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Formation Control with Virtual Leaders and Reduced Communications

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Abstract— **A feedback control law is given that can achieve a pre-specified formation for a group of mobile autonomous agents in an obstacle-free environment. This formation design uses virtual leaders and attractive forces to direct the group to track a desired path and achieve a desired formation, and uses repulsive forces to avoid agent collisions. The feedback control law can lead to reduced communication requirements by allowing an agent, under certain conditions, to cease communication with its neighbors. It is shown that by applying the pre-specified formation design with the feedback control law, the agent group can not only achieve and maintain a desired formation while tracking a desired path, but also avoid agent collisions with reduced inter-agent communications and reduced actuation of the individual agents.**

I. Introduction

Recently, the formation control of multi-agent systems has attracted many researchers from diverse fields in scientific and engineering disciplines. The object of formation control is to drive a group of mobile autonomous agents to move together in a desired formation and accomplish desired tasks. At the individual level, each agent follows certain simple rules that only employ local information. The idea of formation by design is inspired by observations of many examples in biology [2], [3], such as ant swarming, bird flocking, and fish schooling. Formation control of multi-agent systems is useful in many practical applications, including moving in formation for fleets of unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs), satellite clusters and mobile sensor networks.

In 1987, Reynolds [1] implemented in software three local flocking rules [2] for mobile agents. His simulations of objects he referred to as 'boids" mimicked the flocking of birds. The three rules are: *separation*, avoiding collisions with neighboring agents; *alignment*, matching velocity with neighboring agents; and *cohesion*, staying close to the neighboring agents. Since Reynolds' paper, many researchers used these three rules, achieving various designs for the local interaction rules of agents (e.g., [4]-[10]). In references [6]-[10], use was made of a local *attractive/repulsive potential* to set the interactive force between neighboring agents to implement the separation and cohesion rules. The tracking problem that appears in formation control arises when the group of agents needs to follow a desired path. This is addressed by the leader/follower approach in [5]-[7], [11], and the non-leader approach in [8]. The virtual leader approach taken here builds on the work of Leonard and Fiorelli [6].

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In most formation control designs, communications are used by each agent to monitor its environment, especially its neighbors, to guide its motion and avoid collisions [12]. In many applications, reducing communication requirements is a very important issue in formation control, since increased communications means more power consumption in addition to a greater likelihood for the agents to be detected. We also note that communications and actuation for models of flocking are linked. Communication links appear in our models as actuating forces driven by information from neighboring agents. Thus, by reducing communication needs there is also an automatic reduction in actuation of the individual agents over time.

In this paper, we present a distributed control law for a group of agents in an obstacle-free environment, whose function is to make the group not only achieve and maintain a desired formation while tracking a desired path, but also to avoid agent collisions with reduced inter-agent communications and reduced actuation of individual agents (these are linked, as noted above).

The main contribution of this work is to introduce a feedback control law with reduced communication requirements achieving a pre-specified formation as discussed by the authors in [13]. This design is based on virtual leaders and two interactive forces: the *attractive* force between each agent and its virtual leader to achieve *tracking*, *alignment* and *cohesion*; and the *repulsive* force between neighboring agents to ensure adequate *separation*. The feedback control law allows each agent to cease communication with its neighbors under certain conditions, hence reducing communications requirements.

The remainder of the paper proceeds as follows. In Section 2, we review a pre-specified formation design [13], then introduce a "*blind*" area for each agent, in which agents cease communication with their neighbors under certain conditions. Analytical results using Lyapunov theory are presented in Section 3. Section 4 provides some simulation results, and conclusions are summarized in Section 5.

II. Problem Formulation

A. Review of algorithm achieving a pre-specified formation

Consider a group of N identical mobile agents, modeled as point particles, moving in a plane with the following dynamics:

$$\dot{r}_i = v_i. \quad (1)$$

Here, r_i and $v_i \in R^2$ are the position and velocity of agent i. The relative displacement of agents i from j is denoted by $r_{ij} = r_i - r_j$. The immediate neighborhood of agent i is

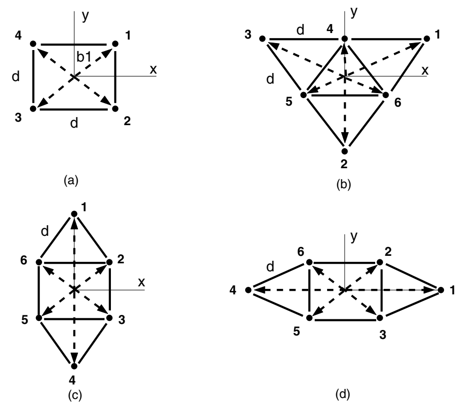

Fig. 1. Four pre-specified desired formations

defined as the interior of a circle of radius d centered at agent i.

