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$\mathcal{H}^\infty$ Hover-to-Cruise Conversion for a Tilt-Wing Rotorcraft

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Abstract— **This paper describes the development of robust, multi-variable $\mathcal{H}^\infty$ control systems for the conversion of the High-Speed Autonomous Rotorcraft Vehicle (HARVee), an experimental tilt-wing aircraft. Tilt-wing rotorcraft combine the high-speed cruise capabilities of a conventional airplane with the hovering capabilities of a helicopter by rotating their wings at the fuselage. Changing between cruise and hover flight modes in mid-air is referred to as the conversion process, or simply conversion. A nonlinear aerodynamic model was previously developed that captures the unique dynamics of the tilt-wing aircraft. An $\mathcal{H}^\infty$ design methodology was used to develop cruise and hover control systems because it directly addresses multi-variable and robust design issues. The development of these control systems was governed not only by performance specifications at each particular operating point, but also by the unique requirements of a gain-scheduled conversion control system. The cruise and hover control designs form the basis for the conversion control system. The performance of the resulting conversion closed-loop systems is analyzed in the frequency and time domains. A tilt-wing rotorcraft Modeling, Simulation, Animation, and Real-Time Control (MoSART) software environment provides 3D visualization of the vehicle's dynamics. The environment is useful for conceptualizing the natural rotorcraft dynamics and for gaining an intuitive understanding of the closed-loop system performance.**

I. INTRODUCTION

Tilt-Wing Aircraft Basics. A tilt-wing is an aircraft that has the ability to hover similar to a helicopter while retaining the high-speed cruise capabilities of a conventional propeller-driven airplane. These two extremely different modes of flight are accomplished by tilt-wing vehicles through the rotation of the wing. The wing is maintained in a position perpendicular to the fuselage major axis for hover flight and in a position parallel to the fuselage major axis for cruise flight. The process of moving between hover (Figure 1a) and cruise (Figure 1b) flight modes is referred to as the conversion process, or simply conversion. Operation between the extremes of cruise and hover is referred to as mid-conversion flight.

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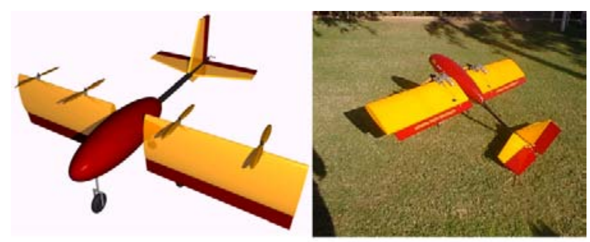
The HARVee Tilt-Wing Aircraft Project. Students and faculty of Arizona State University's Mechanical/Aerospace and Electrical Engineering Departments have been working for several years on the development of a tilt-wing aircraft through the High-Speed Autonomous Rotorcraft Vehicle (HARVee) Project [3], [4], [5], [6], [7], [8], [9], [10]. The purpose of this project is to design, build, and fly a small-scale tilt-wing aircraft. Figure 1a shows a three-dimensional (3D) computer generated model of HARVee in hover mode. Figure 1b, shows a photograph of the actual HARVee aircraft in cruise flight mode. Approximate vehicle parameters are as follows: length, 4.5ft; wing span, 5.7 ft; weight, 26.9 lbs. This vehicle, when flown successfully, will demonstrate the feasibility of the tilt-wing concept for small aircraft applications.



