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Robust Fault Diagnosis for a Satellite System Using a Neural Sliding Mode Observer

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Abstract— **In this paper a nonlinear observer which synthesizes sliding mode techniques and neural state space models is proposed and is applied for robust fault diagnosis in a class of nonlinear systems. The sliding mode term is utilized to eliminate the effect of system uncertainties, and the switching gain is updated via an iterative learning algorithm. Moreover, the neural state space models are adopted to estimate state faults. Theoretically, the robustness, sensitivity, and stability of this neural sliding mode observer-based fault diagnosis scheme are rigorously investigated. Finally, the proposed robust fault diagnosis scheme is applied to a satellite dynamic system and simulation results illustrate its satisfactory performance.**

I. INTRODUCTION

Due to the importance of safety and reliability of the control systems in many complex system applications, fault detection, isolation, identification and accommodation have received considerable attention over the past two decades. Prompt fault detection indicates the occurrence of faults. Correct fault isolation determines the locations of the faults. Precise fault identification specifies the characteristics of the faults. All of this work helps to develop fault accommodation strategies to guarantee failsafe operations of the control systems.

In the categories of fault diagnosis (FD) techniques, analytical redundancy approaches based on linear or nonlinear models have been widely considered. Fruitful contributions are summarized in the books [1], [2], [3]. In general, model-based fault diagnosis methods generate a residual via comparing the measurable output of a system with that of its mathematical model. Then, fault diagnostic decisions are made based on the residual.

Efficient fault diagnosis depends on the robustness of the residual with respect to system uncertainties. For linear systems, robust fault diagnosis can be obtained via unknown input observers and eigenstructure assignment methods, both of which decouple the effect of the uncertainties from the residual. For nonlinear systems, learning approaches based FD schemes, which use neural networks [4], [5] or adaptive observers [6], [7], [8] to estimate faults have been investigated in many literatures. Dead-zone operators are always adopted in the learning algorithms to achieve a robust estimation of the faults [9], [10].

Owing to the inherent robustness to system uncertainties, sliding mode observers have been applied to the fault detection and diagnosis [11], [12], [13]. In order to guarantee the stability of the fault diagnosis scheme, the bound of system uncertainties is usually estimated and involved in the design of the switching gain. However, a large amount of chattering occurs when this method is implemented by digital computers at a given sampling frequency. Thus, a variety of approaches have been proposed to reduce the unnecessary chattering. One method is to use a continuous saturation function rather than the discontinuous *sign* function. Other methods adaptively estimate the bound of the system uncertainties [14] or construct an adaptive switching gain [15].

This work establishes a nonlinear observer and applies it to the fault diagnosis of a class of nonlinear systems. The observer consists of an adaptive sliding mode term and a neural state space (NSS) model. The sliding mode term is used to eliminate the effect of the system uncertainties, and the NSS model is adopted to identify various faults. In this fault diagnosis scheme, the adaptive switching gain avoids unnecessary chattering, and the iterative learning algorithm can be easily implemented. Additionally, This fault diagnosis scheme is not only robust to the system uncertainties, but also able to identify various faults with satisfactory performance. Finally, the application of the proposed FD scheme to a satellite control system demonstrates its effectiveness.

II. PROBLEM FORMULATION

The class of nonlinear dynamic systems under this study is described by

$$\begin{aligned}\dot{x}_i(t) &= \xi_i(x_1, x_2, \cdots, x_n) + B_i(y,u) + \eta_i(x,u,t) \\ &\quad + f_i(y,u,t) \\ \dot{x}_{i+1}(t) &= x_i(t), \qquad (i = 1, 3, \cdots, n-1) \\ y(t) &= [x_2, x_4, \cdots, x_n]^\top, \end{aligned} \tag{1}$$

where $x = [x_1, \cdots, x_n]^\top \in \Re^n$ with $x(0) = x_0$ is the state vector, $u \in \Re^m$ is the control input vector, and $y \in \Re^p$ is the measurable output vector of the system. The vector $\xi(x) = [\xi_1(x), x_1, \cdots, \xi_{n-1}(x), x_{n-1}]^\top$ is defined as the state function, $B(x,u) = [B_1(y,u), 0, \cdots, B_{n-1}(y,u), 0]^\top$ denotes the input function, $\eta = [\eta_1(t), 0, \cdots, \eta_{n-1}(t), 0]^\top$ represents the uncertainty vector, and $f = [f_1(t), 0, \cdots, f_{n-1}(t), 0]^\top$ is the state fault vector.

In a vector form, (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= \xi(x(t)) + B(y,u) + \eta(x,u,t) + f(y,u,t) \\ y(t) &= Cx(t), \end{aligned} \tag{2}$$

where $\eta : \Re^n \times \Re^m \times \Re^+ \to \Re^n$, $f : \Re^p \times \Re^m \times \Re^+ \to \Re^n$ are all smooth vector fields.

