Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Saturation Prevention for MIMO LPV Controllers: An Error Governer Approach

Oguzhan Cifdaloz, Armando A. Rodriguez, Yu-Lung Yi, and Richard Steenis

Abstract— **In many applications, it is essential to limit control signals. This paper presents methods that process accessible system signals (e.g., plant and compensator states, reference commands) to limit control signals. In contrast to "classical techniques" which limit variables by unnecessarily reducing control system bandwidth, the proposed methods limit variables in a nonconservative manner, reducing bandwidth only when necessary. A fundamental contribution of this work is that the methods presented are applicable to linear parameter varying (LPV) and quasi-LPV plants and controllers. The methods presented rely on appropriately "scaling back" key signals. The methods are conservative in that they are applicable to "slowly varying" LPV systems. Although the methods require access to all internal closed loop system state variables, one could use an appropriately designed LPV state estimator, if necessary. The Error Governer (EG) methods are applied to an LPV missile autopilot example.**

I. INTRODUCTION

Motivation: Status-of-Field. During the last decade, the research community has embraced the linear parameter varying (LPV) framework as one way to model and design gain scheduled control systems [1], [2], [3]. Gain scheduling refers to the "gluing" of LTI controllers that have been designed on the basis of LTI plant models at specific operating points for a nonlinear plant. While traditional methods lack guarantees and are not systematic (e.g., LTI controller "gluing" process), the LPV framework offers some nominal guarantees and a natural method for scheduling the LTI controllers. Recently, polytopic system representations and linear matrix inequalities (LMIs) have been used to design LPV controllers which exhibit induced $\mathcal{L}^2$-norm γ-performance [4], [5], [6], [7]. In this paper, we present new saturation prevention methods for "slowly" varying LPV systems.

Typically, initial control system designs are based on linear models and neglect nonlinearities such as saturating actuators and desired variable limits. Traditionally, such nonlinearities and requirements have been addressed in a conservative manner; sacrificing performance to satisfy some "small-gain" design criterion [8]. For single-input single-output (SISO) systems, methods have been developed to improve performance [9]. These methods, however, do not readily extend to multivariable applications [10], [11].

Oguzhan Cifdaloz and Richard Steenis are currently working towards a Ph.D. in Electrical Engineering, Arizona State University. ogu@asu.edu

Dr. Armando A. Rodriguez is a professor of Electrical Engineering at Arizona State University, Tempe, AZ 85282, USA. This research has been supported, in part, by the National Science Foundation (0231440, ECS-0401405) and by the Consortium for Embedded and Inter-Networking Technologies (05012112). aar@asu.edu

Dr. Yu-Lung Yi is currently with Raytheon Missile Systems, CA.

Our Approach: Saturation Prevention for LPV Systems. In an effort to overcome the above deficiencies, we show how an initial multiple-input multiple-output (MIMO) LPV control system design may be systematically modified to accommodate control variable limits and parameter variations without unnecessarily sacrificing performance. The idea is to introduce nonlinear gain operators that use available signals (e.g., plant and controller state variables, error signals, reference commands) and a "look-ahead" function to "scale-back" relevant signals (e.g., error signals). The methods presented assume that a nominal stabilizing LPV controller has been designed for an LPV plant. It is assumed that the nominal controller performs well without saturation nonlinearities. When saturation nonlinearities are inserted into the feedback loop, the nominal controller performs well for external signals that do not result in saturation (or control wind up). For sufficiently "large" external signals (e.g., reference commands, disturbances, and sensor noise), saturation and control wind-up results [12], [13], [14], [15], [16]. As expected, this degrades closed loop performance. The methods presented correct this performance degradation by appropriately "scaling-back" error signals entering the nominal controller. The "scaling-back" is achieved via an *error governor (EG) system.*

- **EG System.** An *error governor (EG) system* is used to appropriately scale-back error signals entering the nominal controller. Scale-back is based on the error signals entering the controller, the controller state, and information provided by a look-ahead function based on the controller's natural (unforced) response. By scaling back error signals, the EG addresses the impact of exogenous reference commands, disturbances, and sensor noises.

The governor methods presented prevent saturation, prevent control wind up, maintain nominal closed loop stability, and maintain, to the extent possible, the multivariable directionality properties of the original MIMO LPV control system [11], [17], [18], [19], [20], [21].

Literature Survey. The following literature survey is intended to establish a technical foundation and perspective for evaluating the contributions of this paper.

- *LPV Systems.* Relevant results on LPV system analysis and design may be found within [22], [1], [23], [5], [4], [24], [7].
- *Saturation Prevention.* The methods presented are based

0-7803-9568-9/05/$20.00

on the novel saturation prevention ideas in [17], the more recent work of [18], [20], [19]. Other references addressing saturation prevention, anti-windup strategies, and variable limiting strategies include [12], [13], [15].