In this section we review a design which begins with the introduction of N virtual leaders at r_{id}, $i = 1,\ldots,N$, one for each agent, and a desired path given by a trajectory $p(t)$. The trajectory of virtual leader r_{id} is given by $r_{id}(t) = p(t) + b_i$, where $b_i \in R^2$ is a constant vector. The vectors b_i, $i = 1,\ldots,N$, are predefined in accordance with the desired formation.

Fig. 1 illustrates four possible desired formations, where bold dots denote agents, and any two agents linked by a solid line are neighbors. In each formation, the distance between any two neighbors is d, the *sensing radius*. The values of the b_is can be calculated by taking the geometric center of each formation to be the origin. In this design, the b_is must satisfy the following two conditions:

1) $\frac{1}{N}\sum_{i=1}^{N} b_i = 0$;
2) If agents i and j are neighbors, then $\|b_j - b_i\| = d$.

In equation (1), v_i consists of two parts as follows

$$v_i = f(\|r_{id} - r_i\|)\frac{r_{id} - r_i}{\|r_{id} - r_i\|} + \sum_{j\in N_i(t)} g(\|r_j - r_i\|)\frac{r_j - r_i}{\|r_j - r_i\|} \tag{2}$$

where $f : R_+ \to R_+$ is a continuous, monotonously increasing function that represents the magnitude of the *attractive* force imposed on agent i by its virtual leader, $g : R_+ \to R_+$ is a continuous function that represents the magnitude of the *repulsive* force exerted on an agent by its neighbors and $N_i(t)$ denotes the label set of agent i's neighbors at time t. We define $f(\|0\|)/(\|0\|) = 0$. Fig. 2 shows the forms of functions f and g. Note that agent collisions are avoided by the requirement that $g(\|r_{ji}\|)$ goes to infinity as $\|r_{ji}\|$ approaches zero. The first term in equation (2) drives agent i to track its virtual leader, hence making the group approach the desired formation. The second term controls the distances between agent i and all its neighbors to avoid collisions.

Next, we modify this control method to reduce requirements for communication between an agent and its neighbors.

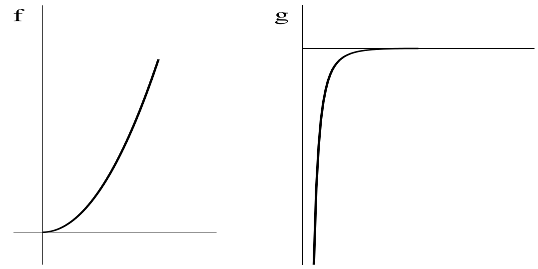

Fig. 2. The functions f and g

B. Feedback control re-design with blind areas

In this section, the design above is modified as follows:

$$\begin{aligned} v_i \;=\; & \omega_i(t) f(\|r_{id} - r_i\|)\frac{r_{id} - r_i}{\|r_{id} - r_i\|} \\ & + (1-\omega_i(t)) \sum_{j\in N_i(t)} g(\|r_j - r_i\|)\frac{r_j - r_i}{\|r_j - r_i\|} \end{aligned} \tag{3}$$

The parameters ω_i will be selected to lie in the interval $(0,1]$ so as to ensure that the inter-agent repulsive terms in this control law vanish when agent i is sufficiently close to its virtual leader. The modified design is called *pre-specified formation design with blind areas*.

Let $\Omega := \{\omega | 0 < \omega \le 1\}$, and require that $\omega_i(t) \in \Omega$, $\forall i$, for $t > 0$.

From equations (1) and (3), the system dynamics can be expressed as:

$$\dot{r}_i = \omega_i(t) f(\|r_{id} - r_i\|) n_{di} + (1-\omega_i(t)) \sum_{j\in N_i(t)} g(\|r_j - r_i\|) n_{ji} \tag{4}$$

where $n_{di} = (r_{id} - r_i)/\|r_{id} - r_i\|$ and $n_{ji} = r_{ji}/\|r_{ji}\|$. Denoting $\tilde{r}_i = r_i - r_{id}$ and $\tilde{r}_{ij} = \tilde{r}_i - \tilde{r}_j$, equation (4) becomes

$$\dot{\tilde{r}}_i = -\omega_i(t) f(\|\tilde{r}_i\|)\tilde{n}_i + (1-\omega_i(t)) \sum_{j\in N_i(t)} g(\|\tilde{r}_{ji} + b_{ji}\|)\tilde{n}_{ji} - q(t) \tag{5}$$

where $\tilde{n}_i = \tilde{r}_i/\|\tilde{r}_i\|$, $b_{ji} = b_j - b_i$, $\tilde{n}_{ji} = (\tilde{r}_{ji} + b_{ji})/\|\tilde{r}_{ji} + b_{ji}\|$ and $q(t) = \dot{r}_{id}(t)$, is the velocity of the virtual leaders. Note that all virtual leaders have the same velocities.