Remark 1: The system (1) only contains modeling uncertainties and state faults, and our work focuses on the robust diagnosis of state faults in the presence of state uncertainties.

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For convenience of analysis, the following assumptions are introduced.

Assumption 1: The state function $\xi(x(t))$ is differentiable at $\hat{x}$, that is

$$A(t) = \frac{\partial \xi}{\partial x}\bigg|_{x=\hat{x}}$$

where $A(t)$ is an $n \times n$ matrix. Hence, the following equation can be obtained through series expansion of $\xi(x)$ at $\hat{x}$.

$$\xi(x) - \xi(\hat{x}) = A(t)\tilde{x}(t) + \psi(\hat{x}, x), \tag{3}$$

where $\|\psi(\hat{x}, x)\| \leq k_\psi \|\tilde{x}(t)\|$, and $\tilde{x}(t) = x(t) - \hat{x}(t)$ is the state estimation error.

Assumption 2: A symmetric matrix $P_e(t)$ satisfies inequality $\beta_1 I \leq P_e(t) \leq \beta_2 I$, where β_1 and β_2 are two positive real numbers. Moreover, $P_e(t)$ is the solution of the following Lyaponov equation:

$$A^\top P_e(t) + \dot{P}_e(t) + P_e(t)A = -Q, \quad Q = Q^\top > 0, \tag{4}$$

where A is defined in Assumption 1.

Assumption 3: The uncertainty vector η is unstructured and bounded, i.e., $\|\eta\| \leq \eta_0$.

III. FAULT DIAGNOSIS STRATEGY

In this section, a nonlinear observer which integrates a sliding mode term and NSS models is established for detecting and identifying faults of the systems represented by (1).

A. Neural Sliding Mode Observer

Based on the system representation (1), a nonlinear diagnostic observer is proposed as follows:

$$\begin{aligned}
\dot{\hat{x}}_i(t) &= \hat{\xi}_i(\hat{x}_1, y_1, \cdots, \hat{x}_{n-1}, y_n) + B_i(y, u) + \hat{\theta}_i(t) \\
&\quad + g_i(t) sign(s_i(t)) \\
\dot{\hat{x}}_{i+1}(t) &= \hat{x}_i(t) + \hat{\theta}_{i+1}(t), \qquad (i = 1, 3, \cdots, n-1) \\
\hat{y}(t) &= [\hat{x}_2, \hat{x}_4, \cdots, \hat{x}_n]^\top,
\end{aligned} \tag{5}$$

where $\hat{x}_i$ is the ith state of the observer, and $\hat{y}$ is the output vector of the observer. The term $sign$ is a signum function, and $\hat{\theta}_i(t)$ is the ith NSS model represented in [17], [18] as

$$\dot{\hat{\theta}}_i(t) = W_{i,1}(t)\hat{\theta}_i(t) + W_{i,2}(t)\sigma(W_{i,3}(t)\hat{\theta}_i(t) + W_{i,4}(t)s_i(t)) \tag{6}$$

where $s_i(t)$ is chosen according to the following rule:

$$\begin{aligned}
s_i(t) &= (\hat{\theta}_{i+1}(t))_{eq} \\
s_{i+1}(t) &= x_{i+1}(t) - \hat{x}_{i+1}(t)
\end{aligned} \tag{7}$$

$W_{i,j}$, $(j = 1, \cdots, 4)$ is the parameters of the NSS model. The activation function is selected to be the tangent hyperbolic function $\sigma(z) = (1 - e^{-z})/(1 + e^{-z})$.

The $(\hat{\theta}_{i+1}(t))_{eq}$ is computed based on the equivalent control method, i.e., $\dot{\tilde{x}}_{i+1}(t) = (x_i(t) - \hat{x}_i(t)) - \hat{\theta}_{i+1}(t)$, when $\dot{\tilde{x}}_{i+1}(t) = 0$.

The dynamics of the observer can be written in a vector form as

$$\begin{aligned}
\dot{\hat{x}}(t) &= \hat{\xi}(\hat{x}, y) + B(y, u) + G(t) sign(S(t)) + \hat{\theta}(t) \\
\hat{y}(t) &= C\hat{x},
\end{aligned} \tag{8}$$

where $B(y, u) = [B_1, 0, \cdots, B_{n-1}, 0]^\top$ is the nonlinear input vector. $G = diag\{g_1, 0, \cdots, g_{n-1}, 0\}$ is a diagonal gain matrix. The sliding mode surface is defined as $S(t) = [s_1(t), \cdots, s_n(t)]^\top$. $\hat{\theta}(t) = [\hat{\theta}_1, \cdots, \hat{\theta}_n]^\top$ is the vector of neural state space models.