(a) (b)

Fig. 1. HARVee Tilt-Wing in (a) Hover and (b) Cruise Configurations

Conversion. Conversion of a tilt-wing aircraft is a difficult maneuver that requires three major items to be developed. First and foremost, a nonlinear model of the aircraft that captures the dynamics of cruise, conversion, and hover flight modes is required. Secondly, the control system itself has to be designed such that acceptable performance and robustness properties are achieved throughout the conversion envelope. Finally, a control input scheduling strategy is needed in order to maintain manageable dynamics during conversion, while keeping the control inputs within their respective ranges

Contributions of Work. This paper addresses several issues that are important in understanding the unique dynamics of a tilt-wing aircraft and in achieving the goal of mid-flight conversion of the HARVee aircraft. In particular,

- the previously developed nonlinear aerodynamic model of the aircraft [9] is analyzed at conversion operating points,

- the previously developed $\mathcal{H}^\infty$ control systems for the cruise and hover operating points has been applied to operating points throughout the conversion process. This has been done in a way such that they form a basis for the development of a gain-scheduled control system that allows mid-air conversion of the tilt-wing aircraft,
- the open and closed loop systems are visualized using 3D animation [8] to better understand unique dynamics of the tilt-wing aircraft and the performance of the control designs. This visualization is made possible through the use of a developed tilt-wing aircraft Modeling, Simulation, Animation, and Real-Time Control (MoSART) environment.

Outline. The remainder of this paper is organized as follows. Section II provides an overview of the nonlinear mathematical model of the HARVee aircraft and presents analysis which illustrates some of the important characteristics of the vehicle. Section III describes the conversion $\mathcal{H}^\infty$ control systems that have been designed and presents analysis of the resulting closed loop systems. Section IV discusses the tilt-wing aircraft Modeling, Simulation, Animation, and Real-Time Control (MoSART) software environment. Section V summarizes the results of this paper and presents directions for future research.

II. MATHEMATICAL MODELS: SYSTEM DYNAMICS

Model Overview. A six (6) degree-of-freedom, nonlinear, rigid-body, 8^{th} order aerodynamic model of the tilt-wing aircraft has been developed to model cruise, hover, and conversion modes of flight [7], [9]. This model is based on a theoretical approach in which the forces and moments on each individual component of the aircraft are calculated and combined together to represent the overall aircraft dynamics.

State Variables. The states of the HARVee tilt-wing aircraft are those that are required for any general six (6) degree-of-freedom vehicle. These twelve (12) states are summarized in Table I.

TABLE I
TILT-WING AIRCRAFT STATES

Symbol	Description	Units
x	Inertial X-Axis Position	ft
y	Inertial Y-Axis Position	ft
z	Inertial Z-Axis Position	ft
ϕ	Euler Roll Angle	deg
θ	Euler Pitch Angle	deg
ψ	Euler Yaw Angle	deg
u	Body X-Axis Velocity	ft/s
v	Body Y-Axis Velocity	ft/s
w	Body Z-Axis Velocity	ft/s
p	Body Roll Angular Velocity	deg/s
q	Body Pitch Angular Velocity	deg/s
r	Body Yaw Angular Velocity	$bi\text{-}deg/s$

Control Inputs. The tilt-wing aircraft employs certain specialized controls in addition to the standard aircraft control inputs. Table II lists all of these inputs and the state which they most affect during cruise and hover flight. Figure 2 illustrates the placement of the controls on the aircraft as well as the conversion process.

Conversion Inputs. The design of a gain-scheduled conversion controller is facilitated by using the same control inputs throughout the conversion envelope. These conversion inputs are T, T_p, δT and δ_a, which were chosen for their effectiveness throughout the conversion envelope [1].

TABLE II
TILT-WING AIRCRAFT CONTROL INPUTS

Symbol	Description	Units	Cruise	Hover
T	Main Thrust	lbs	u, w	w, u
δT	Diff. Thrust	deci-lbs	ψ, v	ϕ, v
T_p	Pitchfan Thrust	lbs	θ	θ
δ_a	Aileron Angle	deg	ϕ, v, ψ	ψ
i_w	Wing Angle	deg	conversion	

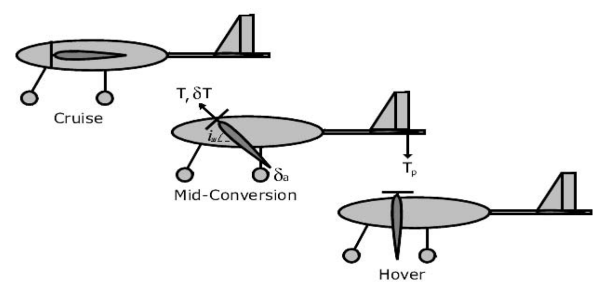


Fig. 2. Conversion Interchange with Aircraft Inputs

Conversion Dynamics. As the wing rotates, the engines rotate with it. By the time the wing is in hover position, the thrust from the main engines supports the weight of the aircraft. However, at low wing angles the thrust vector from the engines are not pointed significantly upward to provide enough force to counteract the weight [3].

