater than 0.05 and less than 10.0. The process model used by the controller is a linearized model of Equation 1 around the initial operating point for each setpoint change respectively.

Without dwelling on the tuning of the controller, the observation made by the development team is easily observed in this example (see Figure 1). The dynamics of the nonlinear process are not compensated for by the controller and appear in the closed-loop response. This observation is not surprising for MPC algorithms that use move suppression as their primary tuning mechanism. Move suppression allows the engineer to indicate how much input energy can be used. Excessive use of MV moves is penalized, sacrificing CV response. This specification amounts to stating that the amount of allowable change in the process inputs does not depend on the operating conditions. In the case of the polymerization problem, this is equivalent to stating that for some products, a slow transition response is acceptable even though a faster response is achievable. Re-tuning the controller for each operating point or, perhaps, parame-



Figure 1: Quasi-linearized MPC simulation responses.

terizing the move suppression factor as a function of the operating point, may provide a way to circumvent this issue.

However, to the credit of the development team, the philosophy driving this effort was to avoid the inclusion of techniques that add to the life-cycle cost of the controller. Techniques like this include the use of:

- multiple linear models to approximate nonlinear models,
- linearization at each sample time to approximate nonlinear models,
- gain or tuning scheduling as a function of operating point.

Each of these techniques requires additional overhead in both the development and the maintenance of the controller. If the controller can be designed and implemented directly, this additional cost can be avoided and the controller is more likely to remain on-line.

To address the nonlinear CV dynamics, a reference system performance specification was added to the NLC design. Reference system synthesis (Bartusiak et al., 1989) is one of a class of differential geometric methods used for nonlinear control design (Lee and Sullivan, 1988; McLellan et al., 1988; Kravaris and Kantor, 1990a,b). Reference system synthesis employs a performance specification on the error trajectory to design a nonlinear control law. In a non-predictive form, this design methodology was studied as way to control product properties in gas-phase polyethylene reactors (McAuley and MacGregor, 1993).

In this development, a reference system specification is added as a soft constraint to a model predictive controller. The resulting control algorithm is tuned by specifying the desired process error trajectory for each CV and the relative weight for each of the CVs. Figure 2



Figure 2: NLC simulation responses.

presents the result of applying this control algorithm to the nonlinear SISO example (Equation 1) for the same two setpoint changes.

The reference system specified describes a secondorder over-damped response with a dominant time constant of 5. The prediction horizon and control horizon are set to 25, the same in the previous simulation. In the unconstrained case, both CV responses are identical and equal to the setpoint change from 0.25 to 0.30. However, the simulation shows the effect of the high MV constraint of 10 on the setpoint change from 0.30 to 0.35(note: u(x=0.30)=2.048, so  $\Delta u = 7.952$  at t = 0 is on the constraint). The quasi-linearized MPC move suppression factor, uwt = 0.04, was selected to generate a MV response peak value for the setpoint change from 0.25 to 0.30 nearly equal to the NOVA Nonlinear Controller (NLC) peak value. Without reducing this move suppression factor, the quasi-linearized MPC controller will not make use of the available input energy to achieve the desired performance in the second setpoint change.

Besides the addition of the reference system tuning, the project team chose a controller design based on a simultaneous optimization/solution algorithm. The implementation was augmented with the necessary code to initialize the controller. With these modification, the project team completed the controller in early 1994 and completed closed-loop testing by year end 1994. All of the original performance issues that motivated the work beginning in 1990 were addressed. Regulatory control achieved performance on par with similar processes. Transition times were reduced by at least a factor of 2 and were no longer limited by controller performance.

## The Nonlinear Control (NLC) Algorithm

The inventors of the control methodology, Fontaine and Bartusiak, were granted a patent (Bartusiak and Fontaine, 1997). The methodology was later commercialized by Dynamic Optimization Technology (DOT) Products and is called NOVA Nonlinear Controller (NOVA NLC, 1997). The algorithm is a nonlinear program (NLP) optimization problem with a multi-objective cost function. The optimization problem is solved using the NOVA DAE solver over a finite time horizon. The solver uses orthogonal collocation to discretize the equations in time. The mathematical formulation of the controller is as follows:

$$\min_{u_{MV}} \Phi = \mu_1 J_1(e) + \mu_2 J_2(y, x, u) + \mu_3 J_3(\Delta u_{MV}) \quad (2)$$