Outline of Paper. The remainder of this paper is organized as follows. Section II discusses the notation, problem definition, and the fundamental assumption. Section III addresses the EG. Sections IVillustrate how the EG may be applied to a quasi-LPV missile.

II. NOTATION AND PROBLEM DEFINITION

Two closed loop feedback systems will be considered. The first, referred to as the *nominal closed loop system* possesses "good" closed loop properties when saturating actuators are neglected. The second feedback system includes saturating actuators and exhibits "poor" behavior when exogenous signals are large. With this motivation, the key issues to be addressed in this paper are discussed.

Nominal System. Consider the feedback system shown in Figure 1 where P represents a MIMO LPV plant (e.g. missile) and K represents a nominal MIMO LPV compensator. The outputs are y, the control signals generated by K are u, the actual inputs to P are u_p , and the error signals are e. The input reference commands are r, the disturbances to be rejected are ξ, and the sensor noise signals to be attenuated are η.

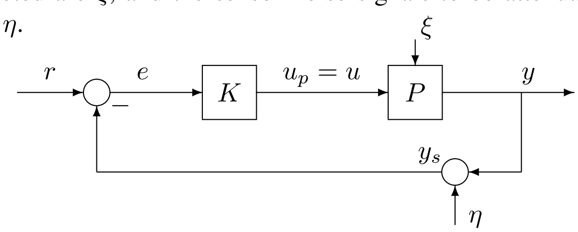

Fig. 1. Nominal Closed Loop System: "Good" Properties

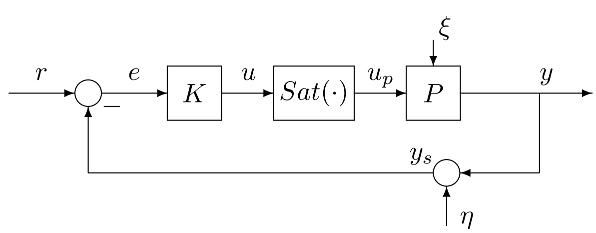

Fig. 2. Unsupervised System with Saturating Actuators: "Bad" Properties

It will be assumed that K is a finite-dimensional LPV controller which has been designed so that the closed loop system in Figure 1 has "desirable" properties (e.g. closed loop stability, induced-$\mathcal{L}^2$ performance).

Comment 2.1: **(Design of Nominal Compensator K)** Here, "desirable" properties can mean that K satisfies some *a priori* specified multivariable robustness and performance specifications.

The compensator K may be designed using any linear design methodology (e.g. $\mathcal{H}^\infty$, $\mathcal{H}^2$, $\mathcal{L}^1$, LQG/LTR, μ-synthesis, etc.) or LPV method [6], [4], [24], [3], [7]. ∎

Saturated System. To motivate the importance of signal limiting in feedback systems, we consider saturating actuators as shown within Figure 2. This system will be referred to as the *Unsupervised System.* In this figure, the plant input $u_p = [\, u_{p_1} \ldots u_{p_m} \,]^T$ is related to the control signal $u = [\, u_1 \ldots u_m \,]^T$ via multiple saturating actuators as follows:

$$u_p = Sat(u) \stackrel{\text{def}}{=} [\, sat(u_1) \ldots sat(u_m) \,]^T \tag{1}$$

where m is a positive integer denoting the number of control channels and

$$u_{p_i} = sat(u_i) \stackrel{\text{def}}{=} \begin{cases} 1 & \mathrm{u_i} \;<\; 1; \\ \mathrm{u_i} & |\mathrm{u_i}| \;\leq\; 1; \\ -1 & \mathrm{u_i} \;<\; -1 \end{cases} \tag{2}$$

The unity slopes and saturation levels are without loss of generality since saturations with different slopes and levels can be accommodated within this framework by scaling the controls u_i and plant inputs u_{p_i} appropriately. It is assumed that the feedback loop in Figure 1 possesses "nice" multivariable feedback properties, it will also be assumed that the system in Figure 2 does not possess "nice" properties. These assumptions establish the philosophical foundation upon which the proposed approach is based. The assumptions are summarized as follows.

