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A State Predictor for Multivariable Systems Including Multiple Delays in Each Output Path

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***Abstract*— Various practical systems include time delays due to measurement and computational delays, and transmission and transport lags. In this paper, the authors propose a novel state predictor for a certain class of multivariable systems including multiple output delays. The predictor consists of full-order observers estimating past state from each delayed output and finite interval integrators compensating the effect of the delays using state transition equations. State prediction error converges to zero at an arbitrary rate, which can be determined by choosing a finite number of poles of the full-order observers. In this predictor, the distance to instability of the state transition matrix is not affected by the delays. This means that large delays have no influence on the numerical stability, whereas that of a conventional observer highly depends on the delays. Numerical examples for an integral process and an unstable process demonstrate the effectiveness of the proposed predictor.**

I. INTRODUCTION

Time periods required for transport of materials through pipes or transmission of information on networks can be handled as time delays included in system dynamics. Measurement delays and computational delays in detection and signal processing existing in some sensors are also treated as output delays.

As controllers for plants including these delays, the Smith controller [1] and the state-predictive controller [2] are well known as effective means. For plants including only a delay of a single length, many studies on analysis and design methods for both controllers have been made[3], [4], [5], [6].

On the other hand, for multivariable systems with multiple delays in manipulated inputs and controlled outputs, modified Smith controllers [7], [8] and an observer compensating the effect of delays [9] have been proposed. However, the formers cannot be applied to unstable plants. The latter can assign poles arbitrarily by tuning an observer gain, but the distance to instability [10] of the state transition matrix of the observer increases with the length of the delays. As a result, the stability of the state estimation is lost for a plant with long delays.

In this paper, we propose a state predictor for a certain class of output delay systems. The predictor consists of full-order observers and finite interval integrators. The state transition matrices of the observers and the distance to instability of the matrices are independent of the length of the delays. Thus the length of the delays has no influence on the stability of the state estimation. Numerical examples are given to illustrate the properties and advantages of the predictor. For simplicity, we explain the proposed state predictor for systems with two delays of different length, but the following argument can be extended to systems including three or more delays of different length.

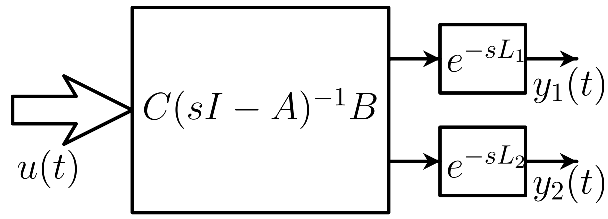

Fig. 1. Output time delay system

II. PROBLEM DESCRIPTION

We consider an m-input 2-output system (Fig.1) including two delays of different length in each output path

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \tag{1}$$

$$y_1(t) = C_1 x(t-L_1), \quad y_2(t) = C_2 x(t-L_2), \tag{2}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is an input vector, and $y_1(t)$ and $y_2(t)$ are scalar outputs. A, B, C_1 and C_2 are constant matrices of appropriate dimensions. L_1 and L_2 are output time delays, and assume

$$0 \le L_1 < L_2, \tag{3}$$

without loss of generality. Moreover, we assume that

$$n_1 = \mathrm{rank}\begin{pmatrix} C_1 \\ C_1 A \\ \vdots \\ C_1 A^{n-1} \end{pmatrix} < \mathrm{rank}\begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix} = n, \tag{4}$$

where $C = (C_1^{\mathrm{T}} C_2^{\mathrm{T}})^{\mathrm{T}}$.

Under these assumptions, the system (1), (2) can be transformed as

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & 0_{n_1 \times n_2} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t), \tag{5}$$

$$y_1(t) = C_{11} x_1(t-L_1), \; y_2(t) = C_{22} x_2(t-L_2), \tag{6}$$

by a coordinate transformation $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = Tx(t)$ [11].

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Here, $n_2 = n - n_1$, $x_1(t) \in R^{n_1}$ and $x_2(t) \in R^{n_2}$ are the states of the transformed system, A_{11}, A_{21}, A_{22}, B_1, B_2, C_{11} and C_{22} are constant matrices whose dimensions are $n_1 \times n_1$, $n_2 \times n_1$, $n_2 \times n_2$, $n_1 \times m$, $n_2 \times m$, $1 \times n_1$ and $1 \times n_2$, respectively, and $0_{i\times j}$ is an $i \times j$ zero matrix. Both (C_{11}, A_{11}) and (C_{22}, A_{22}) are observable. Eqs. (5) and (6) can be rewritten as

$$\left\{ \begin{array}{rcl} \dfrac{dx_1(t)}{dt} &=& A_{11}x_1(t) + B_1u(t), \\ y_1(t) &=& C_{11}x_1(t - L_1), \end{array} \right. \tag{7}$$

$$\left\{ \begin{array}{rcl} \dfrac{dx_2(t)}{dt} &=& A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t), \\ y_2(t) &=& C_{22}x_2(t - L_2). \end{array} \right. \tag{8}$$

