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# Fault Accommodation Strategy for LTI Systems based on GIMC Structure: Additive Faults.

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*Abstract*— **In this contribution a fault accommodation strategy is suggested for LTI systems. The faults and perturbations are considered as additive signals that modify the output measurement. The accommodation scheme is based on the generalized internal model control architecture recently proposed [17] for fault tolerant control. In order to improve the performance after a fault, the compensation is considered in two steps according with a fault detection and isolation algorithm. After a fault scenario is detected a general fault compensator is activated. Finally, once the fault is isolated a specific compensator is introduced. In this setup, multiple faults could be simultaneously treated since their effect is assumed to be additive.**

## I. INTRODUCTION

In the early stages of control applications, the achievement of good closed-loop performance was the main objective. To achieve this goal, the implementation of these feedback configurations involve sensors, actuators, electronic instrumentation, and digital signal processors. However during a normal operation, these parts could fail in some degree, and the resulting performance of the closed-loop will be largely deteriorated or even unstability can be observed. But in some processes besides performance, safety is also a necessary and important objective. Therefore, it is desirable to detect these malfunctions to take proper action in order to avoid a dangerous situation. Nowadays, the advances in the electronics has made possible to have digital signal processors as microcontrollers, DSP's and FPGA boards that can perform in real time very complex algorithms. Hence this extra processing capacity could be applied to perform in parallel fault diagnosis algorithms to the nominal control schemes. The problem of fault diagnosis is indeed a challenging one, and its importance in applications has attracted the attention of the research community in control theory and signal processing [4], [6], [13].

In any process, the faults can be classified in two sets: *unrecoverable* and *recoverable*. The unrecoverable faults represent all the faults that cannot be compensated or repaired while the system is running. On the other hand, the recoverable faults comprise any fault whose outcome can still be compensated safely by the control algorithm with a possible deterioration of performance, but still allowing the necessary conditions to maintain closed-loop stability. Obviously, this classification depends on the problem at hand, and requires knowledge about the operation of the system. From a control point of view, the focus is on the *recoverable faults*, where a degree of robustness or reconfigurability in the control scheme is desirable to accommodate these faults and still preserve closed-loop performance. These ideas have triggered a research line called Fault Tolerant Control (FTC) [1], [2], [3], [12], [14].

FTC can be approached from two perspectives: passive and active. In the passive approach, the faults are treated as disturbances into the closed-loop system. As a result, a single controller is designed to achieve stability and performance against all the faults analyzed. The main drawback of this scheme is the conservativeness that can be incorporated, however no extra complexity in the control implementation is carried out. In LTI systems, the passive approach can be treated as a simultaneous stabilization or robust $H_\infty$ design [14], and for nonlinear systems, a variable structure control (sliding mode) methodology can be applied [8]. On the other hand, the active approach of FTC requires a fault diagnosis stage, followed by a controller reconfiguration or accommodation [2]. Compared to the passive approach, the active one requires more computational power during implementations, since it relies on a fault diagnosis stage, but it can provides less conservative results and overall better closed-loop performance after faults. Applications of the active idea have been suggested for LTI [3], [12], [15], [17], and nonlinear systems [7], [16].

In this work, an active FTC scheme is proposed for LTI systems under an additive faults scenario. Design strategies are proposed for the diagnosis and accommodation schemes based on general optimization criteria. The paper is structured as follows. Section 2 describes the problem formulation. The FTC scheme is presented in Section 3. First, the general methodology is introduced, and the design criteria for the diagnostic, isolation and accommodation are detailed. Section 4 analyzes the effect of model uncertainty in the FTC scheme. Finally, Section 5 gives some concluding remarks.

## II. PROBLEM FORMULATION

The problem addressed in this paper is fault accommodation for LTI systems under additive faults and perturbations. In this way, consider a system $P(s)$ affected by disturbances $d \in R^r$ and possible faults $f \in R^l$, see Figure 1, described

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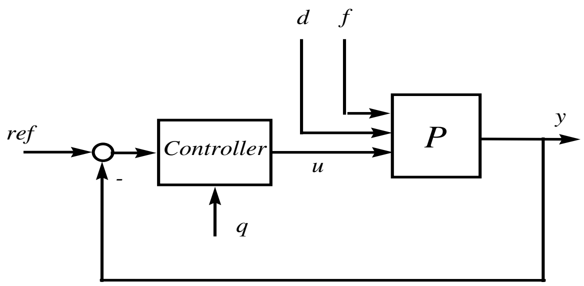


