Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Synchronization of TCP Flows in Networks with Small DropTail Buffers

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Abstract— **A recent fluid-model formulation of an Internet congestion-control problem considers routers with small Droptail buffers. This paper is interested in the oscillatory regime of such networks and considers a topology where two TCP-controlled flows (each regulated by separate (edge) routers) merge to compete for bandwidth at a common (core) router. We describe this dynamic using a weakly-coupled oscillator model and analyze the coherence of oscillation as a function of coupling strength. We show that increased coupling leads to increased coherence and to larger variations in the arrival flow at the core router. The coupling strength can be expressed in terms of network parameters.**

I. INTRODUCTION

The TCP (Transmission Control Protocol) is widely-used for reliable data transmission through the Internet. TCP sources have adjustable sending rates, and on route to destinations, this data is buffered and directed to different links. At the beginning of a session, TCP sources send data at an exponential rate which is called the slow-start phase. As traffic rates approach a link's capacity, packets are queued at the link buffers. Some packets may be dropped or marked at the routers by mechanisms called Active Queue Management (AQM)s, and such packets will trigger TCP receivers to issue negative acknowledgements back to the source. Upon receipt of a negative acknowledgement, a TCP source halves its sending rate and enters the so-called congestion avoidance phase. At this stage, a TCP source increases its rate linearly on receiving a positive acknowledgement and decreases its rate by half after receipt of a negative acknowledgement.

The previously mentioned AQM mechanisms were introduced to anticipate and improve performance such as network throughput. Different types of AQMs include DropTail, Random Early Detection (RED) [1], proportional-integral (PI) [3], Adaptive Virtual Queue (AVQ) [2], Random Early Marking (REM) [4], and their variants. The most widely-implemented AQM in the current Internet is the Droptail queue which is simple; i.e., if the buffer is full, all incoming packets are dropped. Stability issues of TCP/AQM systems are of recent interest, and readers are referred to [5] – [12] and the references contained therein. However, there has been little research devoted to the analysis of Droptail routers, which is most-likely attributable to the discontinuous nature of its feedback. Recently, a model [13] was proposed for the case of small Droptail buffers where it's argued that, for large numbers of flows, the blocking probability of an M/M/1 queue is a suitable model for packet loss. As a result, a fluid model is adopted for analysis where it is demonstrated that limit-cycling occurs when the system becomes linearly unstable.

The present paper is interested in the oscillatory regime of small Droptail buffer networks, and our approach is inspired by the research on oscillator synchronization; e.g., see [14] – [17]. This research has exposed the interesting and rich phenomenon arising when many independent oscillators, with differing intrinsic frequencies, are coupled together. The signature behavior of such networks is characterized by the oscillators spontaneously locking to a common frequency.

The contribution of this paper is a network problem formulation for small Droptail buffers that admits a model of weakly-coupled oscillators. More specifically, we consider a situation where two TCP-controlled flows (each regulated by separate (edge) routers) merge to compete for bandwidth at a common (core) router; see Figure 2. Presumption that the bandwidth of the core router is larger than the edge router implies that the two TCP flows are weakly coupled. If each of the two TCP flows are intrinsically oscillatory (as a result of interaction with their Droptail edge routers), then we have an instantiation of weakly-coupled oscillators. The edge routers induce intrinsic oscillations that are weakly coupled by the core. Our ensuing analysis shows that as coupling strength increases, the synchronizing frequency converges to a constant which is decided only by the round-trip time delays, regardless of the intrinsic frequencies. When the coupling strength exceeds a bound, the synchronizing state becomes unstable.

The rest of this paper is outlined as follows. In Section II, the TCP dynamics at the edge routers are described and the intrinsic frequencies computed. In Section III, a core router is considered and a model for the weekly-coupled oscillators is derived. In Section IV we provide analysis and describe the synchronized state, and its stability, in terms of network parameters. In Section V we provide simulations of both the fluid and discrete-event (ns) network models.

This work is supported in part by the National Science Foundation under Grant ANI-0238299,

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II. Two Uncoupled TCP flows

In this section we consider two uncoupled TCP flows that experience congestion at small-sized Droptail buffers. We assume that each of these loops are linearly unstable and approximate their ensuing limit-cycling with harmonic oscillations. We express their intrinsic frequencies in terms of the Droptail buffer lengths and round-trip times.