In the following, we assume that A, B and C in eq. (1) and (2) are given without loss of generality as follows:

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & 0_{n_1\times n_2} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \\ C &= \begin{bmatrix} C_{11} & 0_{1\times n_2} \\ 0_{1\times n_1} & C_{22} \end{bmatrix}. \end{aligned} \tag{9}$$

If (A, B) is a controllable pair and the present state of the system are available, then we can drive the state to zero at an arbitrary rate by the control law:

$$u(t) = -Fx(t). \tag{10}$$

Here, $F \in R^{m\times n}$ is a feedback gain that assigns eigenvalues of $(A - BF)$ in the left half plane. However, we cannot obtain the actual state. Only the delayed outputs $y_1(t)$, $y_2(t)$ are available. These outputs include the information about the state at the time $t - L_1$ and $t - L_2$, respectively. Thus, in order to realize the control law of eq. (10), we require a state predictor that predicts the present state using only manipulated inputs and controlled outputs.

III. PROPOSED PREDICTOR

The proposed predictor consists of full-order observers and finite interval integrators. Each full-order observer estimates the past state from each output of the plant, the past inputs and the past states of other observers. The finite interval integrators predict the present state from the estimated past states and the past inputs using state transition equations. In the following subsections, a method to predict the present state is shown first under the assumption that the past states are directly obtained. Then, the full-order observers estimating the delayed states are combined with the above mentioned predictor. Furthermore, we show that the prediction error of the present state converges to the origin at a rate according to arbitrarily assigned poles, and compare the proposed predictor with a conventional observer [9] from the viewpoint of robust stability.

A. Prediction of present state from past states

In this subsection, we assume that the past states $x_1(t - L_1)$ and $x_2(t - L_2)$ are obtained. Under this assumption, a prediction method of the present state $x(t)$ is shown.

From eq. (8), we obtain

$$\begin{aligned} \hat{x}_2(t - L_1|t) &= \int_{-(L_2-L_1)}^{0} e^{-A_{22}\tau} B_2 u(t - L_1 + \tau) d\tau \\ &\quad + \int_{-(L_2-L_1)}^{0} e^{-A_{22}\tau} A_{21} x_1(t - L_1 + \tau) d\tau \\ &\quad + e^{A_{22}(L_2-L_1)} x_2(t - L_2), \end{aligned} \tag{11}$$

where $\hat{x}_2(t - L_1|t)$ is an estimate of $x_2(t - L_1)$ at the time t. From eq. (1), we obtain

$$\hat{x}(t|t) = e^{AL_1}x(t - L_1) + \int_{-L_1}^{0} e^{-A\tau} Bu(t + \tau)d\tau, \tag{12}$$

where $\hat{x}(t|t)$ is an estimate of $x(t)$. Replacing $x(t - L_1)$ in eq. (12) with $\begin{bmatrix} x_1^{\mathrm{T}}(t - L_1) & \hat{x}_2^{\mathrm{T}}(t - L_1|t) \end{bmatrix}^{\mathrm{T}}$ and substituting eq. (11), we obtain

$$\begin{aligned} \hat{x}(t|t) &= e^{AL_1} \begin{bmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & e^{A_{22}(L_2-L_1)} \end{bmatrix} \begin{bmatrix} x_1(t - L_1) \\ x_2(t - L_2) \end{bmatrix} \\ &\quad + e^{AL_1} \int_{-(L_2-L_1)}^{0} e^{-A\tau} \begin{bmatrix} 0_{n_1\times n_1} \\ A_{21} \end{bmatrix} x_1(t - L_1 + \tau)d\tau \\ &\quad + e^{AL_1} \int_{-(L_2-L_1)}^{0} e^{-A\tau} \begin{bmatrix} 0_{n_1\times 2} \\ B_2 \end{bmatrix} u(t - L_1 + \tau)d\tau \\ &\quad + \int_{-L_1}^{0} e^{-A\tau} Bu(t + \tau)d\tau. \end{aligned} \tag{13}$$

Using the above equation, we can predict the present state from the past states and the past inputs.