Assumption 2.1: **(Fundamental Assumption)**
It is assumed that the compensator K is a finite-dimensional LPV system that has been designed so that the closed loop system in Figure 1 is $\mathcal{L}^\infty$ finite-gain stable [25] and has "good" multivariable feedback properties. It is also assumed that the feedback system in Figure 2 does not exhibit "nice" properties for sufficiently large exogenous signals. This is because for large signals, the saturating actuators may (1) alter the directionality properties of the original multivariable feedback design, (2) cause wind-up phenomena, (3) induce instability. ∎

III. ERROR GOVERNOR DESIGN METHODOLOGY

This section examines a method for preventing actuator saturation, control wind-up, and general "signal saturation." The method involves the design of a *saturation detection and prevention system*, called an *Error Governor (EG)* [17], [11] which maintains, to the extent possible, the multivariable properties of a nominal MIMO control system design K.

EG Structure. We begin by first addressing performance enhancement in the presence of saturating actuators. More specifically, this means eliminating wind-up phenomena, ensuring closed loop stability, and maintaining to the extent possible, maintaining the directionality properties of the original control system design and the performance of the original design K (Figure 1). Toward this end, the structure in Figure 3 is proposed [17], [11].

In this figure, K has a state space description (A_k, B_k, C_k, D_k) (dependence on ρ suppressed) with input $\lambda_e e$ and output u:

$$\dot{x}_k = A_k x_k + B_k \lambda_e e \qquad u = C_k x_k + D_k e. \tag{3}$$

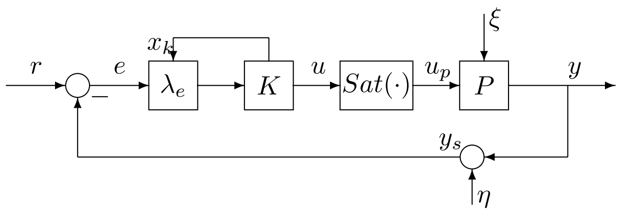

Fig. 3. Visualization of Error Governor (EG)

Given this, it follows that $u_p(t) = Sat(u(t))$ where $\lambda_e(t) \stackrel{\text{def}}{=} \lambda_e(x_k(t), e(t))$ is a nonlinear scalar gain which depends on the compensator state $x_k(t)$ and the error signal $e(t) \stackrel{\text{def}}{=} r(t) - y_s(t)$. Since λ_e directly multiplies the error signal, it (or the structure in Figure 3) is called an *Error Governor (EG)*. Figure 3 is a nonlinear system. This is in contrast to Figure 1, where $u_p(t) = u(t)$ and nominal performance is assured.

In this section, we shall assume that the plant P is stable. (Unstable plants are briefly addressed at the end of the section.) More precisely, we make the following assumption.

Assumption 3.1: **(Stable Plant)**
It will be assumed that the plant P is $\mathcal{L}^\infty$ finite-gain stable [25]; i.e. there exists a constant $c \in [0, \infty)$ such that

$$\|Pu_p\|_{\mathcal{L}^\infty} \ \le \ c \, \|u_p\|_{\mathcal{L}^\infty} \quad \text{for all} \quad u_p \in \mathcal{L}^\infty. \qquad \blacksquare$$

This assumption implies that the plant is bounded-input bounded-output (BIBO) stable. A necessary and sufficient condition on P for this to hold, when P is finite-dimensional, is that all of its poles lie in the open left half plane.

Selecting Nonlinear EG Gain λ_e. In [17], [11], a procedure for computing λ_e is developed. The idea behind the procedure is based on the following intuitive guidelines:

- When the system is operating "nominally" (or linearly if P is linear) as intended (i.e. controls not saturation), the gain λ_e is maintained at unity.
- When the system is on the "verge of saturation," the gain λ_e should be "appropriately" reduced toward zero.

Since λ_e is a scalar, such gain reduction preserves the relative coordination of the controls. Consequently, such gain reduction preserves the directionality properties of the original multivariable design K.

Assumption 3.2: **(Nominal Compensator $K(\rho)$)**
Suppose that the compensator has a state space representation $K = [A_k(\rho), B_k(\rho), C_k(\rho), D_k(\rho)]$, $\rho \in \mathcal{P}$ where $\mathcal{P}$ is a compact set. It will be assumed that $K(\rho)$ is LPV stable; i.e. there exists finite $M > 0$ s.t. $\|x_k\|_{\mathcal{L}^\infty} \le M \|x_k(t_o)\|_\infty$ for any $t_o \ge 0$. $\blacksquare$

To present the procedure for computing λ_e, the following definition is necessary.