Define the *blind area* $B_i(t)$ for agent i at time t as the area bounded by a circle of radius α around $r_{id}(t) + \tilde{r}_i^*(t)$:

$$B_i(t) = \{\; r_i(t) \;\; | \;\; \|r_i(t) - r_{id}(t) - \tilde{r}_i^*(t)\| \le \alpha \;\}$$

where $\alpha \in [0, d/2)$ is a constant and $\tilde{r}_i^*(t)$ is the equilibrium point of equation (5) with $\omega_i(t) = 1$, $\forall i,t$. So $\tilde{r}_i^*(t)$ is the solution of the following equation:

$$f(\|z\|)\frac{z}{\|z\|} = -q(t) \tag{6}$$

Since function f is a predefined continuous, monotonously increasing function, $\tilde{r}_i^*$ is predictable and unique for each time instant t. Solving equation (6), we obtain

$$\tilde{r}_i^*(t) = -\frac{q(t)}{\|q(t)\|} f^{-1}(\|q(t)\|) \tag{7}$$

where f^{-1} is the inverse function of f.

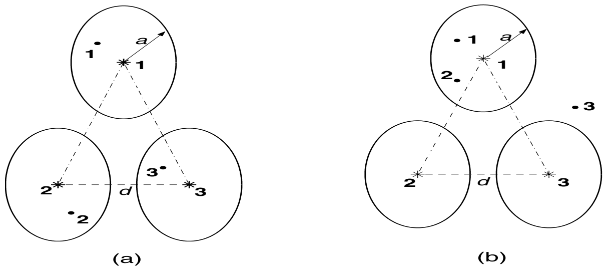

Fig. 3. Examples of a 3-agent group with blind areas

The feedback logic specifying $\omega_i(t)$ is as follows. Whenever agent i is inside its blind area, set $\omega_i(t) = 1$, meaning that agent i no longer communicates with its neighbors. Whenever agent i is outside its blind area, take any $\omega_i(t) < 1$, so that agent i communicates with its neighbors. Later, we will study how the choice of the $omega_i$ affects communication power savings.

Next, we use example of a 3-agent group to demonstrate how, when applying this feedback control law, the collision issue is resolved.

Figs. 3 (a) and (b) show a 3-agent group with a desired equilateral triangle formation at time t. In the figure, the bold dots with numbers $i = 1, 2, 3$ represent the positions of agent i at time t; the three circles denote the boundaries of the blind areas $B_i(t)$ for these agents; and the stars indicate the centers $r_{id} + \tilde{r}_i^*$, $i = 1, 2, 3$, of these regions at time t. Since the value of $\tilde{r}_i^*(t)$ is unique at time t, the formation formed by the centers of $B_i(t)$, $i = 1, 2, 3$ is the same as the desired formation formed by the virtual leaders, as can be seen in both figures.

Fig. 3 (a) shows the case when all the agents are inside their blind areas. So in this case $\omega_i(t) = 1$, $\forall i$, and each agent only experiences attractive force of its virtual leader. Since $\alpha \in [0, d/2)$, the three blind areas do not overlap and no agent collisions occur.

Fig. 3 (b) shows the case when agent *1* is inside $B_1(t)$, but agents *2* and *3* are outside of $B_2(t)$ and $B_3(t)$. Moreover, agent *2* is also inside $B_1(t)$. So $\omega_1(t) = 1$, $\omega_2(t)$, $\omega_3(t) < 1$. Consequently, agent *1* is only attracted by its virtual leader and does not communicate with its neighbors, so does not know of the existence of agent *2*. However agent *2* does know the existence of agent *1* by communication, so it will try to move away from agent *1* at time t to avoid collision. Agent *3* will proceed in a manner similar to agent *2*, if it is a neighbor of agent *1*. This is reminiscent of the fact that a seeing person will make way for a blind person when they approach each other on the sidewalk. The reason we call $B_i(t)$ the blind area of agent i is because agent i can be considered blind to its neighbors when $r_i(t) \in B_i(t)$.

In the next section, detailed analysis is given to see how this design can achieve collision-free tracking while reducing communication requirements.

III. Analysis

A. General parameters ω_i

In this section, we first consider the pre-specified formation design with general parameters ω_i, that might or might not imply the use of blind areas.

Proposition 3.1: Consider system (4). Suppose that the agent group has at least two agents, say agent i and j, satisfying the following condition:

$$\omega_i(t) \neq \omega_j(t) \qquad t \geq 0 \tag{8}$$

Then the agent group cannot achieve and maintain the desired formation.