B. Estimation of past states from present outputs

In this subsection, full-order observers that estimate past states $x_1(t - L_1)$ and $x_2(t - L_2)$ from the present outputs, the past inputs and the states of other observers are shown.

In order to estimate $x_1(t - L_1)$, we introduce a full-order observer

$$\begin{aligned} \frac{dz_1(t - L_1|t)}{dt} &= (A_{11} - K_{11}C_{11})z_1(t - L_1|t) \\ &\quad + B_1u(t - L_1) + K_{11}y_1(t), \end{aligned} \tag{14}$$

where $z_1(t - L_1|t)$ is an estimate of $x_1(t - L_1)$ at the time t, K_{11} is an observer gain that assigns the eigenvalues of a matrix $(A_{11} - K_{11}C_{11})$ into the left half plane.

Next, another observer that estimates $x_2(t-L_2)$ from $y_2(t)$ is introduced. From eq. (8), an ideal observer is given as

$$\begin{aligned} \frac{dz_2(t - L_2|t)}{dt} &= (A_{22} - K_{22}C_{22})z_2(t - L_2|t) \\ &\quad + A_{21}x_1(t - L_2) + B_2u(t - L_2) \\ &\quad + K_{22}y_2(t). \end{aligned} \tag{15}$$

Here, $z_2(t - L_2|t)$ is an estimate of $x_2(t - L_2)$ at the time t, K_{22} is an observer gain that assigns the eigenvalues of a matrix $(A_{22} - K_{22}C_{22})$ into the left half plane. Since $x_1(t - L_2)$ is not actually available, we substitute $z_1(t - L_2|t - (L_2 - L_1))$, an estimate of $x_1(t - L_2)$, into eq. (14) instead of $x_1(t - L_2)$. Then, we have

$$\begin{aligned} \frac{dz_2(t - L_2|t)}{dt} &= (A_{22} - K_{22}C_{22})z_2(t - L_2|t) \\ &\quad + A_{21}z_1(t - L_2|t - (L_2 - L_1)) \end{aligned}$$

$$+B_2u(t-L_2)+K_{22}y_2(t). \quad (16)$$

Combining eqs. (14) and (16), we obtain

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} z_1(t-L_1|t) \\ z_2(t-L_2|t)\end{bmatrix} = \\ \begin{bmatrix} A_{11}-K_{11}C_{11} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & A_{22}-K_{22}C_{22}\end{bmatrix}\begin{bmatrix} z_1(t-L_1|t) \\ z_2(t-L_2|t)\end{bmatrix} \\ +\begin{bmatrix} 0_{n_1\times n_1} \\ A_{21}\end{bmatrix} z_1(t-L_2|t-(L_2-L_1)) \\ +\begin{bmatrix} B_1 \\ 0_{n_2\times m}\end{bmatrix} u(t-L_1)+\begin{bmatrix} 0_{n_1\times m} \\ B_2\end{bmatrix} u(t-L_2) \\ +\begin{bmatrix} K_{11} & 0_{n_2\times 1} \\ 0_{n_1\times 1} & K_{22}\end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t)\end{bmatrix}. \quad (17)\end{aligned}$$

This equation gives the observer estimating the past states from the present outputs, the past inputs and the states of the observer.

C. Proposed predictor

Now, we give the equation of the proposed predictor. Replacing the past states $\left[\begin{array}{cc} x_1^{\mathrm{T}}(t-L_1) & x_2^{\mathrm{T}}(t-L_2)\end{array}\right]^{\mathrm{T}}$ in eq. (13) with the estimates by the observer eq. (17), we obtain

$$\begin{aligned}\omega(t) &= e^{AL_1}\begin{bmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & e^{A_{22}(L_2-L_1)}\end{bmatrix}\begin{bmatrix} z_1(t-L_1|t) \\ z_2(t-L_2|t)\end{bmatrix} \\ &+e^{AL_1}\int_{-(L_2-L_1)}^{0} e^{-A\tau}\begin{bmatrix} 0_{n_1\times n_1} \\ A_{21}\end{bmatrix} z_1(t-L_1+\tau|t+\tau)d\tau \\ &+e^{AL_1}\int_{-(L_2-L_1)}^{0} e^{-A\tau}\begin{bmatrix} 0_{n_1\times m} \\ B_2\end{bmatrix} u(t-L_1+\tau)d\tau \\ &+\int_{-L_1}^{0} e^{-A\tau}Bu(t+\tau)d\tau, \quad (18)\end{aligned}$$

where $\omega(t)$ is an estimate of the present state $x(t)$.

