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Output Tracking of Nonlinear Nonminimum Phase Systems: an Engineering Solution

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Abstract—**In this paper, we review some of design methods for output tracking of nonlinear nonminimum phase systems. Then we propose a control design procedure for output tracking of such systems which are stabilizable in the first approximation. The design procedure consists of two steps. In the first step the standard input output linearization is applied. In the second step, we group a subset of output with the internal dynamics as one subsystem, which is usually nonlinear, and the rest of output as the other subsystem which is linear; the nonlinear subsystem is linearized about its equilibrium; then we apply a two degree-of-freedom linear design procedure to each of two subsystems. The feedback part of the controller is designed to yield proper trade-off between disturbance rejection and robustness while the feedforward controller is designed to improve tracking performance.**

I. INTRODUCTION

There has been significant development in nonlinear control over past couple of decades, particularly the acceptance and application of feedback linearization to aerospace systems. One challenge remains is the output tracking of nonlinear nonminimum phase systems.

The output regulation theory of nonlinear systems of Isidori and Byrnes [1] provides a solution to asymptotic tracking of nonlinear systems to the output of a finite-dimensional exosystem, provided the zero dynamics of nonlinear system has no poles on imaginary axis. The design is a constant gain linear state feedback plus a feedforward of nonlinear function of exosystem states. Due to the very reason of using feedback, disturbance and model uncertainties, asymptotic tracking may not be achievable when applied to practical engineering systems [2,3].

Gurumoorthy and Sanders [4] applied feedback linearization and singular perturbation concept to come up with a high gain feedback design for SISO systems. Feedback linearization is first applied. Then the output variable is regarded as a *quasi* control to stabilize the internal dynamics in the first approximation. Finally, feedforward of inverse solution of internal dynamics together with high gain feedback is used to make the closed system asymptotically stable about its equilibrium at first approximation. A better and simpler approach, as proposed in this paper, is to design the control to stabilize the whole system (internal dynamics plus linear output dynamics) instead of just the internal dynamics, without resorting to high gain feedback and feedforward of inverse solution.

Al-Hiddabi and McClamroch [5] considered output tracking of MIMO nonlinear nonminimum phase systems. It proposed a two step design procedure. In the first step, feedback linearization is applied to linearize I/O dynamics. In the second step, part of the I/O dynamics is grouped together with internal dynamics to form one (generally nonlinear) subsystem and the rest of I/O dynamics the other (linear) subsystem. With the objective of achieving asymptotic (exact) tracking, linear high gain state feedback is designed for the linear subsystem, and linear state feedback together with feedforward of non-causal inverse is used for the other subsystem. The design has some disturbance rejection capability, but not good as simulation results of a PVTOL aircraft with wind of only 0.1 m/s were shown.

While the design methods reviewed above aim to achieve asymptotic (exact) output tracking, the design proposed here is to achieve approximate tracking under external disturbances and internal model uncertainties, as experienced in all engineering systems. Consequently, we do not require feedforward of non-causal inverse solution and high gain feedback. The design proposed here follows the two step procedure of Al-Hiddabi and McClamroch [5], but differs in the techniques used in the second step to achieve output tracking. Standard feedback linearization is applied in the first design step. In the second step, part of I/O linear dynamics is grouped together with the usually nonlinear internal dynamics to form a stabilizable system in the first approximation as in [5]. Then we apply a two-degree-of-freedom design (feedback and feedforward) to each of the two subsystems to achieve output tracking. The feedback controller is designed to have proper trade-off between disturbance rejection and robustness to model uncertainties, while feedforward design, which is a filter on reference trajectory, is designed to enhance tracking performance.

For hovering control of PVTOL aircraft, i.e., regulation about an equilibrium, a nonlinear state feedback was proposed in [6] based on an optimal control approach. However, the design would suffer from poor disturbance rejection as it doesn't include any integral feedback. We could apply the two step design procedure proposed in this paper, applying input output linearization first and followed

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by linear design in the second step. The output tracking design proposed here can be applied to regulation verbatim.

In section II, we describe the two step design procedure and specific design considerations in the second design step. In section III, we will highlight the application of the design procedure to PVTOL aircraft example. Section IV is conclusion.

II. DESIGN PROCEDURE

The nonlinear nonminimum phase systems considered here satisfy two conditions: it is input-output feedback linearizable [7] and the system after the linearization is stabilizable in the first approximation.

There are two steps in the design procedure. In the first step, standard I/O feedback linearization is applied. In the second, part or the whole of linear input-output dynamics is grouped together to form one subsystem and the rest of linear input-output dynamics forms the other subsystem. The subsystem with internal dynamics is linearized. Output tracking control is designed for each of the two subsystems by using a two degree-of-freedom control structure. Both the input-output linearization and output tracking control of linear system are well established. What we contribute is to apply those two methods to output tracking control of MIMO nonlinear nonminimum phase systems.