Successful conversion requires a schedule of control inputs for a series of trim points calculated as the wing incidence angle varies between zero and ninety degrees. It is also important that the dynamics during conversion are not too unstable and are reasonably well-behaved. In the conversion routine, the forward velocity is schedule such that is starts at 15 m/s in cruise, drops off quickly and eventually reaches 0 m/s in hover.

Modal Analysis. The system poles associated with cruise dynamics are consistent with standard airplane modes [2]. Longitudinally, these consist of phugoid modes and a short period modes (Figure 3). Between wing angles of zero and thirteen degrees, the short period mode begins to move inwards toward the origin. At thirteen degrees, the wings of the aircraft begin to stall. After thirty degrees, the plane no longer gains lift from the wings, and the poles move toward the real axis. At a wing angle of sixty seven degrees, the downward thrust from the engine become sufficient to support the weight of the aircraft. As the wing angle moves toward ninety degrees, the poles move towards locations consistent with the vertical modes of a helicopter [2]. The phugoid mode becomes unstable once the aircraft loses lift from the wings. These poles continue to move further into the right-half plane, until the downward thrust from the engine begins to overcome the force due to gravity. They then begin to move back toward the imaginary axis. Their position at ninety degrees is consistent with the dynamics associated with the pitching motion of the aircraft [2].

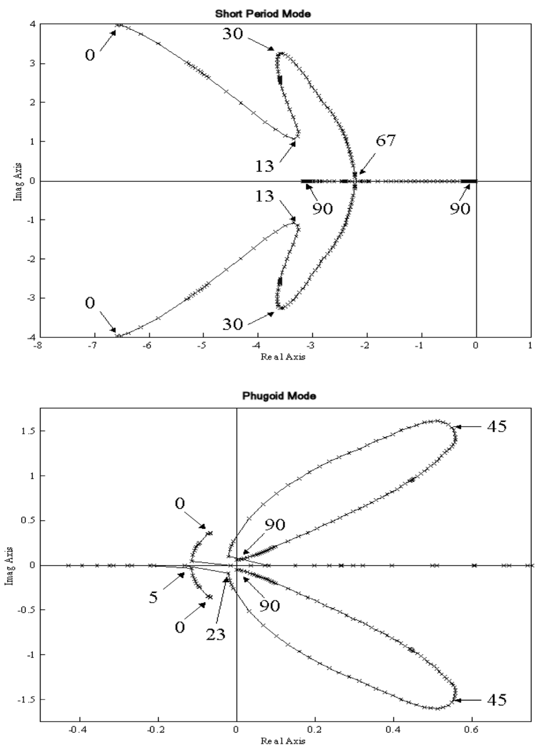


Fig. 3. Short Period and Phugoid Mode Movement During Conversion

The lateral modes associated with the tilt-wing rotorcraft include dutch roll, spiral divergence, and roll subsidence modes (Figure 4). The Dutch roll modes become unstable backflapping modes at wing angles of eighty degrees. At a wing angle of ninety degrees, the modes have moved back into the left-half plane, implying a stable backflapping for the rotorcraft. The stable roll subsidence mode in cruise remains a stable mode in hover. The unstable spiral divergence mode becomes a stable spiral divergence mode in hover.

Additionally, there is a single transmission zero associated the pitchfan thrust control input. For $i_w = 0$ it is located at $s = -1.34$. As i_w approaches ninety degrees, the transmission zero approaches $s = -3.18$.