Defining $\tilde{y}(t) = y(t) - \hat{y}(t)$ as the output estimation error, based on Assumption 1, the dynamics of the estimation error can be derived by subtracting (8) from (2)

$$\begin{aligned}
\dot{\tilde{x}}(t) &= A\tilde{x}(t) + \psi(\tilde{x}) + \eta(x, u, t) - G(t) sign(S(t)) + \tilde{\theta}(t) \\
\tilde{y}(t) &= C\tilde{x}(t),
\end{aligned} \tag{9}$$

where $\tilde{\theta}(t) = f(y, u, t) - \hat{\theta}(t)$ is the fault estimation error.

B. Adaptive Switching Gain

When designing the switching gain of the sliding mode term, we need to avoid unnecessary high-frequency chattering. Some adaptation laws of the switching gain have been introduced in [16]. Here, a *P-type* iterative learning update law is proposed for the switching gain as follows

$$G_{j+1}(t) = G_j(t) + \Phi|S_j(t)| \cdot sign(S_j(t) \cdot S_{j-1}(t)), \tag{10}$$

where j indicates the iteration number at time t. Φ is a positive definite iterative learning gain matrix which determines the rate of convergence. The operator $|\cdot|$ takes the absolute value of each element in a vector. The result of the dot multiplication of two vectors is still a vector whose element is the product of the corresponding two elements in each vector.

Remark 2: From the adaptation law (10), we see that if the system has not reached the sliding surface (the switching gain should be larger), the element of $sign(S_j(t) \cdot S_{j-1}(t))$ is +1, and the switching gain will increase. If the system cross the sliding surface (the switching gain should be less), the component of $sign(S_j(t) \cdot S_{j-1}(t))$ is -1, and the gain will decrease correspondingly.

The convergence property of the proposed switching gain update law is analyzed in the following theorem.

Theorem 1: If the inequality (15) holds, the iterative update law (10) for the switching gain is convergent.

Proof: Subtracted by G^* from both sides of (10) yields

$$\Delta G_{j+1} = \Delta G_j - \Phi|S_j| \cdot sign(S_j \cdot S_{j-1}), \tag{11}$$

where G^* is the ideal switching gain, and $\Delta G_j = G^* - G_j$. Inner products of both sides of (11) with themselves via Φ^{-1}, we have

$$\begin{aligned}
\Delta G_{j+1}^\top \Phi^{-1} \Delta G_{j+1} &= \Delta G_j^\top \Phi^{-1} \Delta G_j + |S_j|^\top \Phi |S_j| \\
&\quad - 2\Delta G_j^\top |S_j| \cdot sign(S_j \cdot S_{j-1}).
\end{aligned} \tag{12}$$

Integration of (12) over the time interval $[0,\ t]$ results in

$$\begin{aligned}
&\|\Delta G_{j+1}\|_{\Phi^{-1}}^2 \\
&= \|\Delta G_j\|_{\Phi^{-1}}^2 + \|S_j\|_\Phi^2 \\
&\quad -2 \int_0^t \Delta G_j^\top(\tau)|S_j(\tau)| \cdot sign(S_j(\tau) \cdot S_{j-1}(\tau)) d\tau \\
&= \|\Delta G_j\|_{\Phi^{-1}}^2 + \|S_j\|_\Phi^2 - 2 \int_0^t |\Delta G_j^\top(\tau)||S_j(\tau)| d\tau,
\end{aligned} \tag{13}$$

where $\|\cdot\|_{\Phi^{-1}}$ is defined as

$$\|\Delta G_j\|_{\Phi^{-1}} = \left(\int_0^t \Delta G_j^\top(\tau) \Phi^{-1} \Delta G_j(\tau) d\tau \right)^{1/2} \tag{14}$$

If the estimation error dynamics satisfies dissipativity, i.e., there exists a positive constant α such that

$$\begin{aligned}\int_0^t |\Delta G_j^\top(\tau)||S_j(\tau)|d\tau &\geq \int_0^t \Delta G_j^\top(\tau)S_j(\tau)d\tau \\ &\geq \frac{1+\alpha}{2}\int_0^t S_j(\tau)^\top \Phi S_j(\tau)d\tau \\ &= \frac{1+\alpha}{2}\|S_j\|_\Phi^2. \end{aligned} \tag{15}$$

Then, it follows from (13) that

$$\|\Delta G_{j+1}\|_{\Phi^{-1}}^2 \leq \|\Delta G_j\|_{\Phi^{-1}}^2 - \alpha\|S_j\|_\Phi^2. \tag{16}$$

This inequality implies that the sequence $\{\|\Delta G_j\|_{\Phi^{-1}}\}$ will monotonously decrease with increasing j as long as $\|S_j\|_\Phi$ is nonzero. Since $\{\|\Delta G_j\|_{\Phi^{-1}}\}$ is bounded from below, the monotonous decrease of $\{\|\Delta G_j\|_{\Phi^{-1}}\}$ means $\|S_j\|_\Phi \to 0$ as $j \to \infty$. Thus, $G_j(t) \to G^*(t)$ as $j \to \infty$, that is, the switching gain update law is convergent.