In the following of this section, we will highlight some details of the design.

A. Input-Output feedback Linearization

We follow the notation of Sastry and Isidori [7].

Consider a p-input p-output and n state nonlinear system of the form

$$\dot{x} = f(x) + g_1(x)u_1 + \ldots + g_p(x)u_p$$
$$y_1 = h_1(x)$$
$$\vdots$$
$$y_p = h_p(x)$$

Differentiate each output until some input appears, we have

$$\begin{bmatrix} y_1^{(\gamma_1)} \\ \vdots \\ y_p^{(\gamma_p)} \end{bmatrix} = \begin{bmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_p} h_p \end{bmatrix} + A(x) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix}$$

If $A(x) \in R^{pxp}$ is bounded away from singularity, the state feedback control law

$$u = A(x)^{-1} \left(\begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} - \begin{bmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_p} h_p \end{bmatrix} \right)$$

yields the closed-loop decoupled and linear

$$\begin{bmatrix} y_1^{(\gamma_1)} \\ \vdots \\ y_p^{(\gamma_p)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \equiv v \qquad (1)$$

The internal dynamics is of dimension

$$m = n - \gamma_1 \cdots - \gamma_p$$

with $m > 0$. Let the states of internal dynamics be

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix}$$

And the n-m states of linear input-output dynamics be

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_1^{(\gamma_1 - 1)} \\ \vdots \\ y_p \\ \vdots \\ y_p^{(\gamma_p - 1)} \end{bmatrix}$$

Then the internal dynamics can be represented as

$$\dot{z} = \psi(z, y, v)$$

And the zero dynamics is

$$\dot{z} = \psi(z, 0, 0)$$

Without loss of generality, we assume the origin is equilibrium. $\psi(0,0,0) = 0$. The linear system about the equilibrium is

$$\dot{z} = A_z z + A_y y + A_v v \qquad (2)$$

$$A_z = \frac{\partial \psi(0,0,0)}{\partial z} \in R^{mxm}$$

$$A_y = \frac{\partial \psi(0,0,0)}{\partial y} \in R^{mx(n-m)}$$

$$A_v = \frac{\partial \psi(0,0,0)}{\partial v} \in R^{mxp}$$

Assume that the original nonlinear system is stabilizable in the first approximation, the n state linear system consists of equations (1) and (2) are stabilizable by v.

B. Output tracking control design of two linear subsystems

In general, we could design output tracking for the complete linear system consists of equations (1) and (2). Then we have transformed tracking control a MIMO nonlinear nonminimum phase system into one of MIMO linear nonminimum phase system. Then we can use linear design method with a feedback controller to achieve trade-off between disturbance rejection and robustness to model uncertainty. Then a feedforward controller (filter of reference) can be designed to enhance tracking.

In some cases, some input-output dynamics can be taken out to form a decoupled subsystem without affecting the stabilizability of rest. Then we can have two linear subsystems and controller of a two degree-of-freedom structure would be used to each of the two linear subsystems to achieve output tracking and robustness.

In feedback design of linear system for output tracking, there are a couple of important points to bear in mind. One is that an integrator should be added for each output. This is to ensure infinite DC gain and zero steady state error to constant disturbance. Internal model of other exogenous signals may also be augmented into design model. The second point is to choose proper loop bandwidth after analysing the properties of the given system in terms of disturbance, model uncertainties and control authority available. In theory, any loop bandwidth can be achieved for controllable systems. However, it is not the higher the bandwidth, the better. Higher bandwidth means higher gains, less robustness, and higher controls required. Define what is to be achieved (objective) for a feedback system is much more critical than choosing what design method to achieve it.

The design procedure will be elaborated further by an example in the following section.

III. EXAMPLE

Planar vertical takeoff and landing (PVTOL) aircraft is considered in the section. The proposed design method of section II can be applied to several other well studied nonlinear nonminimum phase systems, such as inverted pendulum on a cart [4,8] and two link manipulator with last link being long and flexible.

PVTOL aircraft is a well studied flight control problem in control literature. PVTOL represents a simplified model for hovering operation of aircraft like Harrier. The PVTOL model [5] is

$$\ddot{z} = u_z \cos\phi + \varepsilon\, u_l \sin\phi - 1$$
$$\ddot{y} = -u_z \sin\phi + \varepsilon\, u_l \cos\phi$$
$$\ddot{\phi} = \lambda u_l$$

where u_z, u_l are two controls and z and y are (normalized) position coordinates of aircraft center of mass. ϕ is aircraft roll angle. The control objective is to have the aircraft track any given smooth trajectory, that is we have an output tracking problem with the output being z and y.