subject to

$$0 = f(y, \dot{x}, x, u, \theta) \tag{3}$$

$$0 = g(y, \dot{x}, x, u, \theta) \tag{4}$$

$$x(0) = x_0$$
 and  $y(0) = y_0$ 

$$0 = \frac{\tau_i}{4\xi_i^2} \frac{d^2 y_{cv_i}}{dt^2} + \frac{dy_{cv_i}}{dt} + \frac{1}{\tau_i} \left( y_{cv_i} - y_i^{sphi} \right) - e_i^{sphi} + s_i^{sphi}$$
(5)

$$0 = \frac{\tau_i}{4\xi_i^2} \frac{d^2 y_{cv_i}}{dt^2} + \frac{dy_{cv_i}}{dt} + \frac{1}{\tau_i} \left( y_{cv_i} - y_i^{splo} \right) - e_i^{splo} + s_i^{splo}$$
(6)

$$e^{sphi}_i, e^{splo}_i, s^{sphi}_i, s^{splo}_i \geq 0$$

$$u_{mv_j}^{LB} \le u_{mv_{j,k}} \le u_{mv_j}^{UB} \tag{7}$$

$$|u_{j,k} - u_{j,k-1}| \le \Delta u^B_{mv_j} \tag{8}$$

The objective function (2) is composed of three components.  $J_1$  is the cost associated with the dynamic response of the closed-loop system.  $J_2$  is the economic cost associated with each of the output and input variables. Finally,  $J_3$  is the cost of moving each of the individual manipulated input variables. Each of these components is weighted by the  $\mu$  weights in the equation.

Equations 3 to 4 define the nonlinear process model where y are the outputs, x are the states, u are the inputs and  $\theta$  are parameters. The set of outputs, y, is composed of controlled variables,  $y_c v$ , measured outputs, and auxiliary outputs. The set of inputs u, is composed of manipulated variables,  $u_{mv}$ , feed forward variables,  $u_{ff}$ , and disturbance variables,  $u_d$ . The initial conditions on both the outputs and the states are specified.

Equations 5 and 6 describe the reference system performance equations for each of the controlled variables. Changing the tuning parameters  $\tau_i$  and  $\xi_i$ , changes the reference system specifying the desired closed-loop performance for that variable. The errors,  $e_i^{sphi}$  and  $e_i^{splo}$ , are the absolute deviation between the reference system trajectories and the predicted response trajectories. These equations are flexible enough to support a single target setpoint or a setpoint high/low window.

Equations 7 and 8 specify constraints on manipulated input variable values and changes.

In addition to the upper and lower bounds on  $u_{mv}$  and the bound on  $\Delta u_{mv}$ , the user has to specify the prediction horizon, the control horizon, the sampling period, the weights that rank the controlled variable errors (both negative and positive), the costs of any variables in  $J_2$ , the cost of each  $\Delta u_{mv}$ , and finally the relative weight of each of the three objective function components.

Also worthy of note, the implementation of this controller in the NOVA DAE system does not categorize variables this same way. This presentation of the variables focuses on variables from a control perspective. In the NOVA DAE implementation, variables are classified in a much more mathematical way. Specifically, variables are either integrated or non-integrated variables. Integrated variables are generally the dependent variables (outputs and states) while the non-integrated variables are typically the independent variables (inputs and parameters). The specification of both the initial state and output variables in the model is a result of this mathematical view of the problem.

#### **Objective Function Details**

The NLP objective function is a weighted composite of three cost functions. The first component is a measure of the cost to get the closed-loop system to target. This cost is computed by

$$J_1(e) = \frac{1}{n_p} \sum_{i=1}^{n_{cv}} \sum_{k=1}^{n_p} w_i^{sphi} e_{ik}^{sphi} + w_i^{splo} e_{ik}^{splo}$$
(9)

where  $n_p$  is the length of the prediction horizon,  $n_{cv}$  is the number of controlled output variables. The errors are computed at those times corresponding to knots in the collocation grid. This discrete sampling structure also applies to the other components of the objective function,  $J_2$  and  $J_3$ .

The economic cost of the outputs, states, and inputs make the  $J_2$  component of the objective function as follows:

$$J_2(y, x, u) = \frac{1}{n_p} \left( C_y + C_x + C_u \right)$$
(10)

where

$$C_y = \sum_{i=1}^{n_y} \sum_{k=1}^{n_p} c_{y_i} y_{i,k}$$
(11)

$$C_x = \sum_{m=1}^{n_x} \sum_{k=1}^{n_p} c_{x_m} x_{m,k}$$
(12)

$$C_u = \sum_{j=1}^{n_u} \sum_{k=1}^{n_p} c_{u_j} u_{j,k}$$
(13)

Note that any output, input or state can be included in the evaluation of this cost function.