We also examine the singular values for the plant as i_w varies between zero and ninety degrees. The maximum singular values vary greatly as hover is approached and the velocity goes to zero.

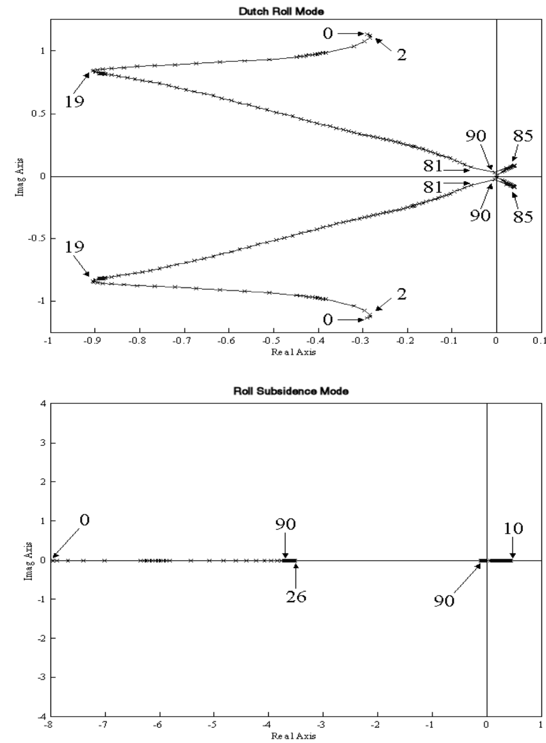


Fig. 4. Dutch Roll , Spiral and Roll Subsidence Mode Movement

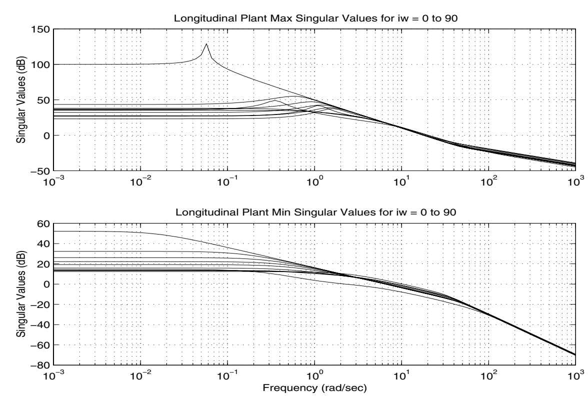


Fig. 5. Longitudinal Plant Singular Values for $i_w = 0^o$ to 90^o

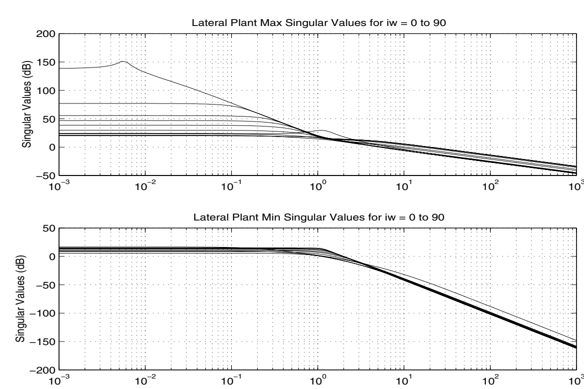


Fig. 6. Lateral Plant Singular Values for $i_w = 0^o$ to 90^o

Finally, examining the direction vectors corresponding to the maximum singular value will be useful in determining the coupling between the inputs and outputs at various points of the conversion.