A physical interpretation of parameter ε is that for a control of u_l which produces a unit of roll acceleration, say $1\ \text{rad/s}^2$, then the control also produces a lateral force which is ε/λ times of aircraft's own weight. Typical value of ε is less than 0.01. Anyhow, we will continue to take the value as 0.5 as in previous studies.

A. Control Design Step One: Feedback Linearization

Applying feedback linearization, we have

$$u_z = (v_1 + 1)\cos\phi - v_2 \sin\phi \qquad (3)$$

$$u_l = \frac{1}{\varepsilon}\left[(v_1 + 1)\sin\phi + v_2 \cos\phi\right] \qquad (4)$$

and the resulting system

$$\ddot{z} = v_1$$
$$\ddot{y} = v_2$$
$$\ddot{\phi} = \frac{\lambda}{\varepsilon}\left[(v_1 + 1)\sin\phi + v_2\cos\phi\right]$$

The zero dynamics of the above system with the given input and output pair is

$$\ddot{\phi} = \frac{\lambda \sin\phi}{\varepsilon}$$

which is unstable. For the given parameter values of $\lambda = 1, \varepsilon = 0.5$, the zeros are at $\pm\sqrt{2}$.

B. Control Design Step Two: Tracking Control

To achieve output tracking and stabilization of internal dynamics, we regroup part of linear output dynamics with internal dynamics. In this example case, we group output dynamics $\ddot{y} = v_2$ with the internal dynamics. So we have two subsystems. The subsystem with internal dynamics is stabilizable in the first approximation and its linear approximation is

$$\ddot{y} = v_2$$
$$\ddot{\phi} = \frac{\lambda}{\varepsilon}(\phi + v_2)$$

Now we have transformed original tracking control problem of a MIMO nonlinear nonminimum phase system into tracking control problem of two SISO linear systems. Linear design methods can be applied. In the following we use state feedback assuming that the states are available for feedback.

For the subsystem $\ddot{z} = v_1$ the state feedback would be PID

$$v_1 = -3\omega_1 \dot{z} - 3\omega_1^2 z - \omega_1^3 \int (z - z_r)\,dt \qquad (5)$$

where feedback loop bandwidth is chosen to have proper trade-off between disturbance rejection and robustness to model uncertainties. z_r is reference trajectory. Once feedback loop is well designed, we can add in a filter to the reference trajectory in the feedforward path to enhance tracking performance. We can simply choose the following filter to extend bandwidth for tracking:

$$\frac{z_r}{z_{r0}} = \frac{\omega_2^4}{\omega_1^3} \frac{(s + \omega_1)^3}{(s + \omega_2)^4}$$

with $\omega_2 > \omega_1$. For simulation, $\omega_2 = 10, \omega_1 = 1$.

For tracking control design of the subsystem with internal dynamics, integral of output is added as the fifth state of the system. LQR design is used to obtain the following design

$$v_2 = -Kx - K_i \int (y - y_r)\,dt \qquad (6)$$

where

$$x = [y \quad \dot{y} \quad \phi \quad \dot{\phi}]'$$

and y_r is the reference for y to track. Given the nonminimum zero at $-\sqrt{2}$, we know the feedback loop bandwidth has to be smaller than $\sqrt{2}$. We designed the following controller with bandwidth close to one as shown in the step response of Figure 1.

$$K = [-8.1533 \ \ -13.1867 \ \ 14.0573 \ \ 9.9400]$$
$$K_i = -2.2086$$

C. Simulation Results

IN the following we present simulation results to illustrate nominal performance without pre-filter, improvement in performance with pre-filter, performance at presence of disturbances, and performance with model uncertainties. A circular reference trajectory is used:

$$y_r = R\cos\Omega t$$
$$z_r = R\sin\Omega t$$

We first used $R = 50$ and $\Omega = 0.02\text{rad/sec}$, as in [5]. Then we used maneuvers of higher frequencies. The reference is shown in Figure 2 as the dashed line and the actual response is shown as the solid line (the two lines are visually indistinguishable).

The nominal performance, without any disturbance and model uncertainties, is shown in Figure 3. The top figure shows tracking error in both outputs, *y* in solid line and *z* in dashed line. The amplitude of tracking errors is around 4. The bottom left figure shows roll angle amplitude during the maneuver is about 1 degree. The top right figure shows the two controls, u_z in solid line and u_l in dashed line. It shows that u_z has a peak of about 1.3 during initial transient period and oscillates about 1 with amplitude of about 0.02. u_l is very small, not visually noticeable. The bottom right plot in all figures shows controls v_1, v_2.

Figure 4 shows improvement in tracking with the addition of pre-filters. The amplitude of tracking errors is now reduced from 4 to less than 0.4. Other data remains about the same except that required control u_z is increased from 0.3 to 0.8 during transient period.