Finally, the last term of the objective function penalizes manipulated variable moves as follows:

$$J_3(\Delta u_{mv}) = \sum_{j=1}^{n_u} \sum_{k=1}^{n_q} \lambda_j \Delta u_{mv_{j,k}}$$
(14)

This term in the objective function serves to ensure a unique solution of the NLP for nonzero weights. Also, this term is much like a move-suppression term in a DMC-type MPC controller. In an unconstrained DMC controller, move-suppression alters the performance of the controller by altering the singular values of the dynamic matrix. The dynamic matrix is the mapping of the control moves to future prediction errors. If this matrix is nearly singular, the inverse mapping will generate large control moves for small predicted errors. Increasing the move suppression factors serves to stabilize this inverse mapping, making it more robust to small errors. Increasing the move suppression factors even further serves to de-tune the controller. The penalty on  $\Delta u_{mv_{j,k}}$  has the same effect on this controller.

### Model Specification

The model described by equations 3 through 4 define a very general structure. The model equations must be compliant with the NOVA DAE format—a continuoustime open-equation residual form. Our experience has been to use fundamental models based on mass and energy balances coupled with empirical correlations and algebraic relationships. Many of the non-empirical nonlinear algebraic relationships are derived from the applicable kinetic, transport, and thermodynamic relationships. Besides supporting these types of models, this framework can easily support algebraic empirical models, neural network models and linear state-space models. Because the DAE system expects to use collocation, using neural network or discrete linear/nonlinear models will require some additional effort by the user. Typically, the empirical correlations used relate product properties to model states and/or other outputs.

Most of these models include parameters that must be specified. The software currently includes that capability to estimate these parameters from steady-state data. The parameter estimation case is defined as a leastsquares fit of the parameters subject to the model and the steady-state data. In practice, getting sufficient data to estimate all parameters is difficult. There is still significant art in estimating parameters for these models.

Fundamental models have provided several advantages over models identified from process data. The fundamental models have extrapolated well to new operating conditions. When the process design changes, these models can be changed more easily than equivalent models identified from process data. This comparison is not necessarily true if the process chemistry changes because of the parameter estimation issues discussed above.

#### **Process Feedback and Initialization**

Since the user is responsible for writing all of the model equations, it is possible to incorporate process feedback in several different ways. The base implementation has a default state correction built into the algorithm. The state estimates at the previous sampling time (from the previous controller execution) are assumed to be known. The state estimates at the current time are computed from this "known" starting point by integrating forward with the measured or known MVs. As implemented, this correction produces corrected state estimates when the actual inputs do not match the values computed at the previous execution.

Given this scheme of updating the state estimates, to start execution of the controller, "cold-start" estimates of the states must be computed to turn the control on. These "cold-start" estimates can be computed by solving the steady-state model equations before the first controller execution.

Output measurements can be used in a variety of ways to provide feedback. Probably the easiest and most common method is to compute a bias between the modeled value and the measured value. This bias can be filtered and used as a feedforward input into the model. This form of output feedback is similar to the approach used by linear MPC controllers.

As indicated in a review of commercial nonlinear MPC offerings (Qin and Badgwell, 2000), this controller provides state estimation through an extended Kalman filter(EKF). While technically correct, the model designer must incorporate the disturbance model into the model equations and augment these equations with the EKF equations. Alternatively, since the initial conditions can be imported into the problem, an external state estimation application can generate new initial state and disturbance estimates at each execution. There are many available methods to incorporate process feedback associated with nonlinear MPC (Muske and Edgar, 1997), however, the burden of implementing these options in the NLC methodology rests with the application/model developer.

### Nonlinear Controller Stability

There are no known stability results for the controller as defined by equations 2 through 8. However, controller stability was one of the most significant areas of academic research during the decade. The formulation of the linear constrained MPC problem by Rawlings and Muske (1993) opened up many new academic studies in the area, see (Lee and Cooley, 1996; Meadows and Rawlings, 1997; Mayne et al., 2000) for reviews. However, even now, very few of these results have found their way into commercially available linear MPC products. However, the academic stability research provides "comfort" to users of the NLC technology. The NLC objective function, equation 2, is very similar to a linear MPC objective function with soft constraints that has been shown to be nominally stable (Scokaert and Rawlings, 1999). Given these similarities, there is guarded optimism that careful selection of horizon lengths and objective function weights will result in stable closedloop behavior.

There is also concern about optimization algorithmic stability, specifically, will the solution algorithm converge every sampling time. Again, academic work in this area (Wright, 1996; Biegler, 2000) would indicate that NLP codes are improving and can be tailored to the MPC problem to improve the convergence properties. Finally, our experience with industrially-used codes for real-time optimization and other on-line applications has been good and would indicate that reliability of this solver will not impact the success of the controller.

