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Likelihood Bounds for Constrained Estimation with Uncertainty

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Abstract— **This paper addresses the problem of finding bounds on the optimal maximum a posteriori (or maximum likelihood) estimate in a linear model under the presence of model uncertainty. We introduce the novel concepts of *at least as likely as the maximum a posteriori* (ALAMAP) estimate, or *at least as likely as the maximum likelihood* (ALAML) estimate. The concept is formulated as a convex optimization problem. We specifically make use of second-order cone programming (SOCP) techniques to compute the likelihood bounds in an efficient manner. The procedure of computing the bounds is illustrated by examples in state estimation (smoothing/filtering), and in system identification.**

I. INTRODUCTION

Many estimation problems in decision and control are ill-conditioned. These include state estimation or trending, identification of the system model parameters, and others. Reliable estimation in the presence of noise, uncertainty and ill-conditioning can be achieved by using a prior knowledge of the unknown state. The *maximum a posteriori* (MAP) estimate is based on the concept of using an a priori known probabilistic distributions of the unknown state. Another method of reliable computation of the estimates is through robust estimation, which makes explicit use of the model uncertainty, see [1], [2], [3], [4]. Tikhonov's regularization is another well known technique to overcome ill-conditioning of the problem, see [5], [6]. The MAP and robust estimation techniques essentially provide a systematic way of choosing the regularization for ill-conditioned estimation problems.

A regularized solution of an inverse problem does not have a 'bad' behavior along small singular values of the forward operator. Yet, it is not clear how reliable is such a regularized estimate. The solution may be very sensitive to noise and perturbations in the problem matrices. What are the confidence bounds for such a solution? The answer is important to understand how far the real parameters can be from the MAP or *maximum likelihood* (ML) estimates. We provide a novel approach for efficient computation of the bounds for a MAP (or ML) solution in the presence of problem perturbations. The main contribution of this work is to introduce the concept of at least as likely as the maximum a posteriori (ALAMAP) or at least as likely as the maximum likelihood (ALAML) solution. The proposed approach is applicable to systems where linear constrained state estimation is required in the presence of bounded data and model uncertainty.

A large class of estimation problems result in a quadratic programming formulation, and can be efficiently solved using numerical optimization to compute the estimates. The proposed approach to the ALAMAP (or ALAML) problems relies on constrained convex optimization based estimation. It provides upper and lower likelihood bounds for the MAP (or ML) estimate under uncertainty instead of computing the robust optimal solution. Constrained estimation using numerical optimization has been studied extensively, see [7], [8], [9]. However most previous work in determining bounds for the ML solution is done in the unconstrained least squares framework, where ellipsoidal sets of all possible states consistent with the given measurements are found. The work presented in this paper is similar in spirit but the implementation in the constrained framework requires additional tools that are provided by second-order cone programming (SOCP) techniques.

The paper is organized as follows. Section II explains the technical problem statement of finding likelihood bounds under uncertainty. The concept of at least as likely as the maximum a posteriori estimate (ALAMAP) is explained using a simple univariate example in Section III. The problem is formulated mathematically in Section IV, where we also propose a solution involving second order conic constraints. Section V shows the application of the concept to monotonic trending using constrained state estimation. The proposed concept is applied to finite impulse response (FIR) model identification in Section VI. Some concluding remarks are given in Section VII.

II. TECHNICAL PROBLEM STATEMENT

In this work, we deal with linear state estimation problems in the presence of sensor noise and data uncertainty. We consider both the constrained and the unconstrained formulations. The objective is to find meaningful confidence bounds for the unknown state given the observed parameters. This paper considers a linear system relating the observed data y to the unknown state x. This data model can be conveniently expressed in the form

$$y = Ax + e, \tag{1}$$

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where $y \in \mathbf{R}^m$ is the vector of observations, $x \in \mathbf{R}^n$ is the unknown state vector, $A \in \mathbf{R}^{m\times n}$ is the known data matrix that defines the linear mapping between the unknown state and the observed vector, and $e \in \mathbf{R}^m$ is the observation noise vector. The noise term in the data model accounts for modeling errors and sensor noise. The linear data model in (1) is useful for a wide class of linear state estimation problems. It appears in many practical applications related to data trending and system identification.