A. *TCP dynamics with small Droptail buffer*

Consider two TCP-controlled flows each passing through a congested edge router as shown in Figure 1. From [7], a

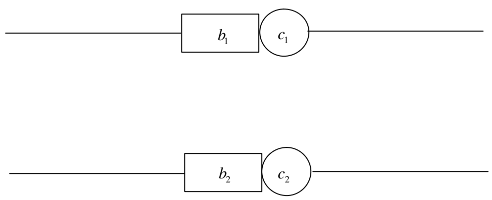

Fig. 1. Two distinct groups of TCP-controlled flows traversing a common edge router

fluid model for the TCP window dynamics is

$$\dot{w}_k(t) = \frac{1}{\tau_k} - \frac{w_k(t)w_k(t-\tau_k)}{2\tau_k} p_k(t-\tau_k); \quad k = 1,2 \qquad (1)$$

where w_k, p_k and τ_k are the window size, edge-buffer packet-loss probability and round-trip time (RTT) for the k-th TCP flow respectively. In [13], it was argued that for large numbers of flows, the blocking probability of an M/M/1 queue is a suitable model for the packet loss incurred by a small Droptail buffer. Thus, our edge routers will be assumed to have the packet loss model

$$p_k(t) = \left(\frac{n_k w_k(t)}{c_k \tau_k}\right)^{b_k}; \quad k = 1,2. \qquad (2)$$

By setting $\dot{w}_k$ in (1) equal to zero, we obtain the equilibrium window size w_k^*:

$$w_k^* = \left(2\left(\frac{c_k\tau_k}{n_k}\right)^{b_k}\right)^{\frac{1}{b_k+2}}; \quad k = 1,2. \qquad (3)$$

Linearizing (1) about this equilibrium point then gives

$$\Delta\dot{w}_k(t) = -\frac{1}{w_k^*\tau_k}(\Delta w + (b_k+1)\Delta w(t-\tau_k)) \qquad (4)$$

where $w_k = w_k^* + \Delta w_k$.

B. *Intrinsic frequencies*

Assume that each of the TCP dynamics described in (1) and 2 are linearly unstable. We approximate their ensuing limit-cycle oscillations with harmonic oscillations. To obtain their frequency, we substitute $\Delta w_k = r_k e^{i\overline{\theta}_k(t)}$ into (4) and obtain

$$r_k e^{i\overline{\theta}_k(t)}\dot{\overline{\theta}}_k = -\frac{1}{w_k^*\tau_k}(r_k e^{i\overline{\theta}_k(t)} + (b_k+1) r_k e^{i\overline{\theta}_k(t-\tau_k)}) \qquad (5)$$

where $\overline{\theta}_k(t) = \omega_k t + \phi_k$ and where ω_k denotes the intrinsic frequency of the kth TCP flow and ϕ_k its phase. From (5):

$$1 + (b_k+1)\cos(\omega_k\tau_k) = 0$$

and

$$\omega_k - \frac{b_k+1}{w_k^*\tau_k}\sin(\omega_k\tau_k) = 0.$$

so that

$$\omega_k = \frac{\sqrt{b_k(b_k+2)}}{w_k^*\tau_k}. \qquad (6)$$

Remark: If the edge router buffer sizes b_k are small, then the intrinsic frequencies in (6) are approximately inversely proportional to their link capacities c_k. Therefore, as technology advances and the c_k increase, the intrinsic frequencies decrease.

III. Weakly coupled TCP flows

In the previous section, we considered uncoupled TCP flows, each passing through distinct edge routers having small Droptail buffers. We assumed that these TCP flows were in limit-cycle oscillation and we approximated their intrinsic frequencies ω_k as in (6). Now, we modify the network configuration in Figure 1 and allow the two edge-regulated flows to pass through a common Droptail (core) router having link capacity C and small buffer size B; see Figure 2.

A. *Weakly coupled TCP dynamics*

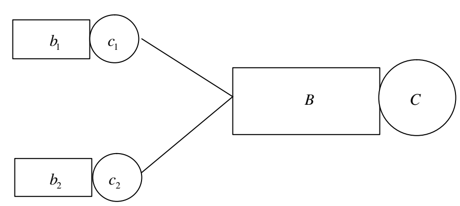

Fig. 2. Modification to the network in Figure 1 wherein the two edge-regulated TCP flows traverse a core router with capacity C and small Droptail buffer size B.