#### **Controller Commercialization**

After successfully implementing the first generation of the in-house version of the control methodology, Exxon Chemical Company had to choose how best to deploy this technology to the rest of the organization and how to keep the controllers on-line. Several of the following factors were studied:

- the cost of maintaining the controller software,
- the competitive advantage gained by keeping the technology proprietary,
- the competitive advantage in the corporate capability to develop process and disturbance models for our processes.

This analysis led Exxon Chemical to commercialize the nonlinear control methodology. After the award of the patent in 1997, a vendor evaluation was conducted. Subsequently, a contract was awarded to DOT Products to develop a commercial version of the software.

The on-line version of this commercial product has been used for subsequent implementations. A PCbased configuration and tuning tool has been recently released to assist with the development of new controller implementations. The configuration tool allows the engineer to specify the controller parameters, and given a nonlinear model, build an off-line controller. This model/controller combination can be used to estimate parameters from steady-state data and to perform steady-state, dynamic, and interactive simulations of the controller. While there is significant room for improvement with the graphics and user interface, the configuration tool represents a significant step forward on the path to new implementations.



Figure 3: Blending drum schematic.

# **Blending Example**

To illustrate the controller, a small blending example is presented. In this example, three flows are blended into a horizontal tank as show in Figure 3.

Two of the flows,  $w_1$  and  $w_2$  are pure components 1 and 2, respectively. The third flow,  $w_D$  is a mixture of the third component and component 1. The weight fraction of component 1 in  $w_D$  is  $x_{11}$ . The controlled variables are the tank level (L), and the weight fraction of components 1 and 2  $(x_1 \text{ and } x_2)$  in the effluent flow, w. The manipulated variables are the three inlet flow rates. The system often sees disturbances in the effluent flow, w, and the weight fraction,  $x_{11}$ , both measured.

The model for this blending process is the overall mass balance and the two component mass balance for components 1 and 2 given by

$$M = \rho V \tag{15}$$

$$V = aL^3 + bL^2 + cL + d (16)$$

$$\frac{dV}{dL} = 3aL^2 + 2bL + c \tag{17}$$

$$0 = -\rho \frac{dV}{dL} \frac{dL}{dt} + w_D + w_1 + w_2 - w$$
(18)

$$0 = -\frac{d(x_1M)}{dt} + x_{11}w_D + w_1 - x_1w$$
(19)

$$0 = -\frac{d(x_2M)}{dt} + w_2 - x_2w \tag{20}$$

where M is the mass of the tank contents, V is the associated volume, L is the associated tank level, and the other variables as shown in Figure 3. The code to model this blending process for the NLC is less than one page. Except for the coefficients of the cubic polynomial relating level to volume, there are no parameters. The volume of the tank as a function of level is known from the vessel strapping chart and can be fit to a cubic polynomial. The error in this fit is less than the expected error in the level measurement. More importantly, there are no parameters that need to be fit to process data or that change with the operating point. Also, there is no special treatment of the model because it contains an integrating mode. The nonlinearities in this problem are mild, resulting from the cubic relationship between level and volume and the bilinear relationships between flows and compositions.



Figure 4: Blending concentration setpoint response.





Figure 5: Effluent flow disturbance response.



Figure 6: Level setpoint change response.

The disturbances in the effluent flow can be as great as 50% of the nominal value. Setpoint changes in the composition,  $x_2$ , are often made for process reasons. Level setpoint changes are made occasionally to reduce the residence time. Under normal operating conditions, this drum has about a one hour hold-up capacity. Level control is not a priority but it is important to keep the inventory to a manageable level without running the drum dry. Control of the effluent concentration is very important. Given these relative priorities, the controller tuning was specified so that the level response time constant would be approximately six minutes and the concentration response time constants would be approximately 1.8 minutes each. The prediction and control horizon are both five, with a sampling time of one minute. The relative weights on the CV response were all specified to be one and the weights on the economic and input move components of the objective function were set to zero.

Figure 4 shows the response to a setpoint change doubling the  $x_2$  concentration, presented as a deviation from normal operating conditions, normalized to the full-scale value. This response is very much like the minimum-time optimal control that one should expect given the tuning. The  $w_2$  flow is driven to a constraint and the other flows are set to achieve the desired concentration responses, temporarily sacrificing the level response. The level response returns to target along the specified error trajectory.

The response of the closed-loop system to change in the effluent flow to 53% of the nominal value is presented in Figure 5. This simulation shows that the expected level deviation is very small, the result of having a perfect model. Since the conventional controls on this unit operation are a combination of PID and ratio controls with no feed forward compensation for this disturbance, the NLC significantly reduces variation for these typical effluent flow changes.