To obtain a statistically optimal estimate of the unknown parameter vector x, such as the ML estimate or the MAP estimate, we make use of the concept of conditional probability. The MAP estimate of the unknown vector x given y is

$$\text{MAP} := \arg\max_x p_{x|y}, \tag{2}$$

where $p_{x|y}$ denotes the conditional probability density of x given the observed vector y. For $p_{x|y} \neq 0$, we define the loss index J as the negative log-likelihood of the conditional probability density, *i.e.*,

$$\begin{aligned} J(x) &:= -\log p_{x|y} && (3)\\ &= -\log p_{y|x} - \log p_x + c, && (4) \end{aligned}$$

where (4) follows from (3) by a direct application of the Bayes rule. The MAP estimate is obtained by minimizing the loss index J. The constant $c = \log p_y$ has no role in determining the MAP estimate. The problem formulation accounts for a probability density on the underlying parameter x. This density represents the prior information about the unknown state, and penalizes choices of x that are unlikely according to this prior density, (*i.e.*, x with p_x small). The prior knowledge about the unknown state may also result in constraints on x (*i.e.*, x with $p_x = 0$). The minimization of the loss index is then subject to these constraints on the unknown state, which can be expressed as

$$x \in \mathcal{C}, \tag{5}$$

The MAP estimation problem (1)–(5) is a convex optimization problem if the negative log-likelihood function J is convex, and the set $\mathcal{C}$ is described by linear equality or convex inequality constraints, as is the case in many estimation problems.

For the purpose of this work, we limit our attention to the probablity distributions for which the constraints set $\mathcal{C}$ (where $p_x > 0$) is a polyhedral set. We also assume that the noise e in the data model (1) is uncorrelated gaussian with zero mean and covariance Q, *i.e.*,

$$e \sim N(0, Q). \tag{6}$$

For the gaussian distributed observation noise e, we can substitute (6) for the conditional probability density $-\log p_{y|x}$ in (4). The MAP estimate is then obtained by minimizing the loss index

$$J(x) = [Ax - y]^T Q^{-1} [Ax - y] - \log p_x \tag{7}$$

Other noise distributions that result in a convex loss index can be handled accordingly. However, the scope of this work is limited to those probability densities of the noise and the unknown state that result in a quadratic objective.

A. Models with Uncertainty

The formulation in (7) assumes that the linear map (1) is known precisely. In most practical applications, there is always some uncertainty in the assumed linear model A. This uncertainty can be caused by a variety of reasons. The most common cause of uncertainty is that all physical systems are inherently nonlinear. Then, linearized model (1) is only an approximation of the actual system. It is therefore natural to deal with the imprecise nature of the linearization by considering uncertainty in the linear map A. Our formulation also accounts for uncertainty in the observed vector y (different from the observation noise).

Assuming that the matrix A and the vector y are not known precisely, we introduce uncertainty parameters ΔA and Δy for the map matrix and the observed data vector respectively. We assume ΔA to be of the same dimension as A. The individual columns of the matrix ΔA account for the uncertainty in the corresponding columns of the matrix A. For the problem with model uncertainty

$$\begin{aligned} y &\longrightarrow y + \Delta y, && (8)\\ A &\longrightarrow A + \Delta A. && (9) \end{aligned}$$

We consider the two uncertainty parameters to be norm bounded, *i.e.*,

$$\begin{aligned} |\Delta y|_p &\leq r_y, && (10)\\ |\Delta A|_p &\leq r_A, && (11) \end{aligned}$$

where $p = \{1, 2, \infty\}$. The choice of the norm for the uncertainty bounds on ΔA and Δy depends on the specific nature of the problem. The examples in the last two sections of this paper use the ℓ_1 norm bound but in some applications the ℓ_2 or ℓ_∞ norm may be more suitable. The question of how far the solution of the problem with uncertainty can be off the nominal solution obtained without uncertainty is addressed in this paper. We consider a novel *at least as likely as the MAP* (ALAMAP) or *at least as likely as the ML* (ALAML) setting. The difference is that the MAP estimate uses a prior knowledge about the distribution p_x, while the ML estimate of the unknown state assumes no information about p_x is available. Assume that the solution of the nominal problem (no uncertainty) obtained by minimizing the loss index $J(x)$ in (7) yields the optimal estimate x^* and the corresponding minimum value of the loss index J^*. In the problem with model uncertainty, the loss index (7) becomes a function of the parameters ΔA and Δy. We consider a set W such that

$$W = \{(x, \Delta A, \Delta y) : J(x; \Delta A, \Delta y) \leq J^*\}, \tag{12}$$