Definition 3.1: **(EG Saturation Detection Function)** Define the function $g_e : R^n \longrightarrow R_+$ as follows:

$$g_e(x) \stackrel{\text{def}}{=} \sup_{\rho \in \mathcal{P}} \left\| C_k(\rho) e^{A_k(\rho)t} x \right\|_{\mathcal{L}^\infty} \tag{4}$$

and the set S_e as follows:

$$S_e \stackrel{\text{def}}{=} \{ \ x \in R^n \ : \ g_e(x) \le 1 \ \}. \tag{5}$$

Comment 3.1: **(Saturation "Detection" Via Look-Ahead Function)**
Homogeneous Response of Compensator: Quantifying Tendency to Saturate. Notice that the function g_e depends entirely on the homogeneous (unforced) response of the compensator, K. Because the $\mathcal{L}^\infty$ norm, in principle, requires access to the entire homogeneous response, it is natural to refer to g_e as a *look-ahead* function. One might characterize $g_e(x_k)$ as quantifying the *natural tendency* that K has to saturate when permitted to evolve from an initial condition x_k with no forcing term (i.e. $e = 0$). The function g_e will be used to look-ahead, detect (or anticipate) control saturation, and prevent control saturation. $\blacksquare$

Algorithm 3.1: **(Construction of λ_e)**

1) If x_k lies within S_e (i.e. $g_e(x_k) < 1$), choose the maximum $\lambda_e \in [0, 1]$ such that $\sup_\rho \|C_k(\rho)x_k + \lambda_e D_k(\rho) e\|_\infty \le 1$, where $\|v\|_\infty = \max_{i=1,2,\ldots n} |v_i|$ and $v = [\ v_1 \ \ldots \ v_n \ ]^T$.
2) If x_k lies on the boundary of S_e (i.e. $g_e(x_k) = 1$), maximize $\lambda_e \in [0, 1]$ such that

$$\lim_{\epsilon \to 0^+} \sup_{0<\delta<\epsilon} \frac{g_e(x_k + \delta[A_k(\rho)x_k + B_k(\rho)\lambda_e e]) - g_e(x_k)}{\delta} \le 0 \tag{6}$$

3) If x_k lies outside S_e (i.e. $g_e(x_k) > 1$), choose $\lambda_e \in [0, 1]$ such that the expression in

$$\lim_{\epsilon \to 0^+} \sup_{0<\delta<\epsilon} \frac{g_e(x_k + \delta[A_k(\rho)x_k + B_k(\rho)\lambda_e e]) - g_e(x_k)}{\delta} \tag{7}$$

is minimized.

$\blacksquare$

Implementation of the algorithm requires online calculation of g_e to determine where the compensator state x_k lies with respect to the boundary of S_e. Algorithm 3.1 provides the following closed loop performance guarantees for stable plants.

Theorem 3.1: **(Closed Loop Properties)**
Suppose that λ_e in Figure 3 is constructed in accordance with Algorithm 3.1 and that P is $\mathcal{L}^\infty$ finite-gain stable. Let $x_0 = x_k(0)$ denote the state of the compensator at $t = 0$. Given this, each of the following holds.

1) If x_0 lies within S_e, then $\|u\|_{\mathcal{L}^\infty} \le 1$ for all e.
2) If x_0 does not lie within S_e, then $\|u\|_{\mathcal{L}^\infty} \le g_e(x_0)$ for all e.
3) The closed loop system in Figure 3 will be $\mathcal{L}^\infty$ finite-gain stable. More specifically, there exists constants $k_{ry}, k_{ru} > 0$ such that

$$\|y\|_{\mathcal{L}^\infty} \le k_{ry} \|r\|_{\mathcal{L}^\infty} \ \text{ and } \ \|u\|_{\mathcal{L}^\infty} \le k_{ru} \|r\|_{\mathcal{L}^\infty} \tag{8}$$

for all $r \in \mathcal{L}^{\infty}$. ■

Proof: A proof of this can be found in [26].

On-Line Computation. The following is a discretized version of Algorithm 3.1.

Algorithm 3.2: **(EG - Discrete-Time Algorithm)**
Suppose $[\tilde{A}_k(\rho), \tilde{B}_k(\rho), \tilde{C}_k(\rho), \tilde{D}_k(\rho)]$, $\rho \in \mathcal{P}$ denotes a discrete-time approximation for K. Let x_n denote the state of the compensator at discrete-time nT where $T > 0$ is the sampling time. Let e_n denote the error signal at time nT. Also, suppose that $\tilde{g}_e$ is an approximation to g_e. The following discrete-time algorithm is proposed for constructing λ_n at each n.