Fig. 1. Problem Formulation for Control.

by

$$\begin{aligned}\dot{x} &= Ax + Bu + F_1 f + E_1 d \\ y &= Cx + Du + F_2 f + E_2 d\end{aligned} \quad (1)$$

where $x \in R^n$ represents the vector of state, $u \in R^m$ the vector of input, and $y \in R^p$ the vector of output. Thus, the matrix $F_1 \in R^{n\times l}$ stands for the distribution matrix of the actuator faults and $F_2 \in R^{p\times l}$ for the sensor faults. Denote as $F_1^i \in R^n$ and $F_2^i \in R^p$ with $i = 1,\ldots,l$ the columns of the fault signature matrices $F_1$ and $F_2$ respectively, i.e.

$$F_1 = \begin{bmatrix} F_1^1 & \cdots & F_1^l \end{bmatrix} \quad (2)$$

$$F_2 = \begin{bmatrix} F_2^1 & \cdots & F_2^l \end{bmatrix} \quad (3)$$

Thus, the matrices $(F_1^i, F_2^i)$ will be represent the signature of the $i$-th component in the fault vector $f$. The nominal system $(A,B,C,D)$ is considered controllable and observable. On the other hand, the system response $y$ can be analyzed in a transfer matrix form (frequency domain):

$$y(s) = P_{uy}u(s) + P_{fy}f(s) + P_{dy}d(s) \quad (4)$$

where

$$\begin{aligned}P_{uy} &= C(sI-A)^{-1}B + D \\ P_{dy} &= C(sI-A)^{-1}E_1 + E_2 \\ P_{fy} &= C(sI-A)^{-1}F_1 + F_2\end{aligned} \quad (6)$$

A left coprime factorization for each transfer matrix can be derived by finding a matrix $L \in R^{n\times p}$ such that $Re\{\lambda_i(A+LC)\} < 0$ [18] as in (5). Consequently, the plants in (6) can be written as

$$P_{uy} = \tilde{M}^{-1}\tilde{N},\ P_{dy} = \tilde{M}^{-1}\tilde{N}_d,\ P_{fy} = \tilde{M}^{-1}\tilde{N}_f \quad (7)$$

where $\tilde{M}, \tilde{N}, \tilde{N}_d, \tilde{N}_f \in RH_\infty$. An initial question about the fault diagnosis and isolation process relies on the necessary conditions to achieve this objective, hence the relations presented in [13] are assumed:

1) For isolation of the fault vector $f$

$$rank(\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}) = l \quad (8)$$

2) For the simultaneous isolation of the faults while having perturbations

$$normrank(\begin{bmatrix}\tilde{N}_d\ \tilde{N}_f\end{bmatrix}) \geq normrank(\tilde{N}_d) + l \quad (9)$$

where $normrank$ stands for the normal rank of the corresponding transfer matrix [18].

Now, it is assumed that a nominal controller $K$ stabilizes the nominal plant $P_{uy}$, and it provides a desired closed-loop performance. The controller $K$ is considered observable, and consequently, it can also be expressed by a left coprime factorization, i.e. $K = \tilde{V}^{-1}\tilde{U}$ where $\tilde{U}, \tilde{V} \in RH_\infty$. The nominal controller can be synthesized following classical techniques or optimal control: Lead/lag compensator, PID, $LQG/H_2$, $H_\infty$ loop shaping design, etc. Consequently, the **control objective** is presented as: *Design an integrated fault-tolerant scheme such that it detects the occurrence of a fault in the closed-loop system, and provides an appropriate compensation signal $q$ to the controller in order to maintain closed-loop performance*, see Figure 1.

## III. Fault Tolerant Control Scheme

The algorithm for Fault-Tolerant Control (FTC) presented in this paper is the so-called *active* [2],[3]. Therefore, the FTC scheme relies on a fault diagnosis and isolation (FDI) algorithm, followed by a fault accommodation into the nominal controller. For LTI systems, several FTC control structures have been suggested [10], [11], [17] departing from the Youla parameterization of all stabilizing controllers [18]. In this configuration, a free parameter $Q \in RH_\infty$ is selected to achieve the fault compensation, with the assurance that closed-loop stability is achieved after the fault accommodation.