We call it the *ALAMAP* or the *ALAML set*. The set contains all the possible uncertainty parameter values that yield an estimate of the unknown state which is at least as likely as the optimal estimate x^* for the nominal model (7). For practical purposes we are interested in the likelihood bounds or extreme points (worst case solution) for a given uncertainty. To obtain the lower bound on components of the estimate vector x for a given uncertainty we do the following minimization

$$\min_x c^T x \tag{13}$$

For the upper bound calculation we simply solve $\min_x -c^T x$. The row vector $c^T \in \mathbf{R}^{1\times n}$ is used to pick point wise the components of x, and can be thought of as a unit vector along the corresponding coordinate of x. The minimization is also subject to any original problem constraints on the unknown state, as given in (5).

The ALAMAP (or ALAML) problem formulation yields a set of plausible estimates under uncertainty that are at least as likely as the original MAP (or ML) estimates. The problem (13) of finding the likelihood bounds is a minimization of a linear objective subject to convex (second order conic) constraints. This convex optimization framework allows for efficient computations of the bounds.

III. Discussion of ALAMAP Estimation

We now explain the concept of finding an ALAMAP estimate for a one dimensional example. Consider a univariate case in which y, A, x, and e in (1) are all scalars. The unknown state x is assumed gaussian with zero mean and covariance r. This implies that the term $-\log p_x$ in the loss index (7) is a quadratic penalty of the form rx^2, where r is the covariance of x. We now introduce uncertainty in this problem setup. We assume the uncertainty ΔA to be bounded by r_A. For the sake of simplicity, we assume no uncertainty in the measurement y, *i.e.*, $\Delta y = 0$ in this example. Rewriting the loss index (7) for this case with uncertainty only in the data matrix A, we get

$$\begin{aligned} J(x, \Delta A) \quad &= \quad [(A+\Delta A)x - y]^T Q^{-1}[(A+\Delta A)x - y] \\ &\quad -rx^2, \end{aligned} \tag{14}$$

The values chosen for the simulation are; observed parameter $y = 10$, $A = 1$, unit noise covariance for the noise e, *i.e.*, $Q = 1$. We choose $r = 10$ for the MAP estimate in this simulation. We assume the uncertainty ΔA to be bounded by $r_A = 0.1$, *i.e.*, at most than 10% uncertainty in the given A. The nominal loss index for this case without uncertainty is plotted using (7) and is shown in Fig. 1. The optimal value for the nominal loss index naturally occurs at the vortex of the parabola. In this example x^* is 0.9 and the corresponding optimal value of loss index is $J^*(x^*) = 90.9$. Limiting our attention to the worst case uncertainty, we first substitute for $\Delta A = r_A = 0.1$, and then $\Delta A = -r_A = -0.1$ in the uncertainty loss index (14). The two corresponding parabolas are shown in Fig. 1 along with the loss index for the nominal case. The introduction of the negative uncertainty, $\Delta A = -0.1$, shifts the parabola upwards. This results in a higher value of optimal loss index, *i.e.*, $J^*(x, \Delta A = -0.1) > J^*$. In this case our ALAML set W in (12) is empty. On the other

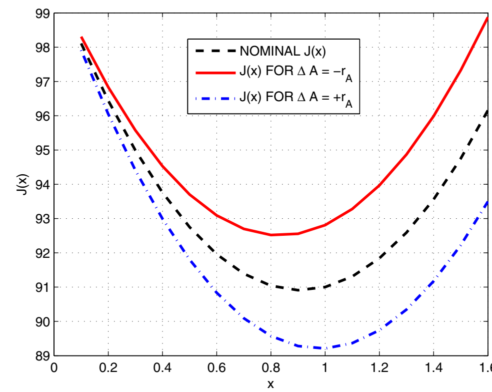

Fig. 1. Nominal loss index $J(x)$ with positive and negative uncertainty

hand, when we consider the positive worst case uncertainty $\Delta A = 0.1$, the parabola $J(x, \Delta A = 0.1)$ is shifted below the nominal parabola $J(x)$. In this case, the ALAML set is not empty and we have a range of solutions that are at least as likely as the nominal MAP solution x^*. It is therefore meaningful to find the worst case likelihood bounds for this point estimate by performing the minimization in (13).