1) If $\tilde{g}_e(x_n) < 1$, then choose the maximum $\lambda_n \in [0, 1]$ such that $\sup_\rho \|C_k(\rho)x_n + \lambda_n D_k(\rho)e_n\|_\infty \leq 1$, where $\|v\|_\infty = \max_{i=1,2,\ldots n} |v_i|$ and $v = [\ v_1\ \ldots\ v_n\ ]^T$.
2) If $\tilde{g}_e(x_n) = 1$, then choose the maximum $\lambda_n \in [0, 1]$ such that
$$\tilde{g}(\tilde{A}_k(\rho)x_n + \tilde{B}_k(\rho)\lambda_n u_n) - \tilde{g}_e(x_n) \leq 0 \tag{9}$$
3) If $\tilde{g}(x_n) > 1$, choose $\lambda_n \in [0, 1]$ such that
$$\tilde{g}_e(\tilde{A}_k(\rho)x_n + \tilde{B}_k(\rho)\lambda_n u_n) - \tilde{g}_e(x_n) \tag{10}$$
is minimized. ■

EG for Unstable Plants. The *EG algorithm* can be adapted for unstable plants. Toward this goal, it is important to note that if λ_e is chosen in accordance with Algorithm 3.1, then λ_e may be reduced to zero and may even remain zero over some time interval. This can be done without compromising closed loop stability only when the plant P is stable (and K is at least stable). When the plant P is unstable, the EG gain λ_e in the feedback loop in Figure 3 cannot be reduced arbitrarily to zero - it must remain greater than some $\lambda_{e_{min}}$ in order to ensure stability. To find $\lambda_{e_{min}} > 0$, one can use the *Multi-Loop Circle Criterion* [8].

IV. APPLICATION OF EG TO MISSILE AUTOPILOT

In this section, we show how our error governor (EG) ideas may be applied to nonlinear systems. We specifically show how the EG may be applied to a quasi-LPV (nonlinear) autopilot example.

Example 4.1: **(Application of EG to Missile Autopilot)**
In this example, we consider the following quasi-LPV (nonlinear) missile model [7], [24]:
$$\dot{x}_p = A(\rho)x_p + B(\rho)u_p \qquad y_p = C(\rho)x + D(\rho)u_p \tag{11}$$
where $x_p = [\ \alpha \quad q\ ]^T$, $y_p = [\ \eta \quad q\ ]^T$,
$$\begin{aligned} A(\rho) &= \begin{bmatrix} K_a\rho_2[a_n\rho_1^2 + b_n|\rho_1| + c_n(2 - \frac{\rho_2}{3})]cos(\rho_1) & 1 \\ K_q\rho_2^2[a_m\rho_1^2 + b_m|\rho_1| + c_m(-7 + \frac{8\rho_2}{3})] & 0 \end{bmatrix}, \\ B(\rho) &= \begin{bmatrix} K_a\rho_2 d_n cos(\rho_1) \\ K_q\rho_2^2 d_m \end{bmatrix} \end{aligned} \tag{12}$$

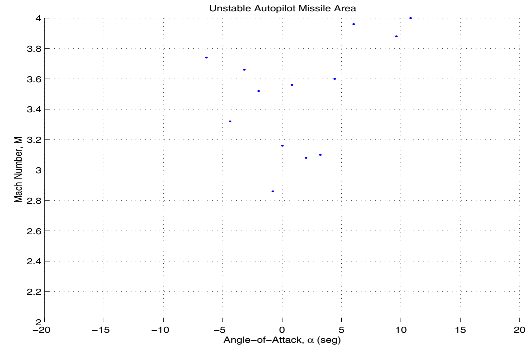

Fig. 4. Quasi-LPV Missile - Frozen Parameter (α, M) Instability

$$\begin{aligned} C(\rho) &= \begin{bmatrix} K_z\rho_2^2[a_n\rho_1^2 + b_n|\rho_1| + c_n(2 - \frac{\rho_2}{3})] & 0 \\ 0 & 1 \end{bmatrix}, \\ D(\rho) &= \begin{bmatrix} K_a\rho_2^2 d_n \\ 0 \end{bmatrix} \end{aligned} \tag{13}$$

and α denotes Angle of attack (deg), q pitch angle (deg/s), M mach number, $u = \delta_c$ commanded tail deflection angle (deg), $u_p = \delta$ actual tail deflection angle (deg), $r = \eta_c$ commanded normal acceleration (g), and η actual normal acceleration (g).

Parameter Set for Quasi-LPV Missile Model. Two scheduling variables are used within the model:
$$\rho_1 = \alpha \qquad\qquad \rho_2 = M. \tag{14}$$
Since $\rho_1 = \alpha$ is an internal state variable, the model is nonlinear - hence the missile is a quasi-LPV system. (It should be noted that the model treats Mach number M as an external variable.) It is assumed that these parameters belong to the following two-dimensional compact set
$$\mathcal{P} \stackrel{\text{def}}{=} \{(\ \rho_1, \rho_2) = [-20.0, 20.0] \times [2.0, 4.0]\ \} \tag{15}$$

Missile Frozen Parameter Instability. Relevant data for our quasi-LPV (nonlinear) missile model is given in [7], [24], [26]. The missile's frozen parameter poles were computed throughout the parameter space. Figure 4 shows that the missile is frozen parameter unstable over the dark triangular (α, M) region in the figure.