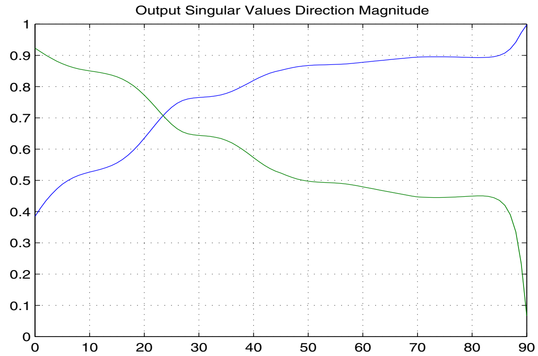


Fig. 7. Output Singular Vector Component for $i_w = 0^o$ to 90^o

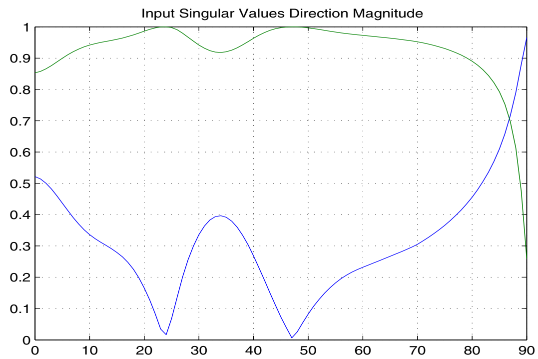


Fig. 8. Input Singular Vector Component for $i_w = 0^o$ to 90^o

III. CONTROL LAWS

This section presents $\mathcal{H}^\infty$ control system designs for the conversion:

Weighted $\mathcal{H}^\infty$ Suboptimal Mixed Sensitivity Problem. The weighted $\mathcal{H}^\infty$ suboptimal mixed sensitivity problem is to find a real-rational (finite-dimensional) proper internally stabilizing controller K that satisfies

$$\|T_{wz}\|_{\mathcal{H}^\infty} = \left\| \begin{bmatrix} W_1 S \\ W_2 KS \\ W_3 T \end{bmatrix} \right\|_{\mathcal{H}^\infty} < \gamma. \qquad (1)$$

where S and T are the sensitivity and complementary sensitivity transfer functions of the closed loop system respectively.

$\mathcal{H}^\infty$ Mixed-Sensitivity Weighting Functions. The selection of the weighting functions used in the $\mathcal{H}^\infty$ design process was kept consistent across the longitudinal and lateral designs for both the cruise and hover operating points. Using the same weighting function structure for each design keeps the order of the controllers the same and allows for interpolation of the weighting function parameters across the gain-scheduled conversion. The weighting functions were selected as follows: $W_i = diag\left(\left(\frac{s/\sqrt{M^i_{e_1}}+\omega^i_{e_1}}{s+\omega^i_{e_1}\sqrt{\epsilon^i_{e_1}}}\right), \left(\frac{s/\sqrt{M^i_{e_2}}+\omega^i_{e_2}}{s+\omega^i_{e_2}\sqrt{\epsilon^i_{e_2}}}\right)\right)$ where $i = 1, 2, 3$. Using these weighting functions, the lateral and longitudinal controller designs produced are each 9^{th} order.

Reference Command Pre-Filter Design. The following reference command pre-filter structure is utilized to reduce the bandwidth of reference signals that would otherwise cause large overshoots. The parameters (a_1 and a_2) were chosen based on location of low frequency controller zeros, which can cause large overshoots to step reference commands. The pre-filter $W = diag(\frac{a_1}{s+a_1}, \frac{a_2}{s+a_2})$ consists of a 1^{st} order low pass filter in each reference command channel.

Design Procedure The following design procedure was employed for the longitudinal and lateral systems at the cruise and hover operating points. Thus, a total of four designs are presented.

- **Step 1:** Augment the plant P with integrators at the plant input, and obtain $\hat{P}$.
- **Step 2:** Apply bilinear transformation to $\hat{P}$ in order to shift all integrators and lightly damped stable plant poles and zeros (if they exist) to the right half plane, to obtain $\hat{\hat{P}}$. (This is a necessary step to avoid cancelling the light damped poles and zeros).
- **Step 3:** Form generalized plant G using $\hat{\hat{P}}$ and weighting functions W_1, W_2, W_3.
- **Step 4:** Obtain $\hat{\hat{K}}$ by applying weighted mixed-sensitivity $\mathcal{H}^\infty$ control design methodology to G.
- **Step 5:** Apply inverse bilinear transformation to $\hat{\hat{K}}$ to obtain $\hat{K}$.
- **Step 6:** Augment $\hat{K}$ with integrators at the controller output(Step 1) to obtain the final compensator K.

