# Assessment of Performance for Single Loop Control Systems

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**Abstract**

Assessment of performance in tracking set-point changes for single loop systems is presented. In contrast to the works that used stochastic performances, the current assessment uses a deterministic performance measure, i.e. the integration of absolute tracking errors (abbr. IAE). A benchmark system that has an open loop transfer function (abbr. OLTF) comprised of one LHP zero, one integrator, and dead time is established. This benchmark system is used to provide a goal of performance that the existing system can practically achieve. For assessing an existing control system, the performance of the system in terms of IAE is computed and an index that indicates the extent of achievement toward this benchmark system is computed. Evaluation of control based on this index can then be made. The model required in computing the aforementioned performance index can be obtained from an auto-tuning procedure.

**Keywords**

Single loop, Assessment, Benchmark system, IAE performance, Auto-tuning

## Introduction

Assessment and monitoring of control systems with stochastic performance has been an active area of research for the last decade(Harris, 1989; Stanfelj et al., 1993; Harris et al., 1996; Qin, 1998; Harris et al., 1999; Leung and Romagnoli, 2000). The developments of research works have been focused on formulating the performance in terms of variance of the systems which are disturbed by stochastic inputs. As a result, with few exceptions, for example: Leung and Romagnoli (2000), controllers are implemented with discrete-time algorithms to pursuit minimum variance. On the other hand, the researches regarding assessment for deterministic performance have been reported, lately. For such assessments, technical developments have been focused on estimating maximum log modules ($L_{c,max}$) of closed-loops, (Chiang and Yu, 1993; Ju and Chiu, 1997), frequency responses (Kendra and Cinar, 1997), process characteristics(Piovoso et al., 1992), rise time (Åström et al., 1992), and settling time (Swanda and Seborg, 1999), etc. But, those works mentioned did not provided any implication regarding the best performance that an existing system can practically achieve.

In this paper, a benchmark system based on an IAE measure for an existing control system of single loop is presented. This benchmark system is to provide a practical goal of performance that the existing system can achieve. It has a loop transfer function comprised of one LHP zero, one integrator, and dead time. An index based on this benchmark system is thus presented to evaluate the existing system comparing with its control limit. This control limit is obtained based on what has been known about the process in terms of a model with specific dynamic order. Models to be required for this purpose can be obtained from an ATV experiment with relay feedback.

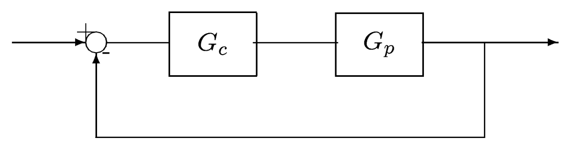


**Figure 1:** The conventional feedback control system.

## Developing Benchmark System for Performance Assessment

According to the IMC design principle, an IMC equivalent controller in a single loop of Figure 1 is given by:

$$\bar{G}_c(s) = \frac{\bar{G}_{p,-}^{-1} F(s)}{1 - \bar{G}_{p,+} F(s)} \tag{1}$$

Where, $\bar{G}_p(s)$ designates the model for the process, and $\bar{G}_{p,-}$ and $\bar{G}_{p,+}$ designate the invertible and noninvertible parts of $\bar{G}_p$. The transfer function $F(s)$ is an IMC filter. For design purpose, the dynamics of a open loop process can in general be represented by a transfer function either of first-order-plus-dead-time (abbr. FOPDT) or of second-order-plus-dead-time (abbr. SOPDT) of the following:

**FOPDT:**

$$\bar{G}_p(s) = \frac{k_p e^{-\theta s}}{\tau s + 1} \tag{2}$$

**SOPDT:**

$$\bar{G}_p(s) = \frac{k_p(as+1)e^{-\theta s}}{\tau^2 s^2 + 2\tau\zeta s + 1}; \qquad a \geq 0 \tag{3}$$

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Notice that the SOPDT process with RHP zero can also be modeled with the one of Equation 3 by making use of the following approximation. Let the RHP zero is given in the form of $1 - \beta s$. Then,

$$-\beta s + 1 \approx e^{-2\beta s} \times (1 + \beta s) \tag{4}$$

By using $G_c$ in Equation 1 and $G_p$ in Equations 2–3, the resulting open loop transfer function (abbr. OLTF) of the system becomes:

$$\begin{aligned} G_{OL}(s) &= G_c G_p(s) \\ &= \frac{G_{p,+}F(s)}{1 - G_{p,+}F(s)} \\ &= \frac{e^{-\theta s}F(s)}{1 - e^{-\theta s}F(s)} \end{aligned} \tag{5}$$

Thus, if $G_c$ is implemented exactly with Equation 1, the performance of the system depends on the choice of $F(s)$. The limiting performance of such a system can be obtained from the result of Holt and Morari (1985) with a slight modification to take into account the additional dead time.