For the case of positive worst case uncertainty $\Delta A = 0.1$, when our feasible set W is not empty, it is useful to get an idea about the convexity of the loss index (14) by plotting the function $J(x, \Delta A)$. The result of the plot of $J(x, \Delta A)$ in the $(x, \Delta Ax)$ plane is shown in Fig. 2. The important thing

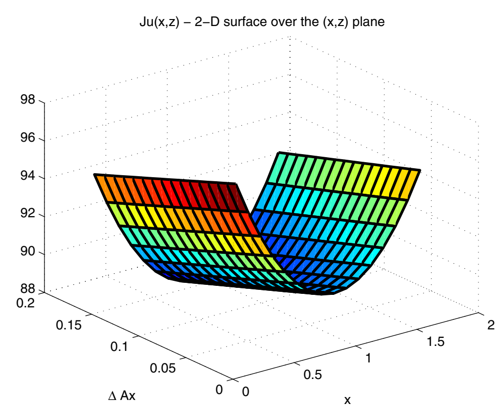

Fig. 2. Quadratic $J(x, \Delta A)$ surface in the $(x, \Delta Ax)$ plane

to note about this region is that it is convex (quadratic). This makes the constrained minimization (13) of finding the point wise confidence bounds for the estimate a convex optimization problem which is computationally feasible.

The problem of finding the ALAML estimate can be easily explained by Fig 3. We find the level sets of $J(x, \Delta A)$ that satisfy $J(x, \Delta A) \leq J^* = 90.9$. A contour that satisfies the equality is shown in Fig 3. The uncertainty bound $\Delta Ax = r_A x$ is superimposed on the contour $J(x, \Delta A) = J^*$ in Fig. 3. The uncertainty bound is satisfied by all the points below the constraint line $\Delta Ax = 0.1x$. As can be seen, there is a range of solutions $(x, \Delta A)$ that satisfy the constraint

$J(x, \Delta A) \leq J^*$ and the set W is not empty. In other words, there is a range of possible values for which the estimate is at least as likely as the nominal MAP estimate, and the extreme points that give the likelihood bounds can be computed using the minimization in (13).

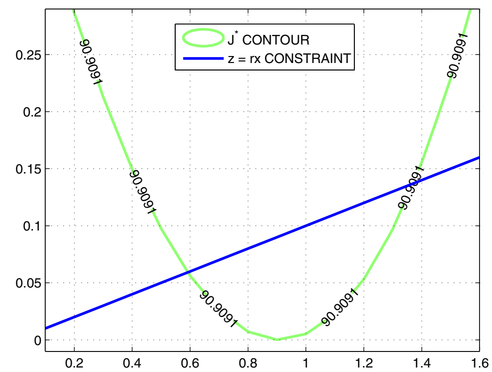

Fig. 3. Feasible quadratic domain in $(x, \Delta Ax)$ plane

IV. Solution Approach

We now mathematically formulate the general problem of finding the likelihood bounds for the unknown state given a prior distribution on x, and bounded uncertainty in the problem data. Introducing the uncertainty parameters ΔA and Δy in the loss index (7), we get

$$\begin{aligned} J(x, \Delta A, \Delta y) &= [(A + \Delta A)x - (y + \Delta y)]^T Q^{-1} \\ & \quad [(A + \Delta A)x - (y + \Delta y)] - \log p_x, \end{aligned} \tag{15}$$

Looking carefully at the above loss index, an alternate interpretation of the bounded uncertainty problem is to think of it in a stochastic sense. If we assume that ΔA and Δy are independent random variables uniformly distributed in the constraint set (10)–(11), we get the same loss index up to an additive constant.