The accommodation scheme is derived from robust control theory [18], where a new implementation of the Youla parameterization called *Generalized Internal Model Control (GIMC)* is used [3],[17]. In this configuration, the nominal controller $K$ is represented by its left coprime factorization, i.e. $K = \tilde{V}^{-1}\tilde{U}$. In addition, the GIMC configuration allows to perform the FDI process and accommodation in the same structure, where these two processes are carried out by selecting two design parameters $Q, H \in RH_\infty$ (see Figure 2). Consequently, the residual $r$ is generated by selecting $H$, and the accommodation signal $q$ by the compensator $Q$, using the filtered signal $f_e$ with the following criteria:

1) $H(s)$: the fault detection filter must diminish the effect of the disturbances or uncertainty into the residual signal, and maximize the effect of the faults.
2) $Q(s)$: the robustification controller must provide robustness into the closed-loop system in order to maintain acceptable performance against faults.

### A. Fault Detection and Isolation

Note that from Figure 2, it can be observed that $f_e \in R^p$ contains the information of the perturbations $d$ and faults $f$:

$$f_e(s) = -\tilde{N}_d\, d(s) - \tilde{N}_f\, f(s) \quad (10)$$

Hence a residual $r$ is naturally constructed by using the information of the coprime factorization of the nominal plant [4]:

$$r(s) = -Hf_e(s) = H\left[\tilde{N}_d\, d(s) + \tilde{N}_f\, f(s)\right] \quad (11)$$

$$\left[\begin{array}{cccc}\tilde{N} & \tilde{M} & \tilde{N}_d & \tilde{N}_f\end{array}\right] = \left[\begin{array}{c|cccc} A+LC & B+LD & L & E_1+LE_2 & F_1+LF_2 \\ \hline C & D & I & E_2 & F_2 \end{array}\right] \quad (5)$$

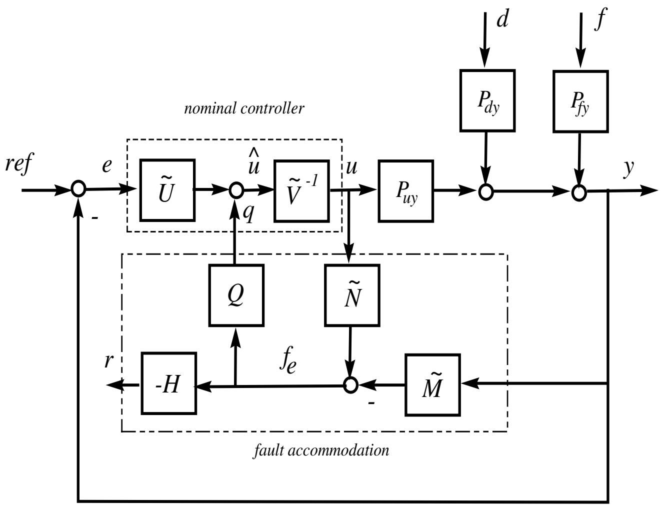


Fig. 2. GIMC with Additive Perturbations and Faults.

The filter $H$ is then designed to isolate the fault vector $f$ and decouple the perturbations $d$, i.e.

- $H\tilde{N}_d(s) \approx 0$,
- $H\tilde{N}_f(s) \approx T$

where $T \in RH_\infty$ is a diagonal transfer matrix. This transfer matriz $T$ is a design parameter, and it should be chosen according with the frequency dynamics of $\tilde{N}_f$, in order to achieve the decoupling objective. The design criterion can be then proposed by combining both objectives measured by a systems norm $j = 1, 2, \infty$:

$$\min_{H \in RH_\infty} \left\| [T \; 0] - H[\tilde{N}_f \; \tilde{N}_d] \right\|_j = \min_{H \in RH_\infty} \| F_l(G_H, H) \|_j \quad (12)$$

where the optimization can be posted using a lower linear fractional transformation (LFT) $F_l(\cdot,\cdot)$ [18], and $G_H$ represents the generalized plant (see Figure 3) given by

$$G_H = \begin{bmatrix} 0 & T & -I \\ \tilde{N}_d & \tilde{N}_f & 0 \end{bmatrix} \quad (13)$$

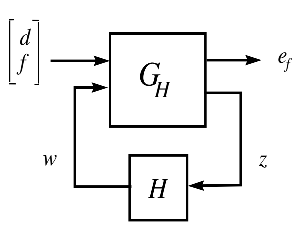


Fig. 3. LFT Formulation for Compensator H.