Remark 3: The inequality (15) is able to be guaranteed by choosing a suitable iterative learning gain Φ. Normally, a small Φ results in a steadily but slowly convergent process. A large Φ leads to a fast convergence rate, though the iterative learning process may be unstable.

In order to make the sliding mode term only eliminate the deviation in the dynamics caused by the system uncertainties, the switching gain $G(t)$ is set to be bounded by G_0, i.e., $\|G(t)\| < G_0$, which is set to $G_0 = \frac{\epsilon(1-\lambda k_\psi)}{\lambda\|C\|} - \eta_0$. The ϵ is a positive constant used to indicate a fault, which implies

$$\begin{array}{ll} \|\tilde{y}(t)\| < \epsilon & \text{no fault occurs} \\ \|\tilde{y}(t)\| \geq \epsilon & \text{fault has occurred} \end{array} \tag{17}$$

where λ is defined to be $\lambda = \int_0^\infty \left\|e^{At}\right\| dt$. The setting of the upper bound distinguishes the effects of faults from those of the system uncertainties.

C. Update Law for Neural State Space Model

The parameters of the NSS model are updated by using a modified extended Kalman filter algorithm:

$$\begin{aligned} K_i(k) &= P_i(k)H_i(k)\left[H_i(k)^\top P_i(k)H_i(k) + R_i(k)\right]^{-1} \\ P_i(k+1) &= P_i(k) - K_i(k)H_i(k)^\top P_i(k) \\ W_i(k+1) &= W_i(k) + K_i(k)D[e_i(k)], \end{aligned} \tag{18}$$

where k denotes discrete sampling time. $W_i(k)$ is the weight vector in the NSS model. $K_i(k)$ is known as the Kalman gain matrix. $P_i(k)$ is the covariance matrix of the state estimation error, and $R_i(k)$ is the estimated covariance matrix of noise. For SISO systems, $R_i(k)$ is recursively calculated by:

$$R_i(k) = R_i(k-1) + \left[e_i(k)^2 - R_i(k-1)\right]/k. \tag{19}$$

$H_i(k)$ is the derivative of the NSS model output with respect to its weight W_i. The estimation error is defined as $e_i = s_i - \hat{\theta}_i$. The dead-zone operator $D[\cdot]$ is defined as

$$D[e_i(k)] = \begin{cases} 0 & \text{if } |s_i(k)| < \delta_{si} \\ e_i(k) & \text{otherwise} \end{cases} \tag{20}$$

Remark 4: The dead-zone operator is introduced to make the observer robust with respect to system uncertainties. When the magnitude of s_i is greater than the bound δ_{si}, which implies the occurrence of a fault f_i, the parameters of the NSS model will be updated to drive the model to approximate the fault. Otherwise, the system is considered to be healthy, and the output of the NSS model remains zero, even though $s_i(k)$ is nonzero.

The convergence property of the modified EKF algorithm is analyzed in the following theorem.

Theorem 2: The parameter update law (18) is convergent, provided that $P_i(k)$ is a positive definite matrix.

Proof: Since for SISO systems, $R_i(k)$ is the estimated variance of noise, it keeps nonnegative in all iterations. Thus, if $P_i(k)$ is positive definite, the following inequality can be guaranteed.

$$H_i(k)^\top P_i(k)H_i(k) > 0. \tag{21}$$

Hence, with (21) we have

$$0 < H_i(k)^\top P_i(k)H_i(k) < 2[H_i(k)^\top P_i(k)H_i(k) + R_i(k)],$$

which is equivalent to

$$0 < H_i(k)^\top P_i(k)H_i(k)[H_i(k)^\top P_i(k)H_i(k) + R_i(k)]^{-1} < 2.$$

Based on the update law (18), we have

$$0 < H_i(k)^\top K_i(k) < 2. \tag{22}$$

Consider a positive Lyapunov function candidate:

$$V(k) = \frac{1}{2}e_i(k)^2. \tag{23}$$

The first order difference of (23) is

$$\begin{aligned} \Delta V(k) &= \frac{1}{2}\left[e_i(k+1)^2 - e_i(k)^2\right] \\ &= \Delta e_i(k)\left[e_i(k) + \frac{1}{2}\Delta e_i(k)\right]. \end{aligned} \tag{24}$$