Define a new uncertainty variable $z \in \mathbf{R}^m$ as

$$z := \Delta Ax - \Delta y. \tag{16}$$

Substituting z in (15) yields

$$\begin{aligned} J(x; z) &= [Ax + z - y]^T Q^{-1} [Ax + z - y] \\ & \quad - \log p_x, \end{aligned} \tag{17}$$

To obtain the likelihood bounds for the individual components of the estimate vector x in the presence of uncertainty, the ALAMAP estimation problem can be formulated as

$$\min_x c^T x, \tag{18}$$

subject to the following constraints

$$J(x; z) \leq J^* \tag{19}$$

$$|z| \leq r_A |x| + r_y. \tag{20}$$

To obtain the upper bound we simply solve $\min_x -c^T x$. The minimization is also subject to any original problem constraints on the unknown state, as determined by the prior knowledge of the probability distribution of x. The exact solution of the ALAMAP problem is not convex due to the constraint (20). To ensure the convexity of the problem, we replace the decision variable x in constraint (20) by the optimal x^* obtained from the nominal problem, *i.e.*,

$$|z| \leq r_A |x^*| + r_y \tag{21}$$

The above simplification will yield an approximate solution of the ALAMAP problem. The approximation can be improved by replacing x^* with the solution of the minimization problem (18) in successive iterations.

We now formulate the approximate ALAMAP estimation as a convex second order conic optimization problem. The objective (18) is linear. The constraint set in (5) is restricted to a polyhedron. The uncertainty bound constraint in (21) is linear. If we can cast (19) as a convex constraint then the likelihood bounds can be easily obtained by solving a linear objective subject to the convex constraints. We now show that (19) can be formulated as a second-order cone constraint. SOCP problems are well known in optimization theory. For a detailed description of the SOCP formulation, see [10]. Rewrite the first term in the loss index (17) for the problem with uncertainty in terms of a new matrix $P \in \mathbf{R}^{m \times (n+m+1)}$ and a vector $v \in \mathbf{R}^{n+m+1}$, where

$$P := \begin{bmatrix} A & I & -y \end{bmatrix}, \tag{22}$$

$$v := \begin{bmatrix} x \\ z \\ 1 \end{bmatrix}, \tag{23}$$

where $I \in \mathbf{R}^{m \times m}$ is the identity matrix. For notational simplicity, assume unit covariance of the noise term e, *i.e.*, $Q = 1$.

The probability distribution of the unknown state determines the second term in (17). In most cases of practical interest, an assumed distribution of x results in either a linear or a quadratic penalty term in the loss index. We define the function $f : \mathbf{R}^n \to \mathbf{R}$ as

$$f(x) := -\log p_x, \tag{24}$$

where $f(x)$ is either a quadratic or a linear function of the unknown state x. Now the constraint (19) can be conveniently written as

$$J(x; z) = \|Pv\|^2 + f(x) \leq J^* \tag{25}$$

In its general form, an SOCP constraint in variable v is expressed as

$$\|Pv + b\| \leq a^T v + d, \tag{26}$$

where $b \in \mathbf{R}^m$, $a \in \mathbf{R}^{n+m+1}$, and $d \in \mathbf{R}$ can be chosen according to the problem at hand. It is straight forward to see that the constraints (21) and (25) can be easily incorporated as one SOCP constraint of the form (26). The solution to the approximate ALAMAP (or ALAML) problem can thus be efficiently computed using convex optimization techniques. Several off the bench solvers are available for solving such problems.

V. Application to monotonic trending

We now apply the concept of ALAML estimate to monotonic trending. Monotonic trends are a priori known to increase (or decrease) with time. This prior knowledge results in constraints on the unknown trend. Monotonic trends are particularly common in a fault estimation setting, where they may represent a gradually accumulating fault state such as mechanical damage during the course of a system operation. For details about monotonic trending and its application to fault estimation, see [11], [12]. This example illustrates the application of the concept of likelihood bounds to the case of constrained state estimation, *i.e.*, where the nominal MAP or ML estimate cannot be obtained using a simple least squares formulation.