For conventional loops, $G_c$ is used to be confined to have the following form:

$$G_c(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{6}$$

To obtain $G_c(s)$ in the above form, $e^{-\theta s}$ in the denominator of Equation 1 should be replaced with a Pade' approximation of proper order. After introducing the same Pade' approximation into the denominator of Equation 5, it is easy to see that the OLTF becomes:

$$G_{OL} = H(s)\frac{e^{-\theta s}}{s} \tag{7}$$

where, $H(s)$ is considered a loop filter and its functional form varies with the choices of $F(s)$ and the Pade' approximation being used. But, in general, $H(s)$ consists of finite number of poles and zeros.

Due to the approximation that has been made for synthesizing $G_c(s)$, the performance limit set by the IMC will no longer be applicable to the system in Figure 1. Thus, the performance limit for a single loop system has to be obtained from a minimization process. The minimization process starts with an $H(s)$ of the following form:

$$H(s) = k_o \frac{\beta s + 1}{\gamma s + 1} \tag{8}$$

In other words, with an OLTF of the following:

$$G_{OL} = k_o \frac{\beta s + 1}{\gamma s + 1}\frac{e^{-\theta s}}{s} \tag{9}$$

If the performance in terms of IAE is used, then the procedure is to find the values of $k_o$ and those of $\beta$ and $\gamma$ that minimize the following integral:

$$J^*_{IAE} = Min_{[k_o,\beta,\gamma]} \int_0^\infty |e(t)|dt \tag{10}$$

where $e(t)$ is given as the inverse transformation of $e(s)$, which is the tracking error of the system, i.e.:

$$e(t) = L^{-1}\left\{\frac{1}{1 + G_{OL}}R(s)\right\} \tag{11}$$

This above optimization problem was solved numerically by simulations. The result turns out to be:

$$G^*_{OL} = \frac{k^*_o(1 + \beta^* s)}{s}e^{-\theta s} \tag{12}$$

where,

$$k^*_o = \frac{0.76}{\theta}, \quad \beta^* = 0.47\theta, \quad \text{and}\gamma^* = 0 \tag{13}$$

Notice that $G_{OL}$ should not have excess number of zeros than poles. Thus, for processes of Equations 2 and 3, the $H(s)$ in the form of Equation 8 is most appropriate for developing the benchmark system. Thus, with the optimization results given above, the benchmark system is selected as the one that has OLTF of the following:

$$G^*_{OL}(s) = \frac{0.76(1 + 0.47\theta s)}{\theta\ s}\ e^{-\theta s} \tag{14}$$

The IAE value of this benchmark system subjected to an unit step set-point change is found to be 1.38 $\theta$. It is also found that this benchmark system has a gain margin of 2.11, and a phase margin of $64.4^o$. The system with such margin values is considered to have acceptable stability robustness.

Based on the FOPDT model of Equation 2, and the the SOPDT processes of Equation 3, the controllers in the form of Equation 6 that yield the benchmark OLTF are given as follows:

| Model or method | $\frac{IAE}{\theta}$ | Remark |
|---|---|---|
| Benchmark system | 1.38 | $k^*_o = \frac{0.76}{\theta}$; $\beta^* = 0.47\theta$ |
| Rovira(PI)* | 1.93 | FOPDT; $\tau = \theta$ |
| Rovira(PID)* | 1.52 | FOPDT; $\tau = \theta$ |
| Sung et al.(PID)† | 2.06 | $G_p(s) = \frac{1}{(s+1)^5}$ |
| Sung et al.(PID)† | 2.22 | $G_p(s) = \frac{e^{-s}}{(9s^2+2.4s+1)(s+1)}$ |
| Swanda and Seborg‡ | 2.0 | PI control |

*Smith and Corripio (1997, pg. 325)
†Sung et al. (1996)
‡Swanda and Seborg (1999)

**Table 1:** The minimum $\frac{IAE}{\theta}$ values for different control systems.

**FOPDT:**

$$G_{c,1}(s) = \left(\frac{0.76}{k_p\theta}\right)\frac{(\tau s+1)(0.47\theta s+1)}{s} \tag{15}$$

**SOPDT:**

$$G_{c,2}(s) = \left(\frac{0.76}{k_p\theta}\right)\frac{(\tau^2 s^2+2\tau\zeta s+1)(0.47\theta s+1)}{s(as+1)} \tag{16}$$

Obviously, these controllers are not physically realizable. For controllers to be realizable, one or two low pass filters with small time constants have to be introduced somewhere in $G_c$. The value of the resulting IAE will thus be degraded. But, the change is really too small to be considered. Thus, if $G_c(s)$ has not been confined to the conventional PID controllers, the minimum achievable IAE will be:

$$IAE^* = 1.38\theta \tag{17}$$

In Table 1, some IAE values of several systems are given. These systems include the benchmark one and some others, which have optimal controllers from different sources. It is to show none of these other systems from different sources has IAE less than the benchmark one.