If $C > c_1 + c_2$, the core link will not be a bottleneck but will nevertheless affect synchronization of the oscillating TCP flows because of the core router's coupling effect. The window dynamics for these coupled TCP window dynamics now becomes

$$\dot{w}_k(t) = \frac{1}{\tau_k} - \frac{w_k(t)w_k(t-\tau_k)}{2\tau_k}(p_k(t-\tau_k) + q(t,\tau_1,\tau_2)); \quad k = 1,2 \qquad (7)$$

where p_k is as in (2) and

$$q(t,\tau_1,\tau_2) = \left(\frac{\frac{n_1 w_1(t-\tau_1)}{\tau_1} + \frac{n_2 w_2(t-\tau_2)}{\tau_2}}{C}\right)^B \qquad (8)$$

where round-trip time delays are assumed to occur in forward path. Substituting (2) and (8) into (7), and then linearizing gives

$$\begin{aligned}\Delta\dot{w}_k &= -\frac{1}{\widetilde{w}_k^*\tau_k}(\Delta w_k + (b_k+1)\Delta w_k(t-\tau_k)) \\ &\quad -\frac{B(\widetilde{w}_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j\widetilde{w}_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{2}\left(\frac{n_j\Delta w_j(t-\tau_j)}{C\tau_j}\right) \\ &\quad -\frac{b_k\widetilde{w}_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j\widetilde{w}_j^*}{\tau_j}}{C}\right)^{B}\Delta w_k(t-\tau_k) \qquad (9)\end{aligned}$$

where $\widetilde{w}_k^*$ is the new equilibrium window length. After some manipulations (see Appendix A for details) we transform (9) into the following phase form:

$$\begin{aligned}\dot{\theta}_k &= \omega_k - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_jw_j^*}{\tau_j}}{C}\right)^{B-1} \\ &\quad \cdot\sum_{j=1}^{2}\left(\frac{n_jr_j}{Cr_k\tau_j}\sin\left(\theta_j(t-\tau_j)-\theta_k(t)\right)\right) \\ &\quad -\frac{b_kw_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_jw_j^*}{\tau_j}}{C}\right)^{B}\sin\left(\theta_k(t-\tau_k)-\theta_k(t)\right). \qquad (10)\end{aligned}$$

Remark: Recall that the r_k are the oscillation amplitudes. They can computed using the techniques followed in [20].

For simplicity, we now focus on the case when the flows experience equal coupling from the core router.

B. Equally-coupled TCP flows

Suppose $n_1 = n_2 = n$, $c_1 = c_2 = c$, $b_1 = b_2 = b$ and that $\tau_1 \approx \tau_2 \doteq \tau$. Then, $w_k^* \doteq w^* = \left(2\left(\frac{c\tau}{n}\right)^b\right)^{\frac{1}{b+2}}$, and $r_1 \approx r_2$. Consequently, (10) can be expressed as the coupled oscillators:

$$\begin{aligned}\dot{\theta}_k &= \omega_k - Kb\sin\left(\theta_k(t-\tau_k)-\theta_k(t)\right) - \\ &\quad \frac{BK}{2}\sum_{j=1}^{2}\left(\sin\left(\theta_j(t-\tau_j)-\theta_k(t)\right)\right); \quad k=1,2 \qquad (11)\end{aligned}$$

where the *coupling strength* K is given by

$$K = \frac{n^B}{2c_0^B}\left(\frac{w^*}{\tau}\right)^{B+1}$$

and where $c_0 = \frac{C}{2}$.

Remark: The model (11) differs from the standard coupled oscillators model in that the coupling term $\sin(\theta_k(t-\tau_k)-\theta_k(t))$ contains the round-trip time delay τ. The work in [16] considers such a time-delay coupled oscillator model and, in the next section, we extend some of [16]'s results to the particular situation in (11).

IV. MAIN RESULT

Our next result gives conditions under which the coupled oscillators (11) synchronize.