Nominal Closed Loop Design Specifications. Nominal closed loop design specifications are as follows:

- *Stability Robustness.* Maintain robust stability for
$$-20^0 \leq \rho_1 = \alpha \leq 20^0 \ \text{ and } \ 2 \leq \rho_2 = M \leq 4. \tag{16}$$
- *Step Reference Acceleration Command Following.* Track step reference acceleration commands in $r = \eta_c$ with time constant $\tau \leq 0.35$sec, maximum overshoot $\leq 10\%$, and steady state error < 0.01
- *Tail Deflection Rate.* Maximum tail deflection rate for 1g step reference acceleration command in $r = \eta_c$ should not exceed 25 deg/sec.

Design Process: Gridding of Parameter Space. Because the model exhibits $\rho_1 = \alpha$ symmetry, the parameter space was gridded as follows for design purposes ($\rho_1 = \alpha$, $\rho_2 = \mathcal{M}$) [7], [24]:

$$\hat{\mathcal{P}} \stackrel{\text{def}}{=} \{(\rho_1, \rho_2) : \rho_1 \in \{0, 4, 8, 12, 14, 20\}, \\ \rho_2 \in \{2.0, 2.4, 2.8, 3.2, 3.6, 4.0\} \quad \} \tag{17}$$

A solution pair (X, Y) can be found to the control and filter LMIs associated with the above points [24]. The associated minimum induced-$\mathcal{L}^2$ norm performance was found to be $\gamma \cong 3.13$. The computed (X, Y) pair was shown to work for a 40×40 grid of $\mathcal{P}$.

Mach Number. For simulation purposes, we considered the following variation for the external mach number signal:

$$M(t) = 2.4 + 0.6e^{-10t}. \tag{18}$$

Nominal System Response. An LPV controller was designed using appropriately selected frequency dependent weighting functions and linear matrix inequality (LMI) methods [5], [6], [24] at the above grid points. Nominal closed loop time responses are provided in Figures 5. The figures suggest that the nominal LPV control system design has desirable closed loop properties.

Saturated (Unregulated) Response. When saturations are inserted within the feedback loop ($satlevel = 12°$), one obtains the saturated or unregulated responses shown in Figures 6. As expected, the controls saturate and wind-up for sufficiently large reference commands $r > 11.14$.

Regulated Response with EG. The EG was applied to the above quasi-LPV closed loop missile-autopilot system for the purpose of control limiting. Nominal closed loop time responses with the EG are provided in Figures 7. The figures suggest that the nominal LPV control system design has desirable closed loop properties. In fact, the saturation prevention has worked up to a reference command $r < 11.19$.

V. SUMMARY AND FUTURE DIRECTIONS

In this paper, methods were presented for limiting control or any other signals within a MIMO feedback control system. The methods are based on so-called *error governor (EG)* method for both linear time-variant (LTI) and linear parameter varying (LPV) systems. Look-ahead functions and on-line optimization are used to appropriately "scale back" signals of interest within the feedback loop. The methods presented satisfy nominal closed loop performance properties. Each method was applied to a quasi-LPV missile model and autopilot. A *reference governor (RG) system* has also been developed to scale-back the rate of reference commands entering the feedback loop. Future work will address computational issues, accommodating other exogenous signals, issues associated with time variations in ρ.