This above equation can be adapted to apply to the case where $G_p(s)$ has $1-\beta s$ as a factor in the numerator (i.e. a RHP zero). In this case, the minimum IAE becomes:

$$IAE^* = 1.38\times(\theta+2\beta) \tag{18}$$

As an example, Consider a system that has open loop transfer function of the following:

$$G_cG_p(s) = \frac{k_o(1+as)(1-0.5s)e^{-s}}{(1+0.05s)s}$$

The minimum IAE of this closed loop system occurs when $k_o = 0.4$ and $a = 0.21$, and has a value of 2.8, which is about $1.38\times(1+2\times0.5)$ (i.e. 2.76).

Thus, assessment for a single loop control system that has $G_p(s)$ of Equation 2 or 3, will be targeting at the minimum IAE values of Equation 17 and 18.

## Assessment for Control Based on IAE

In the previous section, it has been mentioned that the minimum achievable IAE value of a conventional feedback loop is 1.38 $\theta$. In order to measure the achievement of an existing system toward this achievable target for set-point tracking, the following index is defined:

$$\Phi = \frac{1.38\theta}{\int_0^\infty |e(t)|dt} \tag{19}$$

Where, in the denominator, the IAE measure of the existing system for tracking step set-point change is used. This IAE measure can be obtained from experiment or from prediction based on a model for $G_p$. The index $\Phi$,

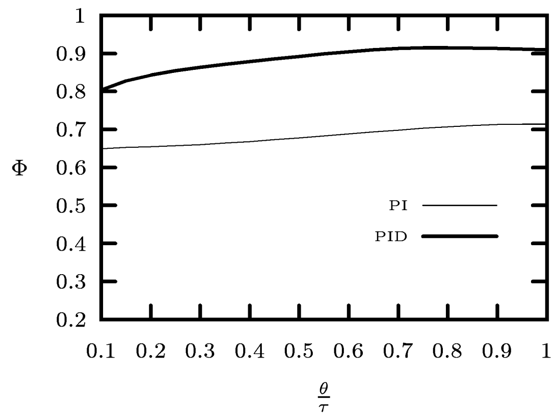


**Figure 2:** The $\Phi$ value for FOPDT process with Rovira's PI and PID controller.

always less than one, is used to represent the ability of an existing system in eliminating the tracking errors. If $\Phi$ is close to one, it indicates that the system is near its performance limit.

To illustrate assessing with this presented index, first, the control of FOPDT process with PID controllers is considered. In order to be more inclusive for the results, the FOPDT model is normalized with dimensionless time unit. The normalized transfer function for the FOPDT model is:

$$\bar{\bar{G}}_p(s) = \frac{k_p e^{-s}}{\bar{\tau}s+1} \tag{20}$$

where, $\bar{\tau} = \tau/\theta$. Then, for each $\bar{\tau}$, Rovira's tuning formula(Smith and Corripio, 1997, pg. 325) are used to tune the PI or PID controllers, which were claimed optimal for the IAE measure. As shown in Figure 2, the values of $\Phi$ resulting from such PI and PID control systems are given. It is thus found that, these optimal PID controllers give values of $\Phi$ higher than 0.8. On the other hand, those for optimal PI controllers, have values sitting between 0.65 and 0.75. Thus, as far as the control for FOPDT processes is concerned, the PID controller is a good choice.

Next, for dynamic systems of SOPDT, a normalized $G_p$ with dimensionless time units are also considered for illustration:

$$\bar{\bar{G}}_p(s) = \frac{k_p(\bar{a}s+1)e^{-s}}{(\bar{\tau}^2 s^2+2\bar{\tau}\zeta s+1)} \tag{21}$$

where, $\bar{a}$ designates $\frac{a}{\theta}$, and $\bar{\tau}$ designates $\frac{\tau}{\theta}$.