Conversion Closed Loop System Analysis. Singular value plots and step responses are used to evaluate the performance of controller designs. Sensitivity at the plant output $S_o = [I+PK]^{-1}$ quantifies the ability of the closed loop system to follow reference commands and attenuate disturbances at the plant output. Sensitivity at the plant input $S_i = [I+KP]^{-1}$ quantifies the ability of the closed loop system to attenuate disturbances at the plant input. S_o plots are provided for the both longitudinal and lateral systems during various stages of the conversion process. (Figures 9 and 10).

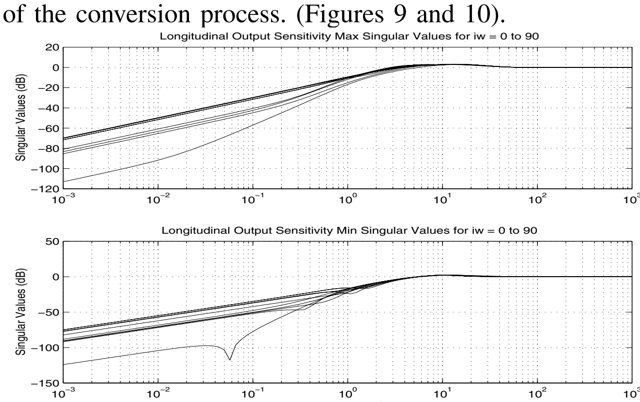


Fig. 9. Longitudinal Sensitivity Singular Values at Plant Output

These sensitivity figures illustrate acceptable nominal properties with respect to low frequency command following and disturbance attenuation. They also illustrate an average closed loop bandwidth of $\omega_{-3dB} \approx 1\frac{rad}{sec}$.

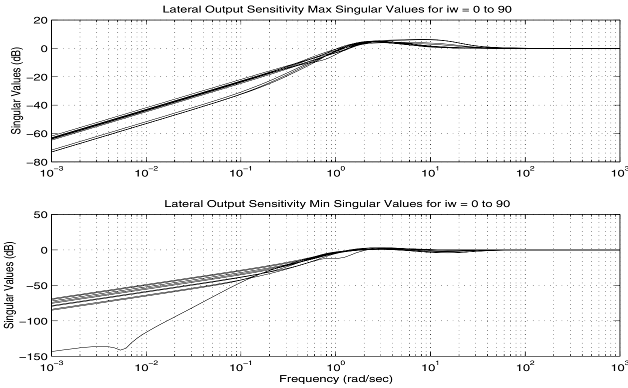


Fig. 10. Lateral Sensitivity Singular Values at Plant Output

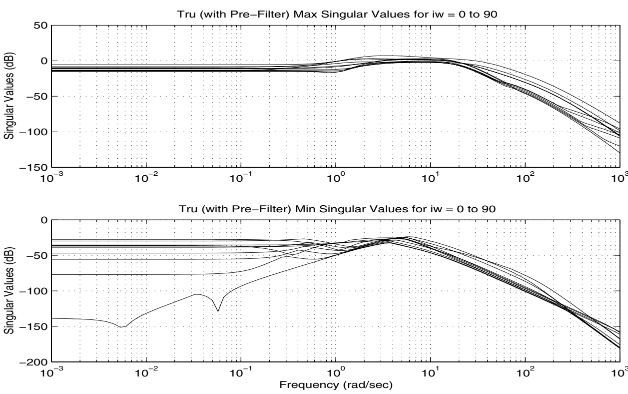


Fig. 11. Reference to Control (Longitudinal)

Reference to control (with pre-filter) singular values at the plant output are provided for both the longitudinal and lateral dynamics. (Figure 11).