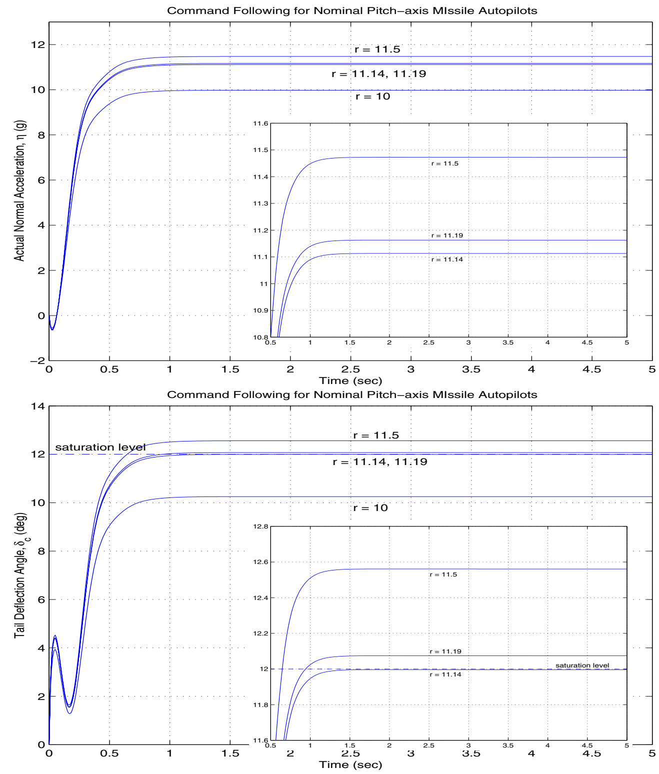

Fig. 5. Missile Autopilot Command Following - Nominal Response

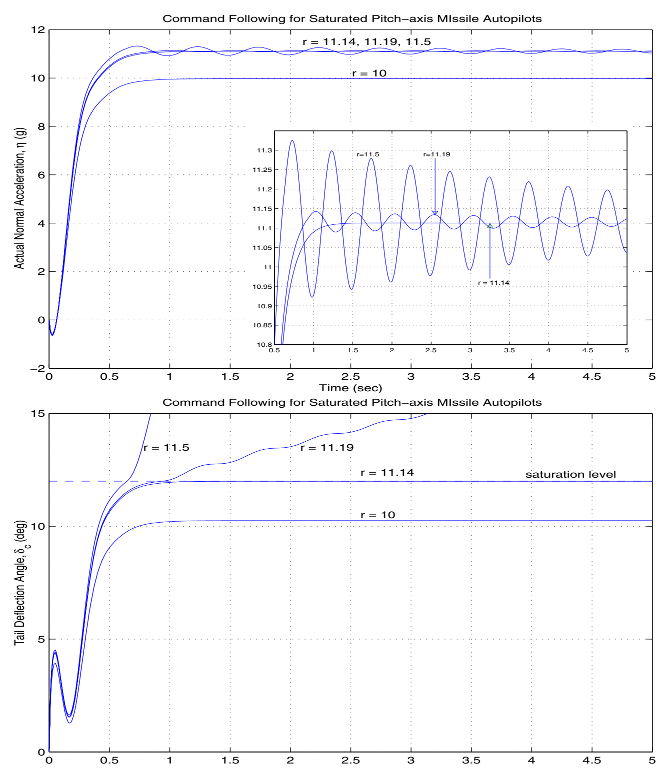

Fig. 6. Missile Autopilot Command Following - Saturated Response

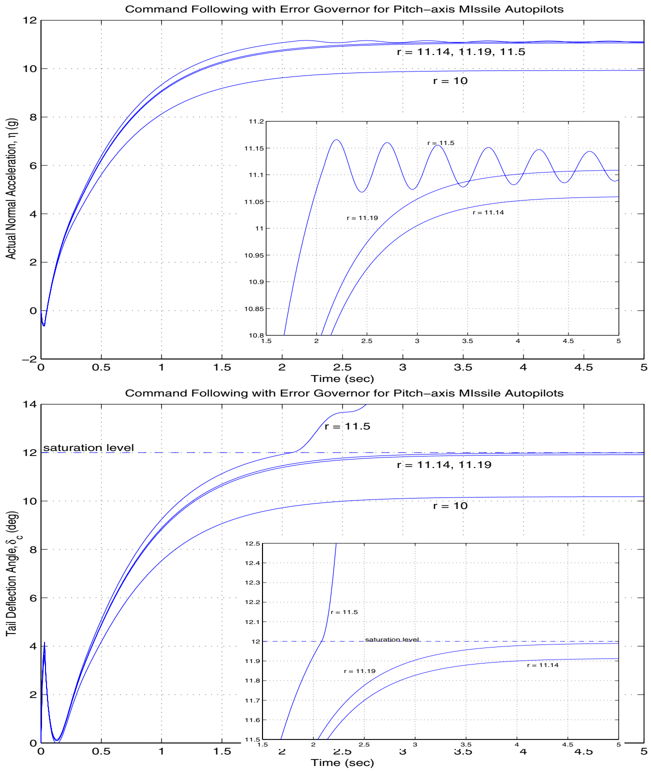

Fig. 7. Missile Autopilot Command Following - Response with EG

References

[1] J. S. Shamma and M. Athans, "Gain scheduling potential hazards and possible remedies," *IEEE Control System Magazine*, vol. 12, pp. 101–107, 1992.

[2] W. J. Rugh, "Analytical framework for gain scheduling," *IEEE Control System Magazine*, vol. 11, pp. 74–84, 1991.