Using eqs. (17) and (18), we can estimate the present state from the controlled outputs and the manipulated inputs. Thus, these equations give the proposed predictor. Note that the state transition matrix of eq. (17)

$$A_o=\begin{bmatrix} A_{11}-K_{11}C_{11} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & A_{22}-K_{11}C_{11}\end{bmatrix} \quad (19)$$

does not depend on the delays.

D. Dynamics of prediction error

In this subsection, the state equation of the error between the present state of the plant and the predicted state of the proposed predictor is shown. The equation shows that the error system has a finite number of poles, and the poles can be assigned at arbitrary positions.

Define the prediction error $e(t)$ as

$$e(t)=\omega(t)-x(t). \quad (20)$$

Differentiating eq. (20) and substituting eqs. (1), (2), (17) and (18) into it, we obtain

$$\frac{d\epsilon(t)}{dt} = \begin{bmatrix} A_{11}-K_{11}C_{11} & 0_{n_1\times n_2} \\ \Phi & A_{22}-K_{22}C_{22}\end{bmatrix}\epsilon(t), \quad (21)$$

where

$$\begin{aligned}\epsilon(t) &= \begin{bmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & e^{-A_{22}(L_2-L_1)}\end{bmatrix} e^{-AL_1}e(t), \quad (22) \\ \Phi &= e^{-A_{22}(L_2-L_1)}A_{21} \\ &\quad -K_{22}C_{22}\int_0^{L_2-L_1} e^{A_{22}(\tau-L_2+L_1)}A_{21} \\ &\qquad e^{-(A_{11}-K_{11}C_{11})\tau}d\tau. \quad (23)\end{aligned}$$

Eqs. (21) and (22) show that $e(t)$ converges to the origin at a rate according to the eigenvalues of $(A_{11}-K_{11}C_{11})$ and $(A_{22}-K_{22}C_{22})$. Since (C_{11}, A_{11}) and (C_{22}, A_{22}) are observable, these eigenvalues can be assigned at any arbitrary positions by K_{11} and K_{22}.

E. Comparison with conventional observer

In [9], an observer for multivariable systems with multiple delays in manipulated inputs and controlled outputs was proposed. The estimation error of this observer theoretically converges to zero as fast as desired.

For the system (1), (2), the full-order version of the observer can be represented by

$$\begin{aligned}\frac{dw(t)}{dt} &= (A-\bar{K}\bar{C})w(t)+Bu(t)+\bar{K}y(t) \\ &+\bar{K}\bar{C}\int_{-L_1}^{0} e^{-A\tau}Bu(t+\tau)d\tau \\ &+\bar{K}\bar{C}\int_{-L_2}^{-L_1} e^{-A\tau}\begin{bmatrix} 0_{n_1\times 2} \\ B_2\end{bmatrix} u(t+\tau)d\tau, \quad (24)\end{aligned}$$

where $w(t)$ is an estimate of $x(t)$,

$$\bar{C}=C\begin{bmatrix} e^{A_{11}(L_2-L_1)} & 0 \\ 0 & I_{n_2}\end{bmatrix} e^{-AL_2}, \quad (25)$$

and $\bar{K}$ is an observer gain that assigns the eigenvalues of $(A-\bar{K}\bar{C})$ into the left half plane.

In this case, the robust stability of the observer can be evaluated by the distance to instability [10] of the state transition matrix $(A-\bar{K}\bar{C})$. For a plant including large delays, the distance might be very small even if "the best $\bar{K}$" [12] is chosen, because $\bar{C}$ includes exponential matrix functions of the delays.

On the other hand, the robust stability of the proposed predictor can be evaluated by the distance to instability of the matrix (19). Since the matrix (19) does not depend on the length of the delays, the proposed predictor is expected to be more stable numerically than the conventional observer especially for a plant with large delays.

IV. Numerical examples

A. State estimation of an integral plant

In order to verify convergence characteristics of the prediction error, we construct the proposed predictor for an evaporator [13]. Although the original model [13] has three outputs, we assume that the first two outputs are available and time delays of the outputs are $L_1 = 12$ and $L_2 = 18$, respectively. The coefficient matrices after the coordinate transformation [11] are as follows:

$$
A = \left[\begin{array}{ccc|cc}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & -0.0592 & 0 & 0 \\
0 & 1 & -0.8496 & 0 & 0 \\
\hline
1.2013 & -0.9637 & 0.7476 & 0 & 0 \\
0 & 0 & 0 & 1 & -0.0380
\end{array}\right],
$$

$$
B = \left[\begin{array}{ccc}
-0.0021 & -0.0045 & 0 \\
-0.0271 & -0.0651 & 0 \\
0 & -0.0766 & 0 \\
\hline
-0.0321 & 0.0030 & -0.0014 \\
0 & 0.0795 & -0.0381
\end{array}\right],
$$

$$
C = \left[\begin{array}{ccc|cc}
0 & 0 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1
\end{array}\right]. \tag{26}
$$

The eigenvalues of A_{11} and A_{22} are $\{0,\ -0.0766,\ -0.7730\}$ and $\{0,\ -0.0380\}$, respectively.