The normalized transfer function shows that $\bar{\tau}$ and $\zeta$ can be used to characterize the dynamic behaviors of such a process. Thus, PID control systems for $G_p$ with different $\bar{\tau}$ and $\zeta$ are used for illustration. For PID control of SOPDT processes, tuning rules of Sung et al. (1996) are used to compute the values of $\Phi$. The results

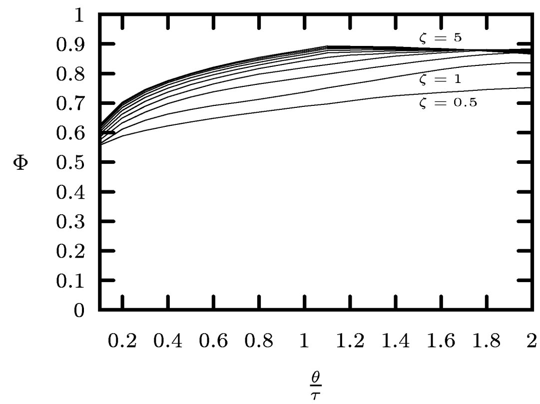


**Figure 3:** The $\Phi$ value for SOPDT process with PID controller given by Sung et al.

are given in Figure 3, where $\Phi$ is plotted along the values of $\theta/\tau$ in the range between 0.1 and 2. The damping factor $\zeta$ has been used as a parameter. The computed $\Phi$ for such systems indicates that such PID controllers are more close to the limit for well overdamped $G_p$. In this figure, it is also observed that the $\Phi$ values computed based on the $IAE^*$ from SOPDT models are lower than those from FOPDT models. This does not imply , for control purpose, an FOPDT model is superior to the SOPDT one. Instead, it indicates that with more knowledge about the dynamics, the control performance would have more stringent limit, and, if achievable, the performance would be superior to those with simpler (such as FOPDT) models.

In general, a $G_p(s)$ of high order may have an FOPDT and an SOPDT models at the same time as approximations of different accuracy. One may be questioned which type of models to be used. To justify, let $\theta_1$ and $\theta_2$ designate two apparent dead times in the FOPDT and SOPDT models, respectively. In general, $\theta_2 \le \theta_1$. Two values of $\Phi$ will be resulted:

$$\Phi_1 = \frac{1.38\theta_1}{\int_0^\infty |e(t)|dt}$$

$$\Phi_2 = \frac{1.38\theta_2}{\int_0^\infty |e(t)|dt}$$

Then, we shall have:

$$\Phi_2 = (1-\eta) \times \Phi_1 \tag{22}$$

where,

$$\eta = \frac{\theta_1 - \theta_2}{\theta_1} \tag{23}$$

The value of $\eta$ implies the portion of apparent dead time, from this FOPDT model, that can be reduced by the controllers based on SOPDT model. Thus, if the value of $\eta$ is too high, that means FOPDT model is not sufficient for designing good control system.

## Assessment with Relay Feedback Experiments

The information needed for assessing the control will be a model of FOPDT or of SOPDT that has an apparent dead time. To obtain this model, a relay feedback system can be used. The use of relay feedback has advantages on a few aspects. The most important one is that the control loop is still operated under closed loop and, hence, is still under control.

The relay feedback system is the same one as has been used in the ATV test of Åström and Hagglund (1984). The experiment consists of two stages. In the first stage, the system is perturbed with a bias to the output of the relay and wait until the system to appear constant cycling at the output. At this time, the cycling period (designated as $P$) and the amplitude (designated as $a$) of the cycles are measured to calculate the process gain:

$$k_p = \frac{\int_{t_0}^{t_0+P} y(t)dt}{\int_{t_0}^{t_0+P} u(t)dt} \tag{24}$$

Then, in the second stage, the bias to the relay is set to zero and wait again until the system appear constant cycling again. The period as well as the amplitude of the cycles are measured for estimating the other dynamic parameters. With these data obtained on-line, parameter estimations are carried out. The procedures and the algorithms for these estimations can be found elsewhere(Huang et al., 1996, 2000).

Thus, with the estimated model, we can calculate the predicted IAE for a set-point change by simulating the following with computer, i.e.:

$$e(s) = \frac{1}{1+G_c(s)\bar{G}_p(s)}\frac{1}{s} \tag{25}$$

Predictions of IAE via this identification procedures and simulations have been carried out over several example processes of high order dynamics, and pretty close results are obtained. In other words, the computation of $\Phi$ for assessment can be performed with an ATV test mentioned above.

## Conclusions

For a single loop control, a benchmark system that aims at minimuming IAE for set-point change has been established. This benchmark system system provides a performance limit for all feasible controllers in the form of Equation 6. It has an open loop transfer function (abbr. OLTF) comprised of one integrator, one simple lead, and dead time. For assessing an existing system, an index for indicating the extent of achievement toward this benchmark system is presented. This index is closely associated with the knowledge of dynamics being available, and the knowledge is usually contained in models of different orders. To obtain these models an auto-tuning experiment with relay feedback can be used. Technically, this assessment reveals how close the existing system is to the benchmark one, and, if the knowledge (model) for design is sufficient.

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