[3] W. J. Rugh and J. S. Shamma, "Research on gain scheduling," *Automatica*, vol. 36, pp. 1401–1425, 2000.

[4] P. Apkarian, P. Gahinet, and G. Becker, "Self-scheduled $\mathcal{H}^{\infty}$ control of linear parameter-varying systems: A design example," *Automatica*, vol. 31, no. 9, pp. 1251–1261, 1995.

[5] G. Becker, "Quadratic stability and performance of linear parameter depedent systems," Ph.D. dissertation, University of California at Berkeley, Berkeley, CA, Dec. 1993.

[6] G. Becker and A. Packard, "Robust performance of linear parametrically varying systems using parameterically-dependent linear feedback," *System & Control Letters*, vol. 23, pp. 205–215, 1994.

[7] F. Wu, "Control of linear parameter varying systems," Ph.D. dissertation, University of California at Berkeley, Berkeley, CA, 1995.

[8] M. G. Safanov and M. Athans, "A multiloop generalization on the circle stability criterion for stability," *IEEE Trans. on Automatic Control*, vol. 26, no. 2, pp. 415–422, 1981.

[9] M. Bodson and S. Sastry, "Small signal I/O stability of nonlinear control systems: Application to the robustness of MRAC scheme," Electronics Research Laborator, University of California, Berkley, CA, Tech. Rep., Sep. 1984, memorandum No. UCB/ERL M84/70.

[10] P. Kapasouris and M. Athans, "Multivariable control systems with saturating actuators antireset windup strategies," in *Proc. of the American Control Conf.*, Boston, MA, 1985, pp. 1579–1584.

[11] P. Kapasouris, "Design for performance enhancement in feedback control systems with multiple saturating nonlinearities," LIDS MIT, March 1988.

[12] J. C. Doyle, R. S. Smith, and D. F. Enns, "Control of plants with input saturation nonlinearities," in *Proc. of the American Control Conf.*, 1987, pp. 1034–1039.

[13] K. J. Astrom and L. Rundqwist, "Integrator windup and how to avoid it," in *Proc. of the American Control Conf.*, Green Valley, AZ, 1989.

[14] K. S. Walgama and J. Sternby, "Inherent observer property in a class of anti-windup compensators," *Int. J. of Control*, vol. 52, no. 3, pp. 705–724, 1990.

[15] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett, "A unified framework for the study of anti-windup designs," *Automatica*, vol. 30, no. 12, pp. 1869–1883, 1994.

[16] N. Kapoor, A. R. Teel, and P. Daoutidis, "An anti-windup design for linear systems with input saturation," *Automatica*, vol. 34, no. 5, pp. 559–574, 1998.

[17] P. Kapasouris, M. Athans, and G. Stein, "Design of feedback control systems for stable plants with saturating actuators," in *Proc. of the Conf. on Decision and Control*, Austin, TX, 1988, pp. 469–479.

[18] A. A. Rodriguez and J. R. Cloutier, "Control of a bank-to-turn-missile with saturating actuators," in *Proc. of the American Control Conf.*, Baltimore, MD, 1994, pp. 1660–1664.

[19] A. Rodriguez and Y. Wang, "Performance enhancement for unstable bank-to-turn (BTT) missiles with saturating actuators," *International Journal of Control*, vol. 63, no. 4, pp. 641–678, 1996.

[20] E. G. Gilbert and K. T. Tan, "Linear systems with state and control constraints: The theory and application of maximal output admissible sets," *IEEE Trans. on Automatic Control*, vol. 36, no. 9, pp. 1008–1020, 1991.

[21] A. Bemporad, "Reference governor for constrained nonlinear systems," *IEEE Trans. on Automatic Control*, vol. 43, no. 3, pp. 415–419, 1998.

[22] J. S. Shamma and M. Athans, "Guaranteed properties of gain scheduled control for linear parameter-varying plants," *Automatica*, vol. 27, pp. 559–564, 1991.

[23] J. S. Shamma and J. R. Cloutier, "A linear parameter varying approach to gain scheduled missile autopilot design," in *Proc. of the American Control Conf.*, Chicago, IL, 1992, pp. 1317–1321.

[24] F. Wu, A. Packard, and G. Balas, "LPV control design for pitch-axis missile autopilots," in *Proc. of the Conf. on Decision and Control*, New Orleans, LA, 1995, pp. 188–193.

[25] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. NY: Academic Press, Inc., 1975.

[26] Y. Yi, "Control saturation prevention for linear parameter varying (LPV) systems," Ph.D. dissertation, Arizona State University, 2003.