We set $K_{11} = [3.84, 7.46, 3.95]^{\mathrm{T}}$ and $K_{22} = [2.52, 3.16]^{\mathrm{T}}$, in order to assign the eigenvalues of $(A_{11} - K_{11}C_{11})$ and $(A_{22} - K_{22}C_{22})$ to $\{-1.2, -1.6, -2.0\}$ and $\{-1.4, -1.8\}$, respectively. The initial state of the system is given as

$$x(\tau) = 0\ (-L_2 \le \tau < 0),\ x(0) = [1\ 0\ 0\ 0\ 0]^{\mathrm{T}},$$

and the initial state of the predictor are set to 0. The input to the plant is set as $u(t) \equiv 0$. The simulation results with sampling time $T = 0.1$ are shown in Fig. 2. Fig. 2(a), (b) and (c) represent the responses of the output $y(t)$, the state $x(t)$ and the prediction error $e(t)$ defined by eq. (20), respectively.

The results of simulations with other sets of K_{11} and K_{22} are shown in Fig. 2(d) and (e). The eigenvalues of $(A_{11} - K_{11}C_{11})$ and $(A_{22} - K_{22}C_{22})$ are set to $\{-0.6, -0.8, -1.0\}$ and $\{-0.7, -0.9\}$ in Fig. 2(d), and set to $\{-0.3, -0.4, -0.5\}$ and $\{-0.35, -0.45\}$ in Fig. 2(e). Fig. 2(c), (d) and (e) show that the predictors cannot suppress the prediction error until the shortest delay time L_1 elapses. However, after the delay times the prediction error returns to the origin at the respective rates according to the assigned poles.

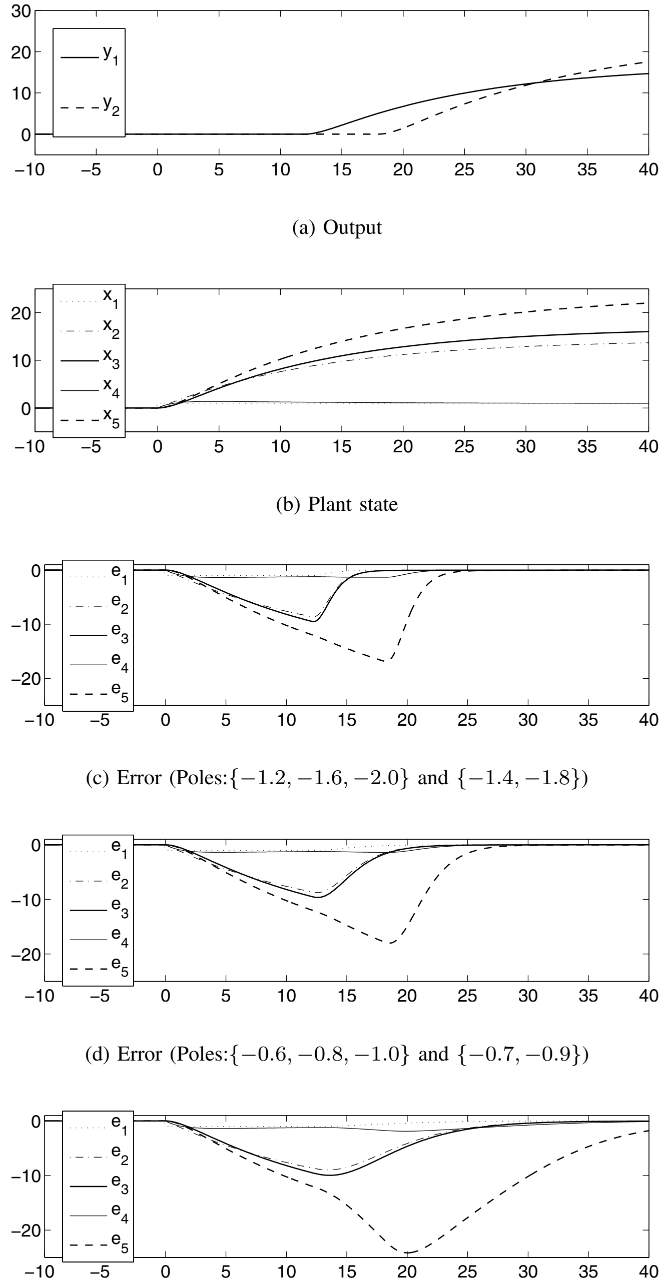
The response of the conventional observer explained in the preceding section are shown in Fig.3. In this case, the observer poles are set to $\{-1.2, -1.4, -1.6, -1.8, -2.0\}$ with