The difference of $e_i(k)$ can be approximated by

$$\begin{aligned} \Delta e_i(k) &= \left[\frac{\partial e_i(k)}{\partial W_i}\right]^\top \Delta W_i \\ &= -H_i(k)^\top K_i(k)D[e_i(k)]. \end{aligned} \tag{25}$$

Hence, based on (22) and (25), when $|s_i| \geq \delta_{si}$, (24) is further derived to be

$$\begin{aligned} \Delta V(k) &= -H_i(k)^\top K_i(k)\left[1 - \frac{1}{2}H_i(k)^\top K_i(k)\right]e_i(k)^2 \\ &< 0, \end{aligned} \tag{26}$$

and when $|s_i| < \delta_{si}$, $\Delta V(k) = 0$. Therefore, in summary, if the condition (21) is guaranteed, then $\Delta V(k) < 0$, which implies the parameter update process using the modified EKF algorithm is convergent.

Remark 5: The convergence of the modified EKF learning algorithm implies that $\lim_{t\to\infty} |f_i(t) - \hat{\theta}_i(t)| = 0$, or $\|f(t) - \hat{\theta}(t)\| \leq \delta_f$ after a finite time, where δ_f is the upper bound of the fault estimation error.

Remark 6: In numerical computation, in order to guarantee (21), $P_i(0)$ is usually set to a large diagonal matrix.

IV. ANALYTICAL PROPERTIES

The purpose of this section is to obtain some theoretical guarantees in the aspects of robustness, sensitivity and stability of the proposed observer based fault diagnosis scheme.

A. Robustness

Robustness of a fault diagnosis scheme refers to its ability to prevent false alarms in the presence of system uncertainties. As for the fault diagnosis scheme described above, robustness is achieved by using a dead-zone operator in the learning algorithm of NSS models and the upper bound of the adaptive switching gain.

Consider the time interval prior to the occurrence of any fault, i.e., $t \in [0, T_x)$, where T_x refers to the beginning time of a state fault.

Theorem 3: The robust fault diagnosis scheme developed in (5) guarantees that $\hat{\theta}(t) = 0$, when $t < T_x$.

Proof: Using Contradiction method, we suppose that there exists a finite time $t_e \in (0, T_x)$ such that $\|\tilde{y}(t)\| < \epsilon$ for $t < t_e$ and

$$\|\tilde{y}(t_e)\| = \epsilon. \tag{27}$$

Prior to any fault, the dynamics of the estimation error is

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + \psi(\tilde{x}) + \eta(t) - G(t)sign(S(t)) \\ \tilde{y}(t) &= C\tilde{x}(t), \qquad \tilde{x}(0) = 0. \end{aligned} \tag{28}$$

By solving this differential equation (28), we obtain

$$\begin{aligned} &\|\tilde{x}\| \\ &\leq \int_0^{t_e} \mathrm{e}^{A(t_e-\tau)} \; \|\psi(\tilde{x}) + \eta - Gsign(S(\tau))\| \, d\tau \\ &\leq \lambda(k_\psi\|\tilde{x}\| + \eta_0 + \|G\|). \end{aligned} \tag{29}$$

Therefore, the output estimation error is

$$\|\tilde{y}\| < \frac{\|C\|\lambda(\eta_0 + G_0)}{1 - \lambda k_\psi} = \epsilon. \tag{30}$$

This contradicts (27). Therefore, It is concluded that for all $t < T_x$, the output estimation error $\tilde{y}(t)$ remains within the bound ϵ, and the output of the NSS model remains zero.

B. Sensitivity

Not only do we want the fault diagnosis scheme to be robust to the system uncertainties, but also we hope that it is sensitive to any faults. However, an inherent tradeoff exists between the robustness and sensitivity of the fault diagnosis scheme, because high sensitivity to faults may reduce its robustness to the system uncertainties. Sensitivity properties focus on the characteristics of the fault diagnosis scheme in the time interval between the occurrence of a fault and the time of its detection.

Theorem 4: Consider the fault diagnosis scheme presented by (5). If there exists a time interval $t_x > 0$ such that the state fault $f(t)$ satisfies

$$\int_{T_x}^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} f(\tau) d\tau \;\; \geq \frac{(\|C\|\|C^\dagger\| + 1)}{\|C\|}\epsilon, \tag{31}$$

then the state fault will be detected, i.e., $\|\tilde{y}(T_x + t_x)\| \geq \epsilon$.