Theorem 1: *Consider the coupled-oscillators described in (11) and assume that the difference in round-trip time delays is small; i.e., $\tau_1 \approx \tau_2$ ($\omega_1 \approx \omega_2$). Suppose there exists nonnegative numbers Ω and ϕ_0 satisfying:*

$$KB\sin\phi_0\cos(\Omega\tau) = \omega_2 - \omega_1 \qquad (12)$$

and

$$\Omega = \omega + K(b+\frac{B}{2})\sin(\Omega\tau) + \frac{KB}{2}\sin(\Omega\tau+\phi_0). \qquad (13)$$

Then, the coupled oscillators (11) synchronize at frequency Ω with phase difference ϕ_0. The synchronized state is locally stable iff

$$K(b+B)\cos(\Omega\tau) < 0. \qquad (14)$$

Proof: From (11) we form the differential equation in the phase difference to give:

$$\begin{aligned}\dot{\theta}_1(t) - \dot{\theta}_2(t) &= \omega_1 - \omega_2 - \\ K(b+\frac{B}{2})&\left(\sin(\theta_1(t-\tau_1)-\theta_1(t)) - \sin(\theta_2(t-\tau_2)-\theta_2(t))\right) - \\ \frac{KB}{2}&\left(\sin(\theta_2(t-\tau_2)-\theta_1(t)) - \sin(\theta_1(t-\tau_1)-\theta_2(t))\right) \qquad (15)\end{aligned}$$

Let $\phi(t) \doteq \theta_1(t) - \theta_2(t)$ define phase difference between the oscillators and let ϕ_0 denote its steady-state value (defined by $\dot{\phi} = 0$). Then, since $\tau_1 \approx \tau_2$:

$$\sin(\theta_1(t-\tau_1)-\theta_1(t)) - \sin(\theta_2(t-\tau_2)-\theta_2(t)) \approx 0,$$

and

$$\theta_1(t-\tau_1) - \theta_2(t) \approx \theta_2(t-\tau_2) - \theta_1(t) + 2\phi_0.$$

(15) now becomes

$$KB\sin\phi_0\cos(\theta_2(t-\tau_2)-\theta_1(t)+\phi_0) \approx \omega_2 - \omega_1 \qquad (16)$$

which implies that $\theta_2(t-\tau_2)-\theta_1(t)$, $\theta_2(t-\tau_2)-\theta_2(t)$, $\theta_1(t-\tau_1)-\theta_1(t)$, and $\theta_1(t-\tau_1)-\theta_2(t)$ are constant. Therefore,

$$\begin{aligned}\dot{\theta}_1 &= \dot{\theta}_2 \\ &= \omega_1 - K(b+\frac{B}{2})\sin\left(\theta_1(t-\tau_1)-\theta_1(t)\right) - \\ &\quad \frac{KB}{2}\sin\left(\theta_2(t-\tau_2)-\theta_1(t)\right) \\ &= \omega_2 - \frac{KB}{2}\sin\left(\theta_1(t-\tau_1)-\theta_2(t)\right) - \\ &\quad K(b+\frac{B}{2})\sin\left(\theta_2(t-\tau_2)-\theta_2(t)\right) \\ &\doteq \Omega.\end{aligned}$$

Consequently,

$$\theta_1(t) = \Omega t,$$

$$\theta_2(t) = \Omega t - \phi_0,$$

$$\Omega = \omega + K(b + \frac{B}{2})\sin(\Omega\tau) + \frac{KB}{2}\sin(\Omega\tau + \phi_0),$$

and

$$KB\sin\phi_0\cos(\Omega\tau) = \omega_2 - \omega_1.$$

As K increases, ϕ_0 approaches zero and (13) becomes

$$\Omega = \omega + K(b + B)\sin(\Omega\tau).$$

(14) then follows directly from [17]. □

Remark 1: If $\tau = 0$, then (12) and (13) are satisfied with $\sin\phi_0 = \frac{\omega_2 - \omega_1}{KB}$ and $\Omega = \frac{\omega_1 + \omega_2}{2}$, which recovers the usual synchronization results for zero time delays.

Remark 2: Let $K \in (K_c, K_u)$ denote the range of coupling strengths K for which (12) – (14) are feasible. When $K < K_c$, no synchronization occurs between two flows. When $K_u > K > K_c$, the phase difference between oscillators is locked. As we increase K, the phase difference decreases, similar to case of non-delayed coupled oscillators. However, the synchronizing frequency approaches $\frac{\pi}{\tau}$ when $K \to \infty$. Indeed, for large K, ϕ_0 is very small, and $\Omega = \omega + K(b + B)\sin(\Omega\tau)$ from (13). To make $K(b+B)\sin(\Omega\tau)$ small for large K, we will have $\Omega\tau \approx \pi$, so that $\Omega \approx \frac{\pi}{\tau}$. Under this situation, the intrinsic frequencies do not affect the synchronizing frequency. Finally, when $K > K_u$, the synchronized state becomes unstable.