*Remark 1:* The assumptions (8) and (9) about the rank properties of the perturbations and faults transfer matrices guarantee that the optimal performance obtained by (12) will provide a good fault isolation property of the residual.

In order to detect a fault, the following residual evaluation criteria can be followed:

$$\|r\| = \|r\|_{2,t,T_o} = \sqrt{\int_{t-T_o}^{t} r^*(\tau) r(\tau) d\tau} \quad (14)$$

$$\|r\| = \|r\|_{\infty,t,T_o} = \sup_{t-T_o \le \tau \le t} \|r\|_2 \quad (15)$$

where $T_o$ is the window length or horizon of evaluation. Hence to avoid a false alarm in the evaluation due to perturbations, a threshold value is selected such that

$$J_{th} = \sup_{f=0,\forall d} \|r\| \quad (16)$$

In the case of the windowed two norm, an assuming that the perturbations satisfy $\|d\|_{2,t,T_o} < \gamma$, then a threshold can be calculated as

$$J_{th} = \gamma \|H\tilde{N}_d\|_\infty \quad (17)$$

## B. Fault Accommodation

In order to derive the fault accommodation scheme, the effect of the compensation signal $q$ in the GIMC structure of Figure 2 is analyzed. Define the nominal closed-loop transfer matrices:

- Input sensitivity $S_i = (I + KP_{uy})^{-1}$,
- Output sensitivity $S_o = (I + P_{uy}K)^{-1}$,
- Complementary output sensitivity $T_o = I - S_o = (I + P_{uy}K)^{-1}P_{uy}K$.

The next lemma characterize the dynamic behavior of the compensated control input $u$, and output $y$ of the closed-loop system.

*Lemma 1:* In the GIMC configuration of Figure 2 considering additive faults, the resulting closed-loop characteristics for the control signal $u$ and output $y$ are given by

$$\begin{aligned} u(s) &= S_i K\, ref(s) - \quad (18) \\ &\quad S_i \tilde{V}^{-1} \left( \tilde{U}\tilde{M}^{-1} + Q \right) \left( \tilde{N}_d\, d(s) + \tilde{N}_f\, f(s) \right) \\ y(s) &= T_o\, ref(s) + \quad (19) \\ &\quad S_o \tilde{M}^{-1} \left( I - \tilde{N}\tilde{V}^{-1} Q \right) \left( \tilde{N}_d\, d(s) + \tilde{N}_f\, f(s) \right) \end{aligned}$$

The resulting closed-loop system is stable provided that $Q \in RH_\infty$ and since the nominal controller $K$ internally stabilizes the nominal plant $P_{uy}$.

■

From equations (18) and (19), two well-known results can be concluded by considering the complete decoupling of the perturbations $d$, and faults $f$ from the control input $u$ and output $y$ of the system.

*Lemma 2:* If the nominal plant $P_{uy} \in RH_\infty$ then $\tilde{M}^{-1} \in RH_\infty$, and complete disturbance and fault decoupling can be achieved at the control signal $u$ by letting $Q = -\tilde{U}\tilde{M}^{-1} \in RH_\infty$ and consequently

$$u(s) = S_i K\, ref(s) \quad (20)$$
$$y(s) = T_o\, ref(s) + \tilde{M}^{-1}\left(\tilde{N}_d\, d(s) + \tilde{N}_f\, f(s)\right) \quad (21)$$

■

Therefore, if the nominal plant $P_{uy}$ is stable by properly choosing the compensator $Q$, the control signal is not affected by faults and perturbations. Moreover, if $P_{uy}$ has also a stable inverse, a complete output decoupling can be achieved.

*Lemma 3:* If the nominal plant satisfies $P_{uy}, P_{uy}^{-1} \in RH_\infty$ then $\tilde{N}^{-1} \in RH_\infty$, and with $Q = \tilde{V}\tilde{N}^{-1} \in RH_\infty$ the resulting output is decoupled perfectly from the perturbations and faults, i.e.

$$u(s) = S_i K\, ref(s) - \tilde{N}^{-1}\left(\tilde{N}_d\, d(s) + \tilde{N}_f\, f(s)\right) \quad (22)$$
$$y(s) = T_o\, ref(s) \quad (23)$$

■

In just particular cases is possible to achieve the perfect decoupling condition, and in general the nominal plant is not stable. Therefore, it is proposed to design the compensator $Q$ looking to reduce the effect of the perturbations and faults at the output, i.e.