$$
\bar{K} = \left[\begin{array}{cc}
-68.9 & 0.563 \\
-636 & 5.20 \\
-707 & 5.79 \\
-1.17 \times 10^4 & 95.2 \\
-1.54 \times 10^5 & 1.26 \times 10^3
\end{array}\right]. \tag{27}
$$

This $\bar{K}$ is calculated by MATLAB standard function *place*. This function uses the algorithm of [12], which determines $\bar{K}$ to minimize the distance to instability of the matrix $A - \bar{K}\bar{C}$.

In Fig.3, there exists a steady state error of the estimation error, which does not appear theoretically. Because this steady state error vanishes when the sampling time is shortened, the cause of this error may be numerical errors due to the discretization.

In order to analyze robust stability of the proposed predictor and the conventional observer, we calculate the distance



(a) Output

(b) Plant state

(c) Error (Poles:$\{-1.2, -1.6, -2.0\}$ and $\{-1.4, -1.8\}$)

(d) Error (Poles:$\{-0.6, -0.8, -1.0\}$ and $\{-0.7, -0.9\}$)

(e) Error (Poles:$\{-0.3, -0.4, -0.5\}$ and $\{-0.35, -0.45\}$)

Fig. 2. State estimation for integral process: proposed predictor

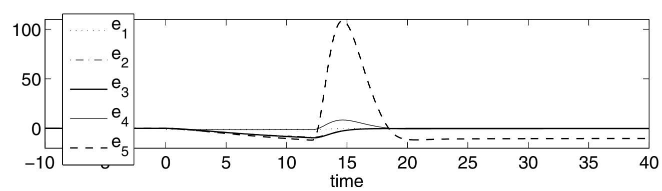


Fig. 3. State estimation for integral process: Error of the conventional observer (Poles: $\{-1.2, -1.4, -1.6, -1.8, -2.0\}$)

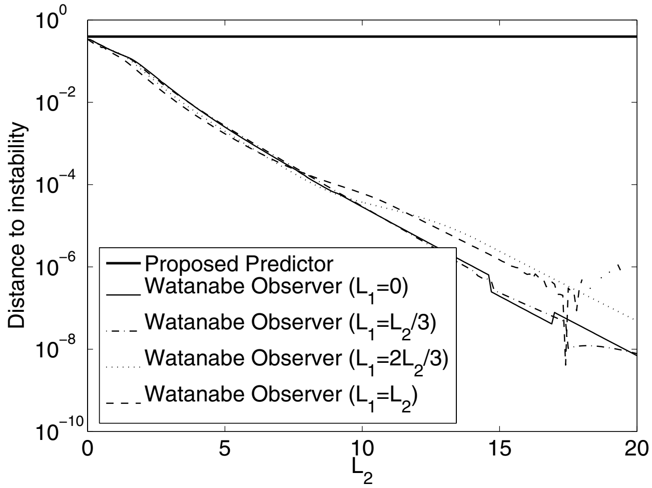


Fig. 4. Distance to instability of observer's state transition matrix

to instability of the state transition matrices of them using the algorithm described in [14]. In this calculation, the system matrices are fixed as eq. (26) and the time delays (L_1, L_2) are changed. The observer poles are assigned to $\{-1.2, -1.6, -2.0\}$ and $\{-1.4, -1.8\}$ in the proposed predictor, and to $\{-1.2, -1.4, -1.6, -1.8, -2.0\}$ for the conventional observer, using MATLAB standard function *place*. In Fig.4, the distances to instability of the proposed predictor and the conventional observers designed for the sets of $(L_1, L_2) = (0, L_2),\ (L_2/3, L_2),\ (2L_2/3, L_2),\ (L_2, L_2)$. Obviously, the distance of the proposed predictor is a fixed value since the state transition matrix eq. (19) is not affected by the length of the delays in this situation. On the other hand, the distance of the conventional observer decreases exponentially with the increase of L_2. This means that the stability of the conventional observer tends to be deteriorated when the plant includes larger delays, as seen in Fig.4.