Remark 3: Let $re^{i\psi(t)} \doteq \frac{1}{2}(\sin(\theta_1(t)) + \sin(\theta_2(t)))$. When the coupled oscillators are synchronized, $r = \cos(\phi_0)$ which increases with coupling strength K. When synchronized, r measures the coherence of the coupled oscillators and is a measure of the variation in TCP traffic at the core router.

V. Simulations

A. Model Simulations

In this section, we present simulations of the coupled oscillators model (11). In these simulations, $\tau_1 = 0.09$s, $\tau_2 = 0.11$s, $\omega_1 = 99$rps, $\omega_2 = 101$rps, $b = B = 20$pkts. We increase the coupling strength K from $K_c = 0.03$ to 40 and plot the synchronizing frequency Ω against coupling strength K in Figure 3. When $K \geq 50$, the synchronized state is unstable. From Figure 3, we observe that the synchronizing frequency Ω converges to $\frac{\pi}{\tau}$. This is quite different from the non-delayed case; see Remarks 1 above. We also plot the coherence r with respect to the coupling strength in Figure 4. We see that flows quickly become coherent.

B. ns simulations

In addition to fluid-model simulations, we also conductted `ns` (discrete-event) simulations of the TCP network described in Figure 1. The parameters are $c_1 = c_2 = 0.4$Mbps, $b_1 = b_2 = 10$pkts, $\tau_1 = 40$ms and $\tau_2 = 44$ms. For $C = 1$Mbps and $B = 10$pkts, the TCP window lengths behave as unsynchronized oscillators as shown in the top plot of Figure 5. For $C = 0.801$Mbps, $B = 2$,pkts the two flows become synchronized as shown in the bottom plot of Figure 5. We also plot the total and average window sizes for the synchronized and unsynchronized cases in Figure 6. The dashed traces

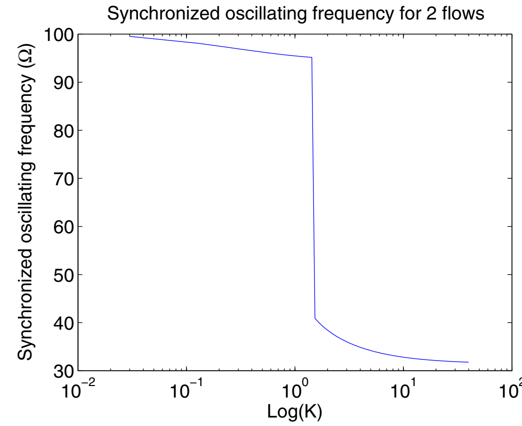

Fig. 3. Synchronizing frequency approaches $\frac{\pi}{\tau}$ with increasing coupling strength K.

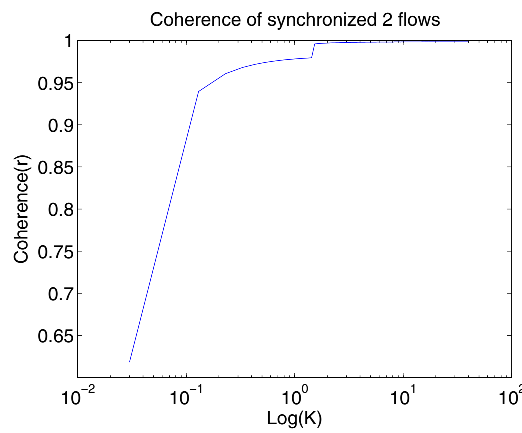

Fig. 4. Coherence r as a function of coupling strength K.

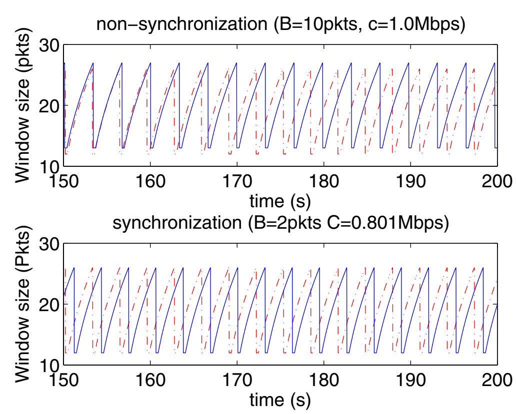

Fig. 5. `ns` simulation of two weakly-coupled TCP flows.

mark the unsynchronized TCP flows, while solid traces are for the synchronized flows. The bolder lines denote the averages. We observe that the average TCP windows for the unsynchronized TCP flows is larger than the synchronized case. Synchronization adversely affects throughput through the core router.