$$\min_{Q \in RH_\infty} \left\| S_o \tilde{M}^{-1}\left(I - \tilde{N}\tilde{V}^{-1}Q\right)\left[\tilde{N}_d \;\; \tilde{N}_f\right] \right\|_j = \min_{Q \in RH_\infty} \left\| F_l(G_Q, Q) \right\|_j \quad (24)$$

where $G_Q$ represents the generalized plant (see Figure 4) given by

$$G_Q = \begin{bmatrix} S_o\tilde{M}^{-1}\tilde{N}_d & S_o\tilde{M}^{-1}\tilde{N}_f & S_o\tilde{M}^{-1}\tilde{N}\tilde{V}^{-1} \\ \tilde{N}_d & \tilde{N}_f & 0 \end{bmatrix} \quad (25)$$

and $j$ can represent the $L_1$, $H_2$ or $H_\infty$ norms [18].

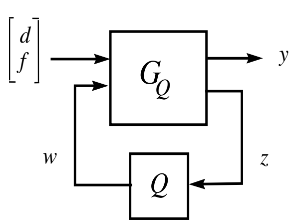


Fig. 4. LFT Formulation for Compensator Q.

*Remark 2:* The optimization criterion for $Q$ in (24) can be interpreted as a normalization process of $\tilde{N}\tilde{V}^{-1}$ by $Q$, with a frequency post-weighting given by the output sensitivity of the nominal plant and controller, and a pre-weighting by the frequency content of the perturbations and faults.

*Remark 3:* Note that the compensator $Q$ designed by the criterion in (24) can be conservative, since it is required to attenuate the effect of all types of faults analyzed in (1).

To improve the performance after faults, it is then proposed to design specific compensators $Q_i \in RH_\infty$ for $i = 1, \ldots, l$ for every fault studied

$$\min_{Q_i \in RH_\infty} \left\| S_o \tilde{M}^{-1}\left(I - \tilde{N}\tilde{V}^{-1}Q\right)\left[\tilde{N}_d \;\; \tilde{N}_f^i\right] \right\|_j \quad (26)$$

where

$$\tilde{N}_f^i = \left[\begin{array}{c|c} A + LC & F_1^i + LF_2^i \\ \hline C & F_2^i \end{array}\right] \quad (27)$$

In this way, the fault accommodation scheme of Figure 5 is proposed, and the overall FTC algorithm consists on three scenarios described next :

1) In the fault-free case, just the nominal control loop is active.
2) After a fault is detected into the system, the general compensator $Q$ designed by (24) is now activated.
3) Finally, after the fault is isolated, the specific compensator $Q_i$ designed by (26) is selected.

*Remark 4:* Since the fault accommodation is based on the Youla parameterization, and since the faults are additive, the closed-loop stability after each reconfiguration is guaranteed provided that $Q, Q_i \in RH_\infty$.

*Remark 5:* In the proposed configuration, multiple and intermittent faults could be handled. Once they are identified by the FDI scheme, the corresponding compensator should be activated to perform its accommodation. However, if FDI algorithm detects that fault is no longer present, the compensation is removed.

## IV. Fault Tolerant Approach under Additive Model Uncertainty

During the implementation of any control strategy, there is always some model uncertainty in the mathematical description used for design. If the description of this uncertainty could be obtained during the problem formulation, this information could be used at the design stage to improve the closed-loop performance, and understand also the practical limitations faced. In this paper, additive model uncertainty is considered [5], [18], as shown in Figure 6, i.e. the actual nominal plant $\hat{P}_{uy}$ is given by

$$\hat{P}_{uy} = P_{uy} + \Delta_{uy} \quad \Delta_{uy} \triangleq W_2 \Delta W_1 \quad (28)$$

where $W_1, W_2 \in RH_\infty$ represent pre and post-uncertainty weighting functions, and $\Delta \in RH_\infty$ a normalized uncertain transfer matrix $\|\Delta\|_\infty < 1$. The consideration of model uncertainty will produce that the signal $f_e$ in the GIMC configuration is no longer decoupled from the control signal $u$ (see Figure 2). The results are summarized in the following lemma.