Proof: Suppose the agent group can achieve the desired formation at time t_f and maintain it for $t > t_f$, then the agents' positions must satisfy:

$$r_i = r_{id} + a(t) \quad \forall i, t \geq t_f$$

where $a(t) \in R^2$ is the relative displacement between each agent and its virtual leader at time t, which should be the same for all the agents. Then equation (5) becomes:

$$\omega_i(t) f(\|a(t)\|) \frac{a(t)}{\|a(t)\|} = -q(t) - \dot{a}(t) \qquad \forall i, t \geq t_f \tag{9}$$

Because f is a continuous, monotonously increasing function and same for every agent, the value of $\omega_i(t)$ that satisfies equation (9) for each time instant must be unique and satisfy:

$$\omega_i(t) = \omega_j(t) \quad \forall i \neq j, i, j \in [1, \ldots, N], t \geq t_f \tag{10}$$

Conditions (10) and (8) are contradictory, so the Proposition 3.1 is proved.

Remark 3.1: If we want to use the pre-specified formation design with blind areas to drive the agent group to achieve the desired formation and maintain it, then $\omega_i(t)$, $\forall i$, must satisfy condition (10).

B. Common design for parameters ω_i with blind areas

We now analyze the pre-specified formation design with a special choice of the parameters ω_i that also implies the use of blind areas. In this design, the logic generating each ω_i is shared by all agents.

In this paper, we define $\omega_i(t)$ as

$$\omega_i(t) = \begin{cases} 1 & \|r_i - r_{id} - \tilde{r}_i^*(t)\| \leq \alpha \\ \omega & \|r_i - r_{id} - \tilde{r}_i^*(t)\| > \alpha \end{cases}$$

where $\omega \in \Omega/\{1\}$ is a constant. Further work will be done in the future using more complicated $\omega_i(t)$. Here, two cases are considered.

Case 1: The virtual leaders have velocity $q(t) = q$, where q is a constant vector. First, we discuss the behavior of the agent group when all the agents are outside their blind areas, i.e. $\omega_i(t) = \omega$, $\forall i$. Rewrite equation (4) in this case as follows:

$$\dot{r}_i = \omega f(\|r_{id} - r_i\|) n_{di} + (1 - \omega) \sum_{j \in N_i(t)} g(\|r_j - r_i\|) n_{ji} \tag{11}$$

Defining $\hat{f} = \omega f$ and $\hat{g} = (1 - \omega) g$, equation (11) becomes:

$$\dot{r}_i = \hat{f}(\|r_{id} - r_i\|) n_{di} + \sum_{j \in N_i(t)} \hat{g}(\|r_j - r_i\|) n_{ji} \tag{12}$$

Note that function $\hat{f}$ ($\hat{g}$) has the same property as function f (g), since $\omega > 0$ ($(1-\omega) > 0$). So (12) does not reduce communications between agents.

Rewriting (12) using $\tilde{r}_i = r_i - r_{id}$ and $\tilde{r}_{ij} = \tilde{r}_i - \tilde{r}_j$, we get

$$\dot{\tilde{r}}_i = -\hat{f}(\|\tilde{r}_i\|)\frac{\tilde{r}_i}{\|\tilde{r}_i\|} + \sum_{j\in N_i(t)} \hat{g}(\|\tilde{r}_{ji}+b_{ji}\|)\frac{\tilde{r}_{ji}+b_{ji}}{\|\tilde{r}_{ji}+b_{ji}\|} - q \quad (13)$$

It is shown theoretically and numerically, in [13], that $\tilde{r}_i(t)$, $\forall i$, converges to one equilibrium of (13), which is the solution of the following equation:

$$\hat{f}(\|z\|)\frac{z}{\|z\|} = -q. \quad (14)$$

Denoting this equilibrium point as $(\tilde{r}_i^*)_\omega$, replacing $\hat{f}$ with ωf and solving (14), we obtain:

$$(\tilde{r}_i^*)_\omega = -\frac{q}{\|q\|} f^{-1}(\frac{\|q\|}{\omega}) \quad (15)$$

So when all the agents are outside their blind areas, the trajectory of $r_i(t)$ converges to the trajectory of $(r_{id}(t) + (\tilde{r}_i^*)_\omega)$. However, reducing agent i's communications requires agent i to be inside its blind area, i.e. $(r_{id}(t) + (\tilde{r}_i^*)_\omega) \in B_i(t)$. Otherwise, it is possible for agent i to converge to some point outside its blind area. So we need

$$\|(\tilde{r}_i^*)_\omega - \tilde{r}_i^*\| \le \alpha$$

where $\tilde{r}_i^*$ in this case is the solution of (6) with $q(t) = q$. So

$$\left| f^{-1}(\frac{\|q\|}{\omega}) - f^{-1}(\|q\|) \right| \le \alpha \quad (16)$$

where $\|q\|/\omega > \|q\|$ because $\omega \in \Omega/\{1\}$. Since $f: R_+ \to R_+$ is a continuous, monotonously increasing function, its inverse f^{-1} is also a continuous, monotonously increasing function. Condition (16) becomes

$$f^{-1}(\frac{\|q\|}{\omega}) \le f^{-1}(\|q\|) + \alpha \quad (17)$$

Condition (17) is a necessary condition on ω and α to guarantee communications reduction by using this design.