Consider the data sequences y and x in (7) on the interval $t = \{1, \dots, N\}$, *i.e.*,

$$y = \{y(1), \dots, y(N)\}, \quad y(t) \in \mathbf{R} \tag{27}$$

$$x = \{x(1), \dots, x(N)\}, \quad x(t) \in \mathbf{R} \tag{28}$$

The monotonic trending problem is to estimate the unknown state x given the observed sequence y and the data matrix A. Each diagonal entry of A represents the linear relationship between the unknown state $x(t)$ and the observed parameter $y(t)$ at a particular instant. For a linear time invariant state estimation problem, all the entries of the diagonal of A are the same. In monotonic trending, we consider a one sided exponential distribution for the unknown state x, *i.e.*, $p_x = \frac{1}{\lambda} e^{-x/\lambda}$ for $x \geq 0$ and 0 otherwise. The penalty term $-\log p_x$ in the loss index (7) thus reduces to

$$-\log p_x = \frac{1}{\lambda} \sum_{t=2}^{N} [x(t) - x(t-1)] \tag{29}$$

In this example, we take A to be an identity matrix of size N. The gaussian noise distribution e is assumed to have covariance $Q = 1$. The sequence y (observed raw data) is generated by adding random noise to an underlying monotonic trend as shown in Fig. 4. There are $N = 25$ time samples for this simulation. We choose $\lambda = 1$ and the initial state covariance is assumed large, *i.e.*, no prior knowledge is available about the state $x(0)$. The loss index (7) is minimized subject to the monotonic state constraints

$$x(t+1) \geq x(t). \tag{30}$$

This is a linearly constrained quadratic programming (QP) problem. Its solution yields the MAP estimate for the nominal uncertain model. The nominal MAP estimate is shown in Fig. 4.

We now introduce uncertainty parameters ΔA and Δy in the given data model. The uncertainty parameters are considered bounded in the ℓ_1 norm, *i.e.*, $\|\Delta y\|_1 \leq r_y$ and $\|\Delta A\|_1 \leq r_A$. The ℓ_1 norm of the matrix here is the maximum absolute column sum norm.

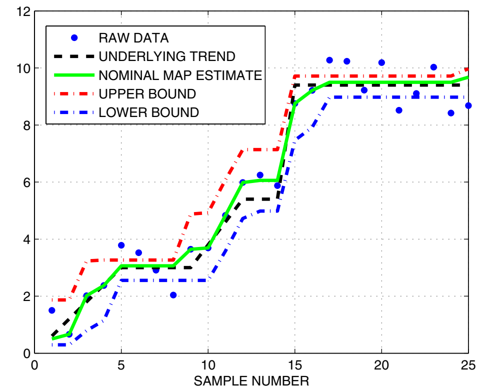

Fig. 4. Likelihood bounds for constrained state estimation

The ALAMAP estimation problem is to find a region of plausible estimates that in the presence of norm bounded uncertainty are at least as likely as the nominal MAP estimate. For this example, we choose $r_y = 1.2$ and $r_A = 0.01$ in (21). The value $r_y = 1.2$ corresponds to about 0.8% of the ℓ_1 norm of vector y. The resulting likelihood bounds obtained by solving the minimization in (13) subject to the constraints (19), (21) and the state constraints (30) are shown in Fig. 4. The likelihood bounds can be made tight or loose depending upon the magnitude of the allowed uncertainty. Fig. 5 shows a comparison of the likelihood bounds as r_y is increased from 1.2 to 1.6, where 1.6 corresponds to about 1% of the ℓ_1 norm of y. The bounds for $r_y = 1.2$ are tighter as expected. The uncertainty bounds r_A and r_y can thus be used as tuning parameters to obtain different likelihood bounds.

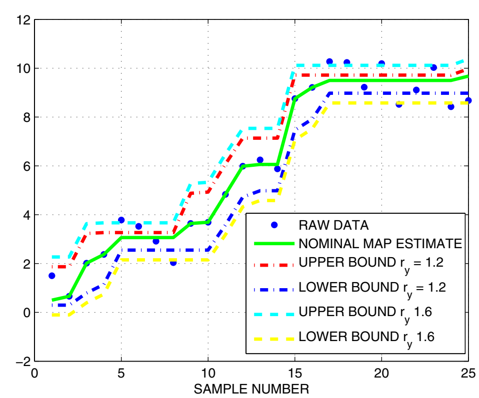

Fig. 5. Likelihood bounds for two uncertainty levels

VI. Application to system identification

We now apply the concept of ALAML to estimate a moving average (MA) or finite impulse response (FIR) model. We measure input $u(t)$ and output $y(t)$ for $t = \{0, \dots, N\}$ of the unknown system. The system identification problem deals with finding a reasonable model for a system based on measured input output data u, y. We illustrate the ALAML concept by an example where the input u and the output y are scalars. The multivariable case is handled readily.