*Lemma 4:* Considering additive model uncertainty in the GIMC configuration of Figure 2, the resulting closed-loop

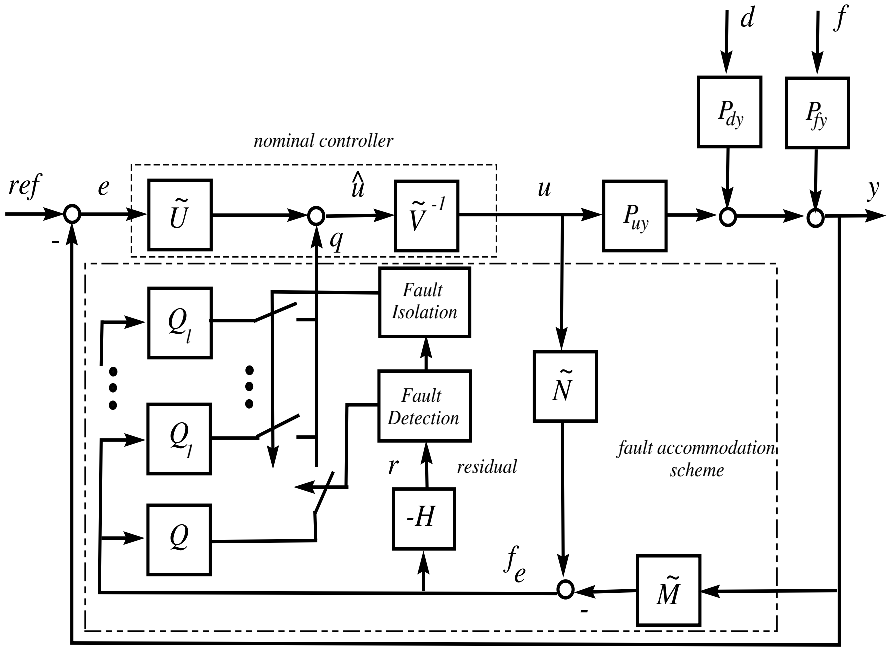


Fig. 5. GIMC Accommodation Setup.

characteristics are given by

$$f_e(s) = \tilde{M}\Delta_{uy}u(s) - \tilde{N}_d\,d(s) - \tilde{N}_f\,f(s) \quad (29)$$

$$u(s) = W_u^{-1}\left[K\,ref(s) - \tilde{V}^{-1}\left(\tilde{U}\tilde{M}^{-1}+Q\right)\left(\tilde{N}_d\,d(s)+\tilde{N}_f\,f(s)\right)\right] \quad (30)$$

$$y(s) = \left(P_{uy}+\Delta_{uy}\right)W_u^{-1}K\,ref(s) + \left[\left(P_{uy}+\Delta_{uy}\right)W_u^{-1}\tilde{V}^{-1}\left(\tilde{U}\tilde{M}^{-1}+Q\right)-\tilde{M}^{-1}\right]\left(\tilde{N}_d\,d(s)+\tilde{N}_f\,f(s)\right) \quad (31)$$

where

$$W_u = I + K\left(P_{uy}+\Delta_{uy}\right) + \tilde{V}^{-1}Q\tilde{M}\Delta_{uy} \quad (32)$$

$$= I + KP_{uy} + \tilde{V}^{-1}\left(\tilde{U}+Q\tilde{M}\right)\Delta_{uy} \quad (33)$$

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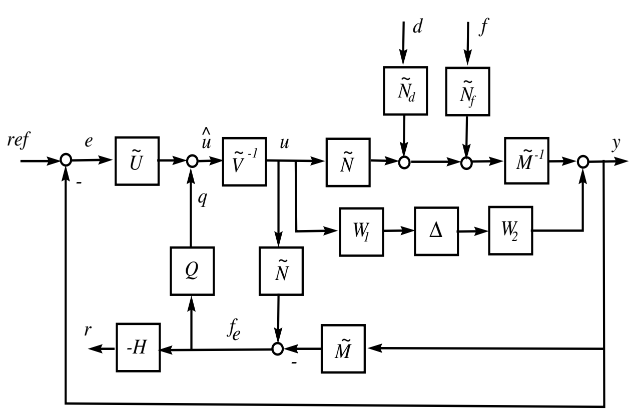


Fig. 6. GIMC with Additive Perturbations, Faults, and Model Uncertainty.