Proof: In the time interval between the occurrence of a state fault and the adaptation of the NSS model, the dynamics of the estimation error satisfy

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + \psi(\tilde{x}) + \eta(t) - G(t)sign(S(t)) + f(t) \\ \tilde{y}(t) &= C\tilde{x}(t). \end{aligned} \tag{32}$$

Solving (32) for any $t_x > 0$ gives

$$\begin{aligned} &\tilde{x}(T_x + t_x) \\ &= \int_0^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} \left[\psi(\tilde{x}) + \eta(\tau) - G(\tau)sign(S(\tau))\right] d\tau \\ &\quad + \int_{T_x}^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} f(\tau) d\tau. \end{aligned} \tag{33}$$

Then, using the triangle inequality and the result from (29), we obtain

$$\begin{aligned} &\|\tilde{x}(T_x + t_x)\| \\ &\geq -\int_0^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} \psi(\tilde{x}) d\tau \\ &\geq -\int_0^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} \eta(\tau) d\tau \\ &\quad - \int_0^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} G(\tau) d\tau \\ &\quad + \int_{T_x}^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} f(\tau) d\tau \\ &\geq -\frac{\lambda(\eta_0 + \|G\|)}{1 - \lambda k_\psi} + \int_{T_x}^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} f(\tau) d\tau \;\; . \end{aligned} \tag{34}$$

Using matrix properties, the output estimation error satisfies

$$\begin{aligned} \|\tilde{y}(T_x + t_x)\| &\geq \frac{\|\tilde{x}(T_x + t_x)\|}{\|C^\dagger\|} \\ &\geq -\frac{\lambda(\eta_0 + \|G\|)}{\|C^\dagger\|(1 - \lambda k_\psi)} \\ &\quad + \frac{1}{\|C^\dagger\|} \int_{T_x}^{T_x+t_x} \mathrm{e}^{A(T_x+t_x-\tau)} f(\tau) d\tau \;\;, \end{aligned} \tag{35}$$

where $C^\dagger$ is the left pseudo-inverse of C.

Therefore, if the state fault function satisfies (31), then $\|\tilde{y}(T_x + t_x)\| \geq \epsilon$, which implies that the state fault will be detected, and the parameters of the NSS model will be updated correspondingly.

C. Stability

In above subsections, the robustness and sensitivity of the proposed fault diagnosis scheme have been discussed respectively. Another key performance of a fault diagnosis scheme is its stability. In this subsection, stability of the fault diagnosis scheme (5) after the occurrence of a state fault is analyzed as follows.

Theorem 5: In the presence of state faults $f(t)$, the proposed nonlinear robust fault diagnosis scheme (5) guarantees that the state estimation error $\tilde{x}(t)$ is uniformly bounded by,

$$\sup \|\tilde{x}(t)\| \leq \sqrt{\frac{\mu^0}{\lambda_{min}(P_e)}} \tag{36}$$

with $\mu^0 = \max\left(V(\tilde{x}(0)), \dfrac{\epsilon^0}{\lambda_{min}(R_e)}\right)$, where $R_e = \rho P_e^{-1}$, and $\epsilon^0 = (\eta_0 + G_0 + \delta_f)\beta_2^2$.

Proof: Construct a Lyapunov function candidate:

$$V(t) = \tilde{x}(t)^\top P_e(t)\tilde{x}(t), \tag{37}$$

where $P_e(t)$ is given in Assumption 2, and Q is chosen such that $\rho = \lambda_{min}(Q) - 2k_\psi\|P_e\| - \eta_0 - G_0 - \delta_f \geq 0$.

Based on Assumption 2, the derivative of $V(t)$ with respect to time t is

$$
\begin{aligned}
&\dot{V}(t)\\
&= \dot{\tilde{x}}(t)^\top P_e(t)\tilde{x}(t) + \tilde{x}(t)^\top \dot{P}_e(t)\tilde{x}(t) + \tilde{x}(t)^\top P_e(t)\dot{\tilde{x}}(t)\\
&= [A\tilde{x}(t) + \psi(\tilde{x}) + \eta(t) - G(t)sign(S(t)) + \tilde{\theta}(t)]^\top P_e(t)\tilde{x}(t)\\
&\quad +\tilde{x}(t)^\top \dot{P}_e(t)\tilde{x}(t)\\
&\quad +\tilde{x}(t)^\top P_e(t)[A\tilde{x}(t) + \psi(\tilde{x}) + \eta(t) - G(t)sign(S(t)) + \tilde{\theta}(t)]\\
&= \tilde{x}(t)^\top (A^\top P_e(t) + \dot{P}_e(t) + P_e(t)A)\tilde{x}(t)\\
&\quad +2\tilde{x}(t)^\top P_e(t)\psi(\tilde{x}) + 2\tilde{x}(t)^\top P_e(t)\eta(t)\\
&\quad -2\tilde{x}(t)^\top P_e(t)G(t)sign(S(t)) + 2\tilde{x}(t)^\top P_e(t)\tilde{\theta}(t)\\
&= -\tilde{x}(t)^\top Q\tilde{x}(t) + 2\tilde{x}(t)^\top P_e(t)\psi(\tilde{x}) + 2\tilde{x}(t)^\top P_e(t)\eta(t)\\
&\quad -2\tilde{x}(t)^\top P_e(t)G(t)sign(S(t)) + 2\tilde{x}(t)^\top P_e(t)\tilde{\theta}(t)\\
&\leq -\lambda_{min}(Q)\|\tilde{x}(t)\|^2 + 2k_\psi\|P_e\|\|\tilde{x}(t)\|^2\\
&\quad +2\|\tilde{x}(t)\|\|P_e\|(\eta_0 + G_0 + \delta_f)\\
&\leq -(\lambda_{min}(Q) - 2k_\psi\|P_e\|)\|\tilde{x}(t)\|^2\\
&\quad +(\|\tilde{x}(t)\|^2 + \|P_e\|^2)(\eta_0 + G_0 + \delta_f)\\
&= -\rho\|\tilde{x}(t)\|^2 + (\eta_0 + G_0 + \delta_f)\|P_e\|^2\\
&\leq -\rho\|\tilde{x}(t)\|^2 + \epsilon^0.
\end{aligned}
\tag{38}
$$