Unlike the response in Figure 4, the response to a level setpoint change, shown in Figure 6, does not appear to have the same minimum-time optimal controller appearance. Instead, the controller responds smoothly according to the specified error trajectory. Note that neither of the concentration responses deviate from target to achieve this closed-loop performance in the level.

While this example is somewhat simplistic, it illustrates how the reference system tuning can be specified to achieve different desired responses in various controlled variables.

# **Polymerization Reactor Example**

Finally, the control of a polymerization reactor is used to demonstrate the application of the NLC to a largerscale industrial example. This polymerization reactor process is actually two reactors in series. Each reactor has independent feed and cooling systems. Catalyst is fed only to the first reactor. The model for each reactor includes mass balances for as many as seven species and multiple phases as well as energy balances around the reactor and cooling systems.

The controlled variables for this application are the polymer melt viscosity and the polymer comonomer incorporation in each reactor. The manipulated variables are setpoints in the distributed control system (DCS) that affect the addition of the comonomer and a transfer agent into the feed to each reactor. The current goal for this application is to control the transition to a desired trajectory.

The model for this 4 input  $\times$  4 output problem has on the order of 50 state variables and is described by approximately 120 DAEs. In this application, a simple output feedback scheme is used. Lab measurements of the polymer properties are made on product collected at a sampling point well downstream of the second reactor. When new results are received, they are compared to the predicted value at the time the process was sampled. The difference is filtered and used to modify the output predictions until new feedback is received. Figures 7 through 10 show a typical transition response achieved with the NLC controller. Note that the laboratory data is shown on an as-measured basis. To compare the laboratory measurements with the measurement estimate, the laboratory data must be shifted back in time by the time period required to complete the analysis.

The controller is tuned so that the ratio of closed-loop settling time to open-loop settling time is close to one for the fastest transitions and significant improvement is achieved for slower transitions. The control and prediction horizons are equal and approximately two-thirds of the nominal process residence time. The controller is executed every six minutes on a DEC Alpha System 1000 processor running at 266 MHz (circa 1997). Normally, controller execution completes in a two to three minute range, less than half of the six minute control period.

The characteristics of the reference system tuning are most evident in the polymer melt viscosity responses. The computed transfer agent command signal for the second reactor becomes constrained at its maximum value. The polymer comonomer incorporation illustrates the effect of corrupted measurements on the controller performance. Overall, the performance achieved in this particular transition represents a significant improvement over past performance.

Perhaps most importantly, this example demonstrates that nonlinear MPC problems of industrial significance can be solved in real-time on modest computing hardware. This demonstration should not discourage the innovation of techniques designed to permit implementation of nonlinear predictive control to even larger problems. However, it provides a counter example for those who claim that modifications are required to solve industrial nonlinear MPC problem in general.



Figure 7: Polymer melt viscosity transition response.



Figure 8: Transfer agent transition command signal.



Figure 9: Polymer comonomer incorporation transition response.



Figure 10: Comonomer transition command signal.

# **Concluding Remarks**

There are no signs that the activity in the area of nonlinear model predictive control are slowing. There is continued academic study of reference system synthesis applied to model predictive control (Kalra, 1997). The presentations at the CPC V conference (Kantor et al., 1996) and the 1998 Workshop on Nonlinear Model Predictive Control at Ascona, Switzerland (Allgower and Zheng, 2000) continue to reinforce the industrial need and academic interests in these areas. A survey of nonlinear model predictive control products by vendors was presented at Ascona (Qin and Badgwell, 2000). Besides the controller described in this paper, four other vendor offerings were available. Of the five, no two were pursuing exactly the same approach to solve this problem.

As stated earlier in this paper, there are very active academic research programs examining nonlinear control, model-predictive control, parameter estimation, and optimization that can be used to further the development of the NLC. Several academic programs have been working on demonstrating nonlinear MPC techniques on industrial-like models (Doyle III and Wisnewski, 2000; Schley et al., 2000; Nagy et al., 2000; Tenny et al., 2001; Findeisen et al., 2001). ExxonMobil Chemical Company actively participates in several university industrial consortia and interacts with several individual faculty members, knowing this participation is important to our continued success in this endeavor.

The NLC controller technology described in this paper is by no means mature. Reminiscent of linear MPC packages of the late 1980's, significant insight and detailed knowledge is required to successfully implement a NLC. However, this control methodology provides a way to pursue opportunities that have been previously beyond the reach of industrial process control engineers.

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