## A. Robust Fault Isolation

Note that by including the additive uncertainty description, an extra requirement is evident, the detection filter $H$ should cancel the effect of the uncertainty at the output of the residual $r$ for a robust detection and isolation, i.e. $H\tilde{M}\Delta_{uy} \approx 0$. Since the description of the uncertainty is posed in terms of the $\infty$-norm, the optimization problem for $H$ is also proposed in terms of this norm. As a result, the following robust performance criterion is adopted

$$\min_{H\in RH_\infty} \left\|F_l\left(F_u(G_H^\Delta,\Delta),H\right)\right\|_\infty \quad \|\Delta\|_\infty < 1 \quad (34)$$

where $F_u(\cdot,\cdot)$ stands for an upper LFT [18], and the respective generalized plant $G_H^\Delta$ is given by

$$G_H^\Delta = \begin{bmatrix} 0 & W_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & T & -I \\ \tilde{M}W_2 & 0 & \tilde{N}_d & \tilde{N}_f & 0 \end{bmatrix} \quad (35)$$

The optimization problem in (34) can then be solved by using $\mu$-synthesis design or LMI's [5], [18]. On the other hand, at the residual evaluation, it is observed that the uncertainty $\Delta_{uy}$ is affected by the control signal $u$ at (29), then an adaptive threshold can be used in order to reduce the conservativeness in the fault detection process introduced by the uncertain term:

$$J_{th}(t) = \|H\tilde{M}W_1W_2\|_\infty\|u\|_{2,t,T_o} + \gamma\|H\tilde{N}_d\|_\infty \quad (36)$$

where $\gamma$ is the bound on the energy of the perturbations.

## B. Robust Fault Accommodation

In general no guarantee of the closed-loop stability is granted, although $Q \in RH_\infty$ as in the uncertainty free case.

$$G_Q^\Delta = \begin{bmatrix} -W_1 S_i K W_2 & -W_1 S_i K \tilde{M}^{-1}\tilde{N}_d & -W_1 S_i K \tilde{M}^{-1}\tilde{N}_f & W_1 S_i \tilde{V}^{-1} \\ S_o W_2 & S_o \tilde{M}^{-1}\tilde{N}_d & S_o \tilde{M}^{-1}\tilde{N}_f & S_o \tilde{M}^{-1}\tilde{N}\tilde{V}^{-1} \\ -W_2 & -\tilde{N}_d & -\tilde{N}_f & 0 \end{bmatrix} \quad (37)$$

From the results in Lemma 4, it can be seen that for a special case (stable nominal plant), the uncertainty can be decoupled from the control signal as in Lemma 2, and closed-loop stability can be deduced if the nominal controller internally stabilizes the nominal plant.

*Lemma 5:* If the nominal plant $P_{uy} \in RH_\infty$ then complete disturbance, fault and uncertainty decoupling can be achieved at the control signal $u$ by letting $Q = -\tilde{U}\tilde{M}^{-1}$ and consequently:

$$u(s) = S_i K\, ref(s) \quad (38)$$

$$y(s) = (P_{uy} + \Delta_{uy}) S_i K\, ref(s) + \tilde{M}^{-1}\left(\tilde{N}_d\, d(s) + \tilde{N}_f\, f(s)\right) \quad (39)$$

Moreover, the closed-loop is stable.

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For a general design case, a robust criteria (performance and stability) should be targeted, i.e.

$$\min_{Q \in RH_\infty} \left\| F_l\left(F_u(G_Q^\Delta, \Delta), Q\right)\right\|_\infty \quad \|\Delta\|_\infty < 1 \quad (40)$$

where the generalized plant $G_Q^\Delta$ including uncertainty information is given by (37).

The robust stability condition is very important, since it is needed that the fault accommodation scheme will preserve closed-loop stability after the compensation despite model uncertainty. However, the size of the uncertainty and its frequency content will dictate the degree of conservativeness introduced.

Once more in order to improve the closed-loop performance after the fault has been isolated, a specific compensator $Q_i \in RH_\infty$ can be designed using the same criteria as in (40), just replacing from the generalized plant $G_Q^\Delta$ in (37), $\tilde{N}_f$ by the information of the analyzed fault $\tilde{N}_f^i$ for $i = 1, \ldots, l$.

## V. Conclusions

In this paper, a control methodology for fault accommodation in LTI systems has been detailed. The FTC scheme is based on the GIMC configuration [17] which extends the use of the Youla parameterization to FTC. Design strategies were presented for the FDI process and accommodation. Multiple and intermittent faults can be treated in FTC scheme. Closed-loop stability is always guaranteed after each configuration. Moreover, the analysis of the design scheme under model uncertainty was carried out. A fixed threshold is suggested for the nominal case in the detection process, and an adaptive one is considered when model uncertainty affects the output measurement.

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