Denote by t_1 as the first time instant when there is at least one agent inside its blind area, and by t_2 the last time instant when there is at least one agent outside its blind area. The behavior of the group in the time interval $[t_1, t_2]$ is very complicated, which results in difficulty in doing classical analysis.

At this stage, the authors have not found a sufficient condition on ω and α to guarantee that every agent will enter its blind area. However, if we assume that the initial position of the group center and $p(t_0)$ coincide, by doing a large number of simulations, it appears that condition (17) is enough to guarantee that every agent will enter its blind area. This assumption about the initial condition is reasonable because the starting point of the agent group, defined as the initial position of the group center, and the starting point of the desired path are better taken as close to each other as possible for the purpose of reducing the convergence time.

Next, we analyze the behavior of agent i subsequent to its entry of its blind area.

Proposition 3.2: Consider system (5) with $q(t) = q$, a constant vector. Suppose f is a convex function and agent i is inside its blind area. Then every solution of this system converges asymptotically to the equilibrium point $\tilde{r}_i^*$ of the system.

Proof. Since agent i is inside its blind area, (5) becomes

$$\dot{\tilde{r}}_i = -f(\|\tilde{r}_i\|)\frac{\tilde{r}_i}{\|\tilde{r}_i\|} - q \quad (18)$$

Introduce the scalar Lyapunov-type function

$$V := \frac{1}{2}\dot{\tilde{r}}_i^T \dot{\tilde{r}}_i$$

and note that $V(\tilde{r}_i^*) = 0$ and $V(\tilde{r}_i) > 0$, $\forall\ \tilde{r}_i \ne \tilde{r}_i^*$. The time-derivative of V is

$$\begin{aligned}
\dot{V} &= \dot{\tilde{r}}_i^T \ddot{\tilde{r}}_i \\
&= \dot{\tilde{r}}_i^T \{-\dot{f}(\|\tilde{r}_i\|)\frac{\tilde{r}_i^T \dot{\tilde{r}}_i \tilde{r}_i}{\|\tilde{r}_i\|^2} \\
&\quad - f(\|\tilde{r}_i\|)\frac{\dot{\tilde{r}}_i}{\|\tilde{r}_i\|} + f(\|\tilde{r}_i\|)\frac{\tilde{r}_i \tilde{r}_i^T \dot{\tilde{r}}_i}{\|\tilde{r}_i\|^3}\} \\
&= -\dot{f}(\|\tilde{r}_i\|)\frac{\dot{\tilde{r}}_i^T \tilde{r}_i^T \dot{\tilde{r}}_i \tilde{r}_i}{\|\tilde{r}_i\|^2} \\
&\quad - f(\|\tilde{r}_i\|)\frac{\dot{\tilde{r}}_i^T \dot{\tilde{r}}_i}{\|\tilde{r}_i\|} + f(\|\tilde{r}_i\|)\frac{\dot{\tilde{r}}_i^T \tilde{r}_i \tilde{r}_i^T \dot{\tilde{r}}_i}{\|\tilde{r}_i\|^3} \\
&= -\left(\dot{f}(\|\tilde{r}_i\|) - \frac{f(\|\tilde{r}_i\|)}{\|\tilde{r}_i\|}\right)\frac{\|\tilde{r}_i^T \dot{\tilde{r}}_i\|^2}{\|\tilde{r}_i\|^2} \\
&\quad - f(\|\tilde{r}_i\|)\frac{\|\dot{\tilde{r}}_i\|^2}{\|\tilde{r}_i\|}
\end{aligned} \quad (19)$$

where $\dot{f} = df(\|z\|)/d\|z\|$. From equation (19), it is known that since f is a convex function we can guarantee

$$\dot{f}(\|\tilde{r}_i\|) \ge \frac{f(\|\tilde{r}_i\|)}{\|\tilde{r}_i\|}. \quad (20)$$

Since $f(\|\tilde{r}_i\|) \in R_+$, we therefore have that $\dot{V}$ is non-positive, $\forall t$.