Closed-loop reference to output (with pre-filter) singular values are provided for both the longitudinal and lateral dynamics (Figure 13).

Output responses to step reference commands are provided for the longitudinal (Figure 14) and lateral (Figure 15) systems as well as the longitudinal (Figure 16) and lateral (Figure 17) controls required to achieve these step responses.

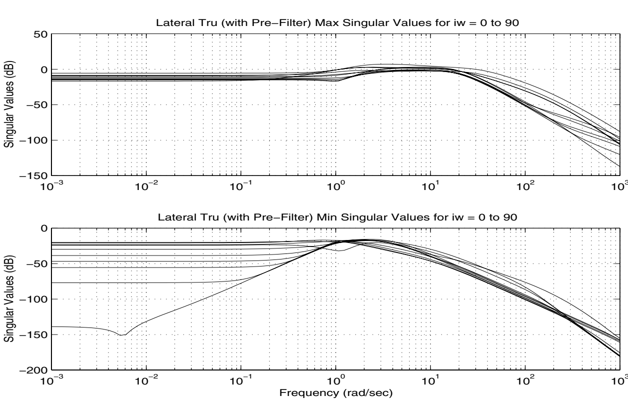


Fig. 12. Reference to Control (Lateral)

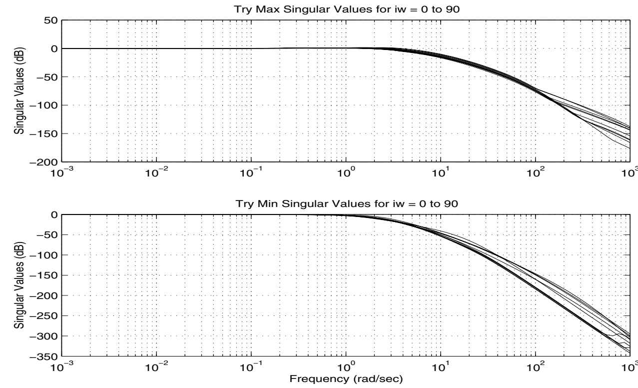


Fig. 13. Longitudinal and Lateral Closed Loop Singular Values

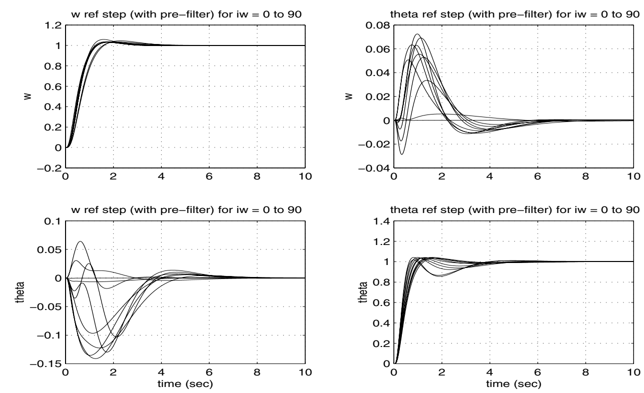


Fig. 14. Longitudinal Step Responses

The longitudinal variables are vertical velocity (w) and pitch angle (θ). The lateral variables are yaw rate (r) and sideslip velocity (v). For all designs, step response overshoot is less than 7% and associated control inputs are very reasonable given the corresponding step size.

IV. TILT-WING AIRCRAFT MOSART ENVIRONMENT

The tilt-wing aircraft Modeling, Simulation, Animation, and Real-Time Control (MoSART) environment is an interactive application for simulating and visualizing complex tilt-wing aircraft systems. The environment is developed with