Therefore, when $\|\tilde{x}\| > \sqrt{\epsilon^0/\rho}$, $\dot{V} < 0$, i.e., the state estimation error is uniformly bounded. Rewrite (38) to be

$$
\begin{aligned}
\dot{V}(\tilde{x}(t)) &\leq -\rho\tilde{x}(t)^\top\tilde{x}(t) + \epsilon^0\\
&= -\tilde{x}(t)^\top P_e^{1/2}(\rho P_e^{-1})P_e^{1/2}\tilde{x}(t) + \epsilon^0\\
&\leq -\lambda_{min}(R_e)\tilde{x}(t)^\top P_e\tilde{x}(t) + \epsilon^0\\
&= -\lambda_{min}(R_e)V(\tilde{x}(t)) + \epsilon^0.
\end{aligned}
\tag{39}
$$

Solving the differential equation (39) obtains the following inequality

$$
\begin{aligned}
V(\tilde{x}(t)) &\leq V(\tilde{x}(0))\mathrm{e}^{-\lambda_{min}(R_e)t}\\
&\quad +\epsilon^0\int_0^t \mathrm{e}^{-\lambda_{min}(R_e)(t-\tau)}d\tau\\
&= V(\tilde{x}(0))\mathrm{e}^{-\lambda_{min}(R_e)t}\\
&\quad +\frac{\epsilon^0}{\lambda_{min}(R_e)}\left(1 - \mathrm{e}^{-\lambda_{min}(R_e)t}\right).
\end{aligned}
\tag{40}
$$

From (37), we have

$$
V(\tilde{x}(t)) \geq \lambda_{min}(P_e)\|\tilde{x}(t)\|^2. \tag{41}
$$

Then, the following inequality can be obtained

$$
\|\tilde{x}(t)\| \leq \sqrt{\frac{V(\tilde{x}(t))}{\lambda_{min}(P_e)}} \tag{42}
$$

Thus,

$$
\sup\|\tilde{x}(t)\| \leq \sqrt{\frac{\sup V(\tilde{x}(t))}{\lambda_{min}(P_e)}} \tag{43}
$$

From (40), due to the first order exponential nature of its right-hand side, the upper bound of $V(\tilde{x}(t))$ is

$$
\sup V(\tilde{x}(t)) \leq \max\left(V(\tilde{x}(0)), \frac{\epsilon^0}{\lambda_{min}(R_e)}\right) \equiv \mu^0. \tag{44}
$$

Therefore,

$$
\sup\|\tilde{x}(t)\| \leq \sqrt{\frac{\mu^0}{\lambda_{min}(P_e)}} \tag{45}
$$

V. AN APPLICATION EXAMPLE

In this section, the proposed robust fault diagnosis scheme is applied to a fourth-order satellite dynamic system, which is described in [10]. The original model of the system is

$$
\begin{aligned}
\dot{r} &= v & r(0) &= r_0\\
\dot{v} &= rw^2 - \frac{k}{mr^2} + \frac{u_1}{m} & v(0) &= 0\\
\dot{\phi} &= w & \phi(0) &= 0\\
\dot{\omega} &= -\frac{2v\omega}{r} + \frac{u_2}{mr} & \omega(0) &= \omega_0
\end{aligned}
\tag{46}
$$

where $m = 200$kg is the mass of the satellite, (r, ϕ) are the polar coordinates of the satellite, v is the radial speed, and ω is the angular speed. Control inputs u_1 and u_2 are the radial and tangential thrust forces, respectively.

In the simulation, the mass of the satellite is supposed to be underestimated by $\varsigma_x = 3\%$ $(m^* = m(1 - \varsigma_x), k^* = k(1 - \varsigma_x))$.