B. Stabilization of an unstable plant

We consider the following unstable plant:

$$A = \left[\begin{array}{cc|cc} 0 & 8 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ \hline 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right], \quad B = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \\ \hline 0 & 1 \\ 0 & 0 \end{array}\right],$$

$$C = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad L_1 = 0.34, \quad L_2 = 0.47.$$

The eigenvalues of A_{11} and A_{22} are $\{-2, 4\}$ and $\{-1, 2\}$, respectively.

The observer gains K_{11} and K_{22} are chosen as

$$K_{11} = \begin{bmatrix} 23 \\ 10 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} 10 \\ 7 \end{bmatrix},$$

to assign the eigenvalues of $(A_{11} - K_{11}C_{11})$ and $(A_{22} - K_{22}C_{22})$ at $\{-3, -5\}$ and $\{-2, -4\}$, respectively. The input to the plant $u(t)$ is given by

$$u(t) = -F\omega(t), \tag{28}$$

where $\omega(t)$ is the predicted state of the proposed predictor and F is a feedback gain, and set

$$F = \begin{bmatrix} 8.16 & 32.5 & 0.36 & 0.73 \\ 0.36 & -0.56 & 4.27 & 8.69 \end{bmatrix},$$

using a standard LQ-design method with $Q = I_4$ and $R = I_2$, where Q and R are the weighting matrices for the state error and manipulated inputs, respectively. The eigenvalues of $(A - BF)$ are $\{-4.10, -2.25, -1.96, -1.12\}$.

Initial conditions of the plant and the observers are set as

$$\begin{aligned} u(\tau) &= 0\ (-L_2 \le \tau < 0),\ x(\tau) = 0\ (-L_2 \le \tau < 0), \\ x(0) &= [0\ 1\ 0\ 0]^{\mathrm{T}}, \\ z_1(\tau) &= 0\ (-L_2 \le \tau \le 0),\ z_2(\tau) = 0\ (-L_2 \le \tau \le 0). \end{aligned}$$

The results of a simulation with sampling time $T = 0.01$ are shown in Fig. 5. Fig. 5(a), (b), (c) and (d) represent the responses of the output $y(t)$, the input $u(t)$, the state $x(t)$ and the prediction error $e(t)$ defined by eq. (20), respectively. From Fig.5, the state of the plant and the prediction error increase at the rate of the unstable pole of the plant until the shortest delay time L_1 elapses. However, after the delay times, the prediction error tends to the origin, and the controller provides appropriate inputs, and then the state returns to the origin.

Under the same conditions, the closed-loop response using the conventional observer with the observer gain

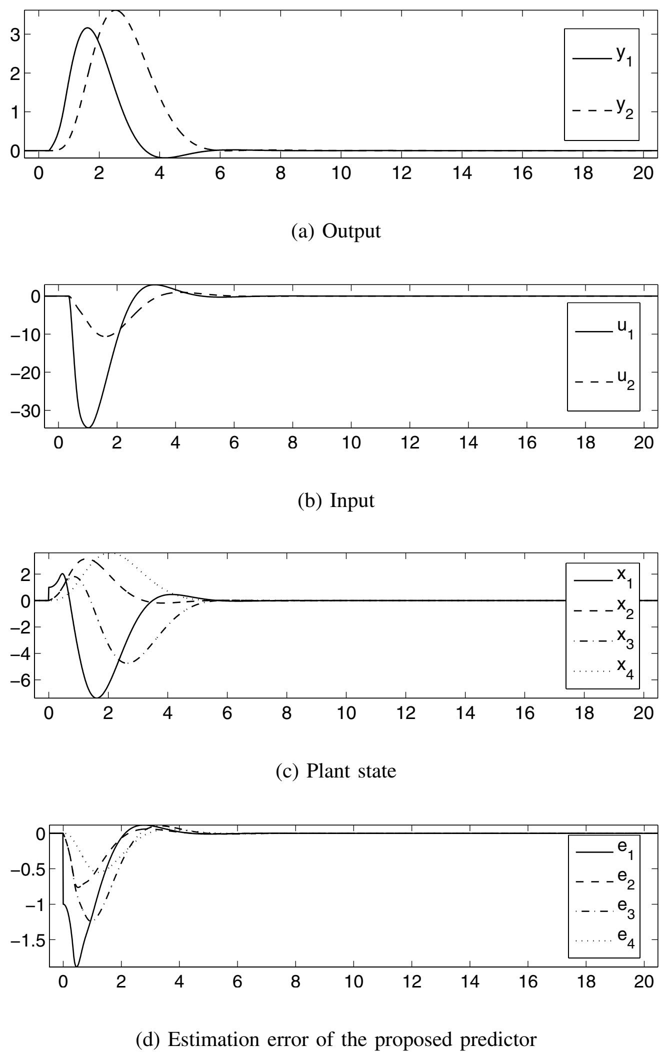
$$\bar{K} = \begin{bmatrix} 73.8 & 36.5 & 46.7 & 15.2 \\ 13.1 & 6.1 & 33.9 & 25.3 \end{bmatrix}^{\mathrm{T}}$$