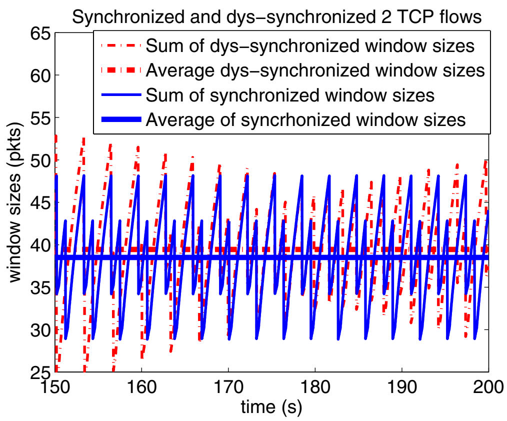

Fig. 6. ns simulation showing that synchronized TCP flows have lower throughput.

VI. Conclusion

Typical stability analysis of the congestion-avoidance phase of TCP networks has been primarily concerned with motion about the equilibrium and, for the most part, has been confined to local behavior. In contrast, this paper was interested in the oscillatory regime of DropTail networks, and in the synchronization and coherence of TCP flows. One application of such results is towards the sizing of DropTail buffers.

Appendix

Since the coupling is weak, i.e., q defined as (8) is small. So $\widetilde{w}_k^* \approx w_k^*$, where w_k^* is defined as (3). Therefore, (9) becomes

$$\Delta\dot{w}_k = -\frac{1}{w_k^*\tau_k}(\Delta w_k + (b_k+1)\Delta w_k(t-\tau_k)) - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{2}\left(\frac{n_j\Delta w_j(t-\tau_j)}{C\tau_j}\right) - \frac{b_k w_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B}\Delta w_k(t-\tau_k)$$

Next, substitute $\Delta w_k = r_k e^{i\theta_k(t)}$ into above equation and obtain

$$i r_k e^{i\theta_k(t)}\dot{\theta}_k = -\frac{1}{w_k^*\tau_k}\left(r_k e^{i\theta_k(t)} + (b_k+1)r_k e^{i\theta_k(t-\tau_k)}\right) - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{2}\left(\frac{n_j r_j e^{i\theta_j(t-\tau_j)}}{C\tau_j}\right) - \frac{b_k w_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B} r_k e^{i\theta_k(t-\tau_k)}.$$

Then,

$$i\dot{\theta}_k = -\frac{1}{w_k^*\tau_k}\left(1 + (b_k+1)e^{i\theta_k(t-\tau_k)-i\theta_k(t)}\right) - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{2}\left(\frac{n_j r_j e^{i\theta_j(t-\tau_j)-i\theta_k(t)}}{C r_k\tau_j}\right) - \frac{b_k w_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B} e^{i\theta_k(t-\tau_k)-i\theta_k(t)}.$$

Since the coupling is weak, at the fast time scale, the oscillators behaves like independently. So, as for the first term, let $\theta_k \approx \overline{\theta}_k$. Therefore, the above equation becomes

$$i\dot{\theta}_k = -\frac{1}{w_k^*\tau_k}\left(1 + (b_k+1)e^{i\overline{\theta}_k(t-\tau_k)-i\overline{\theta}_k(t)}\right) - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{N}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{N}\left(\frac{n_j r_j e^{i\theta_j(t-\tau_j)-i\theta_k(t)}}{C r_k\tau_j}\right) - \frac{b_k w_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{N}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B} e^{i\theta_k(t-\tau_k)-i\theta_k(t)}.$$

After substitution we obtain

$$i\dot{\theta}_k = i\omega_k - \frac{B(w_k^*)^2}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B-1}\sum_{j=1}^{2}\left(\frac{n_j r_j e^{i\theta_j(t-\tau_j)-i\theta_k(t)}}{C r_k\tau_j}\right) - \frac{b_k w_k^*}{2\tau_k}\left(\frac{\sum_{j=1}^{2}\frac{n_j w_j^*}{\tau_j}}{C}\right)^{B} e^{i\theta_k(t-\tau_k)-i\theta_k(t)}.$$

By equating the imaginary part, we obtain the coupled oscillators model (10)

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