Using a local diffeomorphism, (46) can be converted into a state space model where (A, C) are given by

$$
A = \begin{matrix} 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 \end{matrix}, \quad C = \begin{matrix} 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{matrix}
$$

The nonlinear term $B(y, u)$ is expressed as

$$
B(y, u) = \begin{matrix} -\frac{y_1}{(y_1^2 + y_2^2)^{3/2}}\frac{k^*}{m^*} + \frac{u_1y_1 + u_2y_2}{(y_1^2 + y_2^2)^{1/2}}\frac{1}{m^*}\\ 0\\ -\frac{y_2}{(y_1^2 + y_2^2)^{3/2}}\frac{k^*}{m^*} + \frac{u_1y_2 - u_2y_1}{(y_1^2 + y_2^2)^{1/2}}\frac{1}{m^*}\\ 0 \end{matrix}
$$

and the state uncertainty is represented by

$$
\eta(x, u, t) = \begin{matrix} -\frac{u_1x_2 + u_2x_4}{(x_2^2 + x_4^2)^{1/2}}\frac{\varsigma_x}{m^*}\\ 0\\ -\frac{u_1x_4 + u_2x_4}{(x_2^2 + x_4^2)^{1/2}}\frac{\varsigma_x}{m^*}\\ 0 \end{matrix}
$$

Based on the nominal model and (5), a neural sliding mode observer is designed, where NSS models $\hat{\theta}_1(t)$ is used to estimate state fault $f_1(t)$ which occurs in the first state. Since the sliding mode minimizes the output error caused by the system uncertainties, the dead-zone values can be set to be very small without losing robustness. In this simulation, the dead-zone in NSS models is set to $\delta_{si} = 2 \times 10^{-5}$.

The simulation results are shown in the Fig. 1 and Fig. 2, respectively. Fig. 1 illustrates the system outputs y_1 and y_2. Fig. 2 demonstrates the characteristics of $f_1(t)$ and NSS model outputs. In Fig. 1, the practical system output deviates from that of the nominal system when a state fault occurs, but the observer outputs follow the system outputs for all time. Moreover, all the outputs of the NSS models remain zero prior to the occurrence of a fault. After the occurrence of a state fault, only the NSS model that associates with the faulty state identifies the fault quickly, and other NSS model outputs are close to zero. These two figures shows that the proposed FD scheme is a reliable fault detection and diagnosis method.

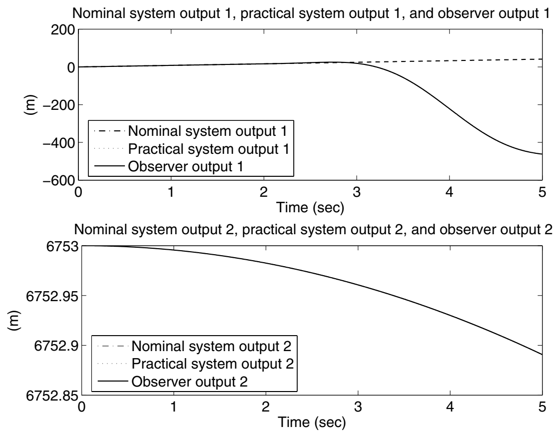

Fig. 1. Time-behavior of system outputs under fault 1

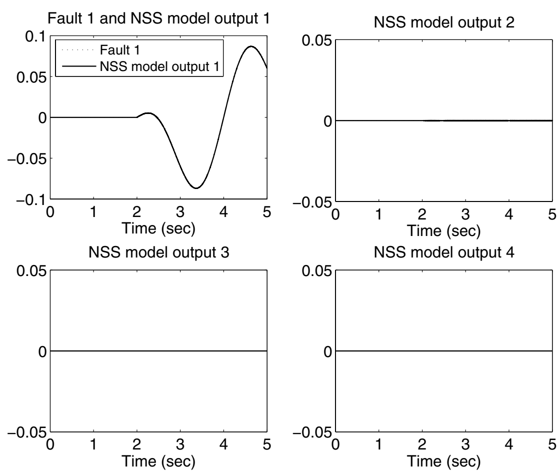

Fig. 2. Time-behavior of fault 1 and outputs of NSS model

VI. CONCLUSIONS

In this paper, a neural sliding mode observer-based fault diagnosis scheme for a class of nonlinear systems is investigated. In this scheme, an adaptive sliding mode term is used to diminish the effect of the state uncertainties, and NSS models are adopted to identify state faults. The switching gain of the sliding mode is updated via an iterative learning algorithm, and the parameters in the NSS models are updated through a modified EKF algorithm. Theoretically, the convergence of these two update laws are proved respectively. Moreover, the robustness, sensitivity and stability properties are all rigorously analyzed. Practically, the proposed FD scheme is applied to a satellite, and the simulation results demonstrate its effectiveness.

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