So $\dot{V}(\tilde{r}) \le 0$ and $\dot{V}(\tilde{r}) = 0$ if and only if $\dot{\tilde{r}} = 0$. Note that when $\|\tilde{r}_i\| \to \infty$, $V(\tilde{r}_i) \to \infty$. From Lyapunov stability theory, the equilibrium $\tilde{r}_i^*$ is globally asymptotically stable. So every solution of (18) converges asymptotically to its equilibrium point $\tilde{r}_i^*$, which is

$$\tilde{r}_i^* = -\frac{q}{\|q\|} f^{-1}(\|q\|)$$

Remark 3.2: Proposition 3.2 shows that the assumption that f is a convex function is very important for the feedback design with blind areas. So this assumption is taken for every such design in this paper, not just for Proposition 3.2.

From Proposition 3.2, we can say that once the agent i moves inside its blind area, its trajectory $r_i(t)$ converges to the trajectory of $(r_{id}(t) + \tilde{r}_i^*)$ asymptotically.

Proposition 3.3: For each agent in the agent group using the blind areas design with $q(t) = q$, once it moves inside its blind area, it will exit.

Proof. In this case, the center of agent i's blind area at time t is at $(r_{id}(t) + \tilde{r}_i^*)$, so Proposition 3.2 shows that every agent, once it moves inside its blind area, will moves asymptotically to the center of its blind area.

Since the centers of all the agents' blind areas form the desired formation, we conclude that by using this design, eventually, the agent group can achieve the desired formation and maintain it.

Case 2: Consider the case that $q(t)$ is continuously differentiable and $\|\dot{q}(t)\|$ is sufficiently small, i.e., $q(t)$ is slowly varying. To analyze this system, we can treat q in short time intervals as a frozen parameter. Then the frozen system with each fixed $q(t) = q$ is identical to the system discussed in Case 1. Note that the properties of the system found in Case 1 are uniform in q, so it is reasonable to expect that this slowly varying system will possess a similar property.

We assume that the initial position of the group center and $p(t_0)$ coincide in this case. When all the agents are outside their blind areas, the system dynamics can be written as equation (5) with $\omega_i(t) = \omega$. In [13], it is shown that when $q(t)$ varies slowly, every solution of equation (5) converges to its equilibrium point, which is denoted as $(\tilde{r}_i^*)_\omega(t)$ and

$$(\tilde{r}_i^*)_\omega(t) = -\frac{q(t)}{\|q(t)\|} f^{-1}\left(\frac{\|q(t)\|}{\omega}\right) \tag{21}$$

By doing the similar analysis as what we did in Case 1, we get the necessary condition on ω and α at time t as

$$\left(f^{-1}\left(\frac{\|q(t)\|}{\omega}\right) - f^{-1}(\|q(t)\|) \right) \leq \alpha \tag{22}$$

So we modify $\omega_i(t)$ as follows:

$$\omega_i(t) = \begin{cases} 1 & \|r_i - r_{id} - \tilde{r}_i^*(t)\| \leq \alpha \\ \omega(t) & \|r_i - r_{id} - \tilde{r}_i^*(t)\| > \alpha \end{cases}$$

where $\omega(t)$ is required to satisfy condition (22). Further work needs to be done to determine sufficient condition on ω and α at time t to guarantee that every agent enters its blind area. However, again, no exceptions have been found in the simulations to the hypothesis that condition (22) suffices.

Next, consider the behavior of agent i after it enters its blind area. Denote the first time instant that $r_i \in B_i(t)$ as $\hat{t}_0$. Consider the system (5) with slowly varying $q(t)$ and initial time $t_0 = \hat{t}_0$. Let $t_0, t_1, t_2, \ldots$ be an increasing sequence of time instants. Suppose that in each time interval $[t_{j-1}, t_j]$, $j = 1, 2, \ldots$, the change of $q(t)$ is small enough that it can be taken as a constant vector q_j. Also assume that function f is chosen such that in each interval $[t_{j-1}, t_j]$, $\tilde{r}_i \to (\tilde{r}_i^*)_j$ as $t \to t_j$, where $(\tilde{r}_i^*)_j$ is

$$(\tilde{r}_i^*)_j = -\frac{q_j}{\|q_j\|} f^{-1}(\|q_j\|)$$

Then we expect that the trajectory of every solution of this system converges to the path $\tilde{r}_i^*(t)$, which is the solution of (6).

IV. Simulation Results

In this section, we present several simulation results using the proposed design with blind areas. In each simulation, the initial conditions of the agents are given by a set of N random initial positions (uniform distributions in a 50×50 area), and zero initial velocities. The starting point of the desired path is set at $\left(\sum_i^N r_i(0)\right)/N$. In all the simulations, we use $d = 20$ and the nonlinear function $f(\|z\|) = 0.05\|z\|^2$. In all the plots, bold dots denote the agents, cross marks denote the virtual leaders, dotted circles denote the boundaries of the blind areas, and a dashed line denotes the desired path.