is shown in Fig.6. This $\bar{K}$ is determined by MATLAB standard function *place* to set the poles of the observer to the same as the above example. In Fig.6, both of the state of the plant and the prediction error cannot return to the origin, and a steady-state error appears.

In these cases the distances to instability of the state transition matrices are 0.5405 in the proposed predictor and 0.1628 in the conventional observer, respectively. This means that a relatively small disturbance might make the conventional observer unstable. Thus the steady-state error appears in numerical computation, although it does not appear theoretically. Furthermore, when the plant includes larger delays, the closed-loop system using the conventional observer becomes unstable while the system using the proposed predictor remains stable.

V. CONCLUSION

In this paper, we propose a state predictor that predicts the present state of a multivariable system including multiple delays in each output path. The error of the predicted state in the predictor converges to zero at the rate according to the poles, which can be adjusted arbitrarily by observer gains. The distance to instability of the state transition matrix of the proposed predictor are independent of the delays, while that of the conventional observer is highly dependent on the delays. This means that large delays have no influence on the stability of the proposed predictor, whereas that of the conventional observer significantly depends on the



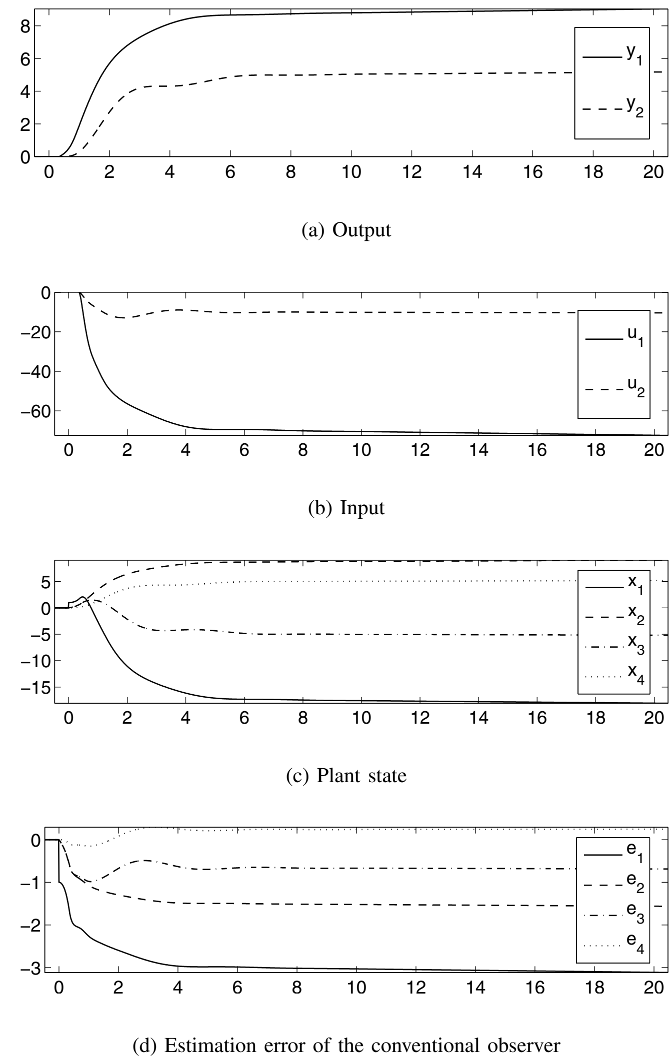
(a) Output

(b) Input

(c) Plant state

(d) Estimation error of the proposed predictor

Fig. 5. Stabilization of unstable process: with proposed predictor



(a) Output

(b) Input

(c) Plant state

(d) Estimation error of the conventional observer

Fig. 6. Stabilization of unstable process: with the conventional observer

delays. Numerical examples show that the proposed predictor provides better performance than the conventional observer.

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