Consider a moving average model with n delays

$$y(t) = h_0 u(t) + h_1 u(t-1) + \cdots + h_n u(t-n) + e(t) \quad (31)$$

where $\{h_0, \ldots, h_n\}$ is the system impulse response. Rewriting the above model in matrix form yields

$$\begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} u(n) & \cdots & u(0) \\ u(n+1) & \cdots & u(1) \\ \vdots & \vdots & \vdots \\ u(N) & \cdots & u(N-n) \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_n \end{bmatrix} + e \quad (32)$$

The above model is in the standard linear state estimation form (1). The data matrix in the system identification setting is often referred to as the auto regressor matrix. The objective is to find the FIR kernel h. We consider gaussian noise distribution, and assume no prior information about the probability density of the impulse response model, *i.e.*, $-\log p_x = 0$ in (7). The solution in this case can be easily obtained by regularized least-squares.

As an example we consider an FIR model with 6 delays and 100 data points, *i.e.*, $n = 6$ and $N = 100$. The excitation to the system is a pseudo random binary signal (PRBS). The input-output pair used for the simulation is shown in Fig. 6. Assuming a unit covariance for the nose e, we estimate

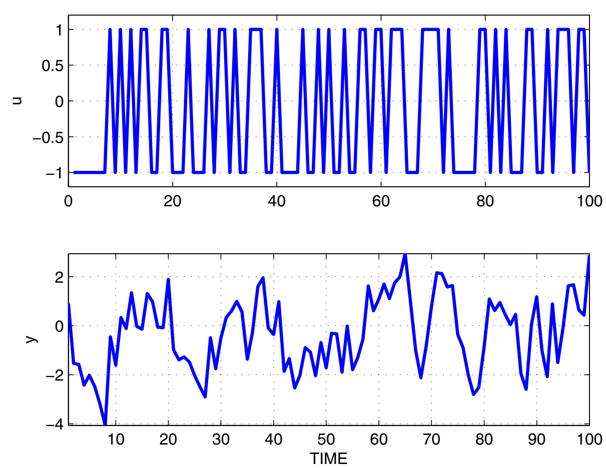

Fig. 6. Input and output sequence for FIR model

the true FIR model $h = [1 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.1]$. The estimate obtained through an over determined least-squares (LS) solution is shown in Fig. 7.

We now introduce uncertainty in the measurement vector y and the auto regression matrix of (32). The uncertainty bounds r_y and r_A are chosen as 0.5% of the ℓ_1 norm of y and A respectively. The likelihood bounds are obtained by solving the minimization in (18) subject to the constraints (19), (21). The upper and lower bounds are shown in Fig. 7 along with the outputs from the actual MA model. In practice, the model order selection is an important consideration and may effect the likelihood bounds as well. However here

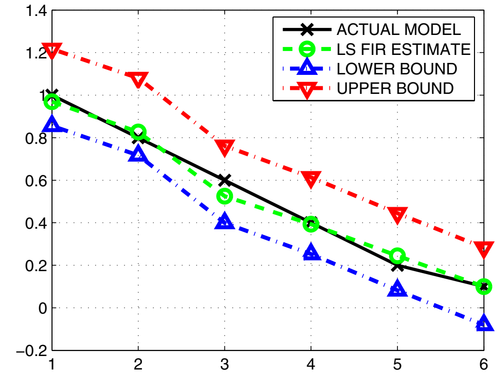

Fig. 7. Likelihood bounds for FIR model

we knew the actual model had $n = 6$ delays and so the issue of model order selection was not explored.

VII. Conclusion

In this paper we introduce the concept of ALAMAP (or ALAML) estimate in the presence of data uncertainty for constrained linear state estimation problems. The presented concepts are illustrated by application to FIR model identification and monotonic trending. The approach is based on convex optimization techniques and formulates the problem in terms of minimization of a linear objective subject to second order cone constraints. The computed likelihood bounds can be tuned by varying the uncertainty bounds specified for a particular problem.

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