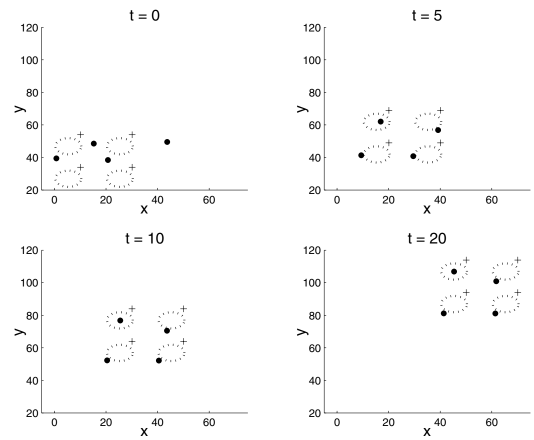

Fig. 4. Failed formation control of a 4-agent group with $\omega = 0.3$

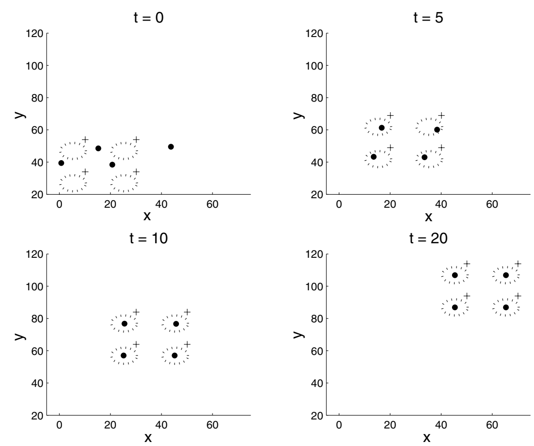

Fig. 5. Successful formation control of the same agent group in the first simulation with $\omega = 0.5$

The first simulation is for a 4-agent group with the desired formation shown in Fig. 1(a). We set the virtual leaders' velocities $q = [2, 3]^T$ and $\alpha = d/4$. So the necessary condition on ω is $\omega \geq 0.3962$. Fig. 4 shows the results by setting $\omega = 0.3$, where eventually only one agent moved inside its blind area and converged to the center. So the desired formation was not achieved.

Fig. 5 shows the second simulation results, obtained with ω changed to 0.5. We see that all the agents have entered their blind areas at time $t = 5$ and converge to the centers eventually. So finally the agent group achieves the desired formation and maintains it. Fig. 6 shows the trajectories of the agents along with the desired path (trajectories begin in the lower left corner of the figure).

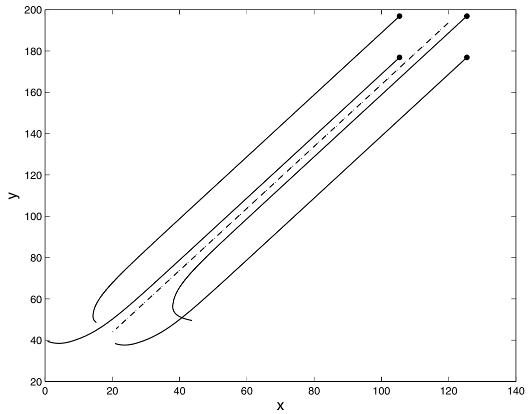

Fig. 6. Agent trajectories in the second simulation

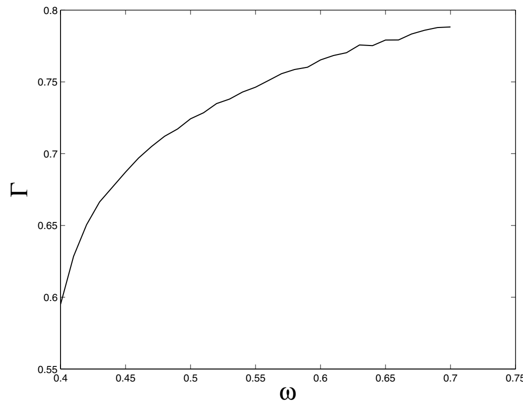

Fig. 7. The relationship between Γ and ω given $\alpha = d/4$

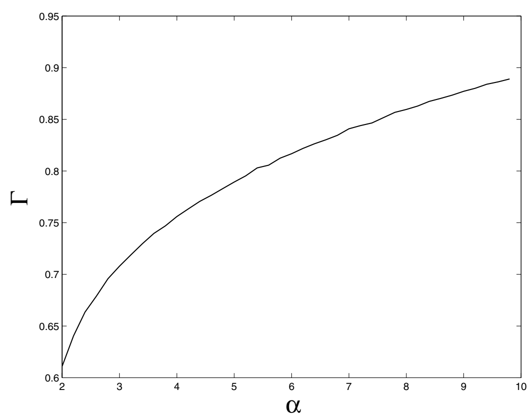

Fig. 8. The relationship between Γ and α given $\omega = 0.7$

In the third simulation, we compared the communication power consumed by an agent group applying the blind areas design, with that consumed by the same agent group applying the original design. We assume that the communication power used by agent i is $\triangle p$ per unit time, if $\omega_i(t) < 1$. So the total power consumed by a N-agent group in the time interval $[0,T]$ when applying the blind areas design is