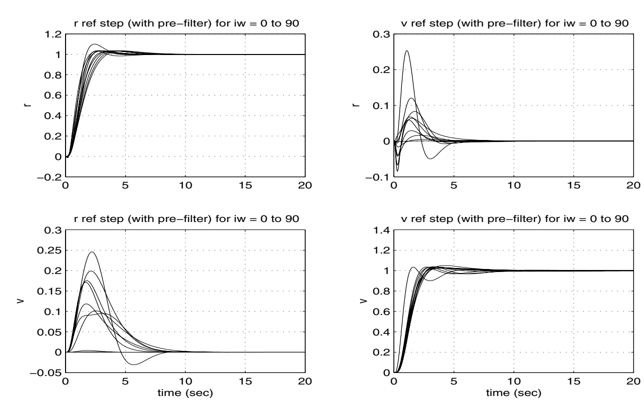


Fig. 15. Lateral Step Responses

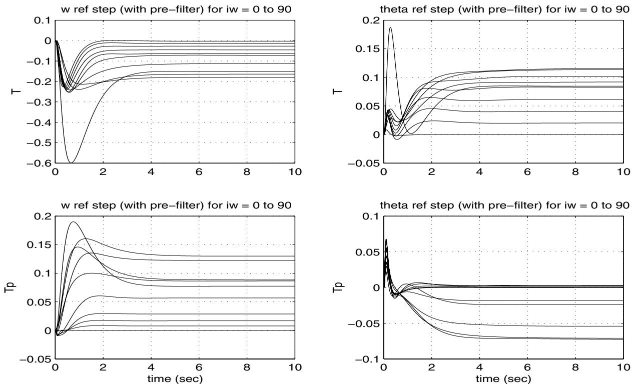


Fig. 16. Longitudinal Control Inputs

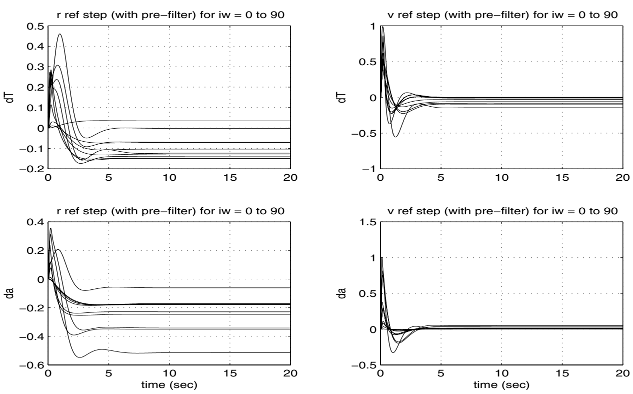


Fig. 17. Lateral Control Inputs

Microsoft Visual C++ [14] for use in Microsoft Windows XP/2000/NT. It relies on the Windows General Drawing Interface (GDI) [14] and Direct-3D [15] for rendering. Microsoft's Active X technology interfaces MoSART with MATLAB/Simulink.

MoSART Functionality: Visualizing Tilt-Wing Dynamics The tilt-wing aircraft MoSART environment can be used to visualize open-loop dynamics in order to convey the instability of the vehicle and the need for a control system. The MoSART environment can also be used for analyzing the performance of a closed-loop system which makes use of a designed controller. The aircraft can be "flown" in real-time using a joystick in order to provide the user with an immediate and intuitive understanding of the vehicles performance and handling properties.

V. SUMMARY AND FUTURE DIRECTIONS

This paper describes an experimental tilt-wing aircraft that is under development at Arizona State University. The lateral and longitudinal dynamics of this aircraft are examined and $\mathcal{H}^\infty$ control designs have been developed at the cruise and hover operating points. The choice of weighting functions is influenced not only by the desired performance at each operating point, but also by the desire to create a gain-scheduled conversion control system using these designs as a basis. The four designs were analyzed in the frequency and time domains to validate their performance. A Modeling, Simulation, Animation, and Real-Time Control (MoSART) environment helps visualize open loop tilt-wing dynamics and closed loop system performance.

Topics to be address in the future include the following:

- updating the model to improve fidelity, including the design of an LPV model for the aircraft,
- including i_w as state in order to capture rotating wing dynamics, [1],
- reduced order controller design,
- gain-scheduling to permit hover-to-cruise conversion; traditional and LPV gain-scheduling techniques will be considered [16], [17].

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