$$P_{B.A.} = \sum_{i=1}^{N} \triangle p \int_0^T U(\omega_i(t))\, dt$$

where $U(\omega_i(t)) = 1$, when $\omega_i(t) < 1$ and $U(\omega_i(t)) = 0$, when $\omega_i(t) = 1$. The total power consumed by a N-agent group in the time interval $[0,T]$ when applying the original design is

$$P = \sum_{i=1}^{N} \triangle p \cdot T$$

We use $\Gamma := (P - P_{B.A.})/P$ to indicate the percentage of communication power saved by using the blind areas design over the original design.

As an illustration, we consider a 6-agent group with the desired formation shown in Fig. 1(b). We set $q = [2,3]^T$ and $T = 15$. Fig. 7 shows the relationship between Γ and ω for $\omega \in \{0.4, 0.41, 0.42, \ldots, 0.7\}$, given $\alpha = d/4$. Fig. 8 shows the relationship between Γ and α for $\alpha \in \{0.10d, 0.11d, \ldots, 0.49d\}$, given $\omega = 0.7$. In both figures, for each ω or α value, we run the simulation 1000 times from random initial conditions to obtain the average Γ. From both figures, we see that when increasing the values of ω and α, we can save more power. For example, Fig. 7 shows that when using $\omega = 0.7$ and $\alpha = 0.49d$, we can save more than 85% communication power in this case.

In the fourth simulation, we use a 6-agent group with slowly varying $q(t) = [5cos(\frac{\pi}{50}t), 10sin(\frac{\pi}{50}t)]^T$ and the desired formation of Fig. 1(c). We set $\alpha = d/4$ and

$$\omega_i(t) = \begin{cases} 1 & \|r_i - r_{id} - \tilde{r}_i^*(t)\| \le \alpha \\ \frac{\|q(t)\|}{\left(\sqrt{\|q(t)\|} + \sqrt{0.05}\alpha\right)^2} + 0.02 & \|r_i - r_{id} - \tilde{r}_i^*(t)\| > \alpha \end{cases}$$

to satisfy the necessary condition on ω. Fig. 9 shows the successful flocking of this group. Fig. 10 shows the trajectories of these agents along with the desired path (trajectories begin in the lower left corner of the figure).

V. Conclusion

In this work, we introduced a feedback control law for pre-specified formations, which is designed to achieve flocking of a group of mobile autonomous agents in an obstacle-free environment. Virtual leaders and two different interactive forces are used in this formation design to achieve a desired formation and direct the group to track a desired path, while avoiding agent collisions. The feedback control law is used to achieve low communication (and actuation) requirements by having an agent cease communication with its neighbors under certain conditions. It is shown that by applying this design, the agent group can not only achieve and maintain a desired formation while tracking a desired path, but also avoid agent collisions with reduced inter-agent communications as well as reduced actuation of individual agents over time.

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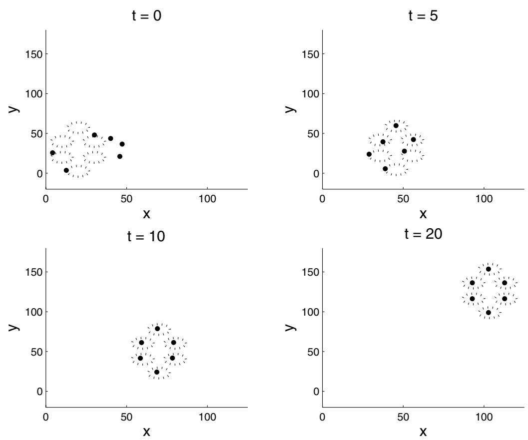

Fig. 9. Successful formation control of six agents with slowly varying $q(t)$

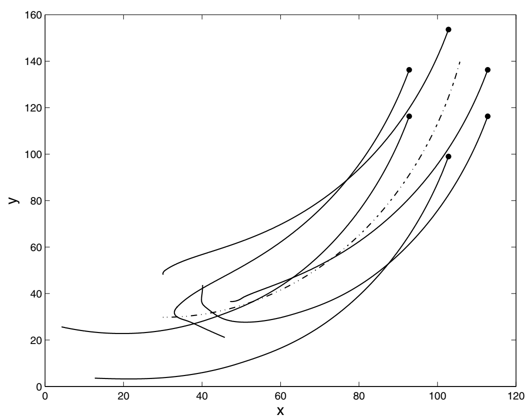

Fig. 10. The trajectories of 6 agents in the fourth simulation

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