Figure 5 shows the disturbance rejection capabilities. The disturbances on two controls u_z and u_l are shown in figure 5, with amplitudes of 0.3 and 0.2, respectively. We see in Figure 6 that tracking errors are the same with and without the disturbance. What have changed after onset of the disturbance are, of course, controls.

Figure 7 shows the case of actual ε is 0.3 instead of 0.5 used in control design, and control gain for u_z is twice of what we used for control design, and control gain for u_l is half of what we used for control design. Again no noticeable changes in performance. The actual trajectory and reference trajectory are shown in Figure 2.

Figure 8 shows the tracking performance is maintained when the frequency of ref. trajectory is 0.1 rad/s. , and ϕ is about 30 degree. Further increasing the frequency to 0.12 rad/s., we see the system is at the verge of instability with ϕ at about 50 degree, as shown in Figure 9.

Figure 10 show step response in lateral position, which is about the same as the designed (Figure 1). For comparison, the response of pure linear design, e.g., replacing equations. (3) and (4) with

$$u_z = (v_1 + 1)$$
$$u_l = (\phi + v_2)/\varepsilon$$

is superposed onto the same figure as dotted lines. The main difference is in vertical response. It is zero for the proposed design, 0.3 for the linear design. We see the advantage of the proposed design is the decoupling of vertical motion from lateral. When there is model uncertainties, decoupling may not be achieved with I/O linearization. Figure 11 shows the step response for actual ε is 0.3 instead of 0.5 used in control design, and control gains for u_z and u_l are 80% of what we used for control design. For the proposed design, there is a vertical displacement of maximum amplitude of about 0.02. A constant disturbance equal to 0.5 in vertical acceleration and 0.1 in lateral acceleration is added from 20 second onwards. The effect of constant disturbance disappears within 10 second.

D. Remarks

The output tracking design described here is local. For a given input-output system, although the internal dynamics is fixed, how nonlinear (or close to linear) the internal dynamics is has much to do with how the system is operated. For the PVTOL aircraft, for example, the nonlinearity is determined by roll angle ϕ. When ϕ is very small during mild manoeuvres, the internal dynamics, and hence the system after I/O linearization can be well considered as linear. For manoeuvres with larger ϕ, the nonlinearity becomes more significant, and the design presented here could fail. Simulation investigation shows that the design works well for operations with ϕ less than 30 degrees, and likely fails with ϕ greater than 50 degrees. In an actual system, an outer loop controller, such as mission planner for an aircraft, would be designed in such a way that the system operates within a safe boundary.

IV. Conclusion

A design procedure is proposed for output tracking of nonlinear nonminimum phase systems which are stabilizable in the first approximation. It does not aim for asymptotic (exact) tracking; rather it aims to achieve close approximate tracking under disturbances and model uncertainties. The design solution is local as linear approximation of internal dynamics is used in design.

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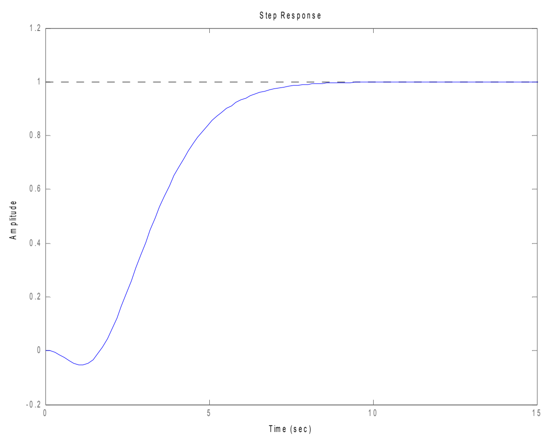

Figure 1 Close loop step response of the linear nonminimum phase system

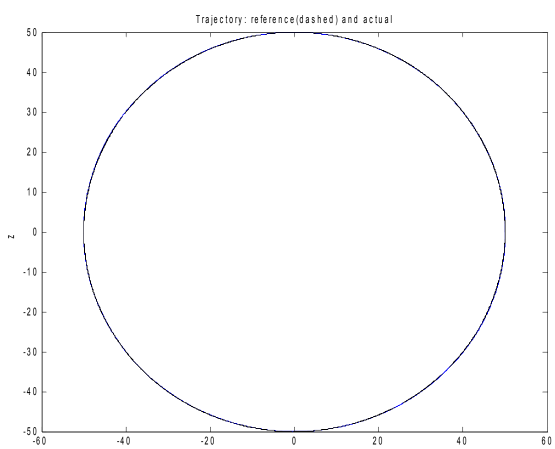

Figure 2 Trajectory corresponding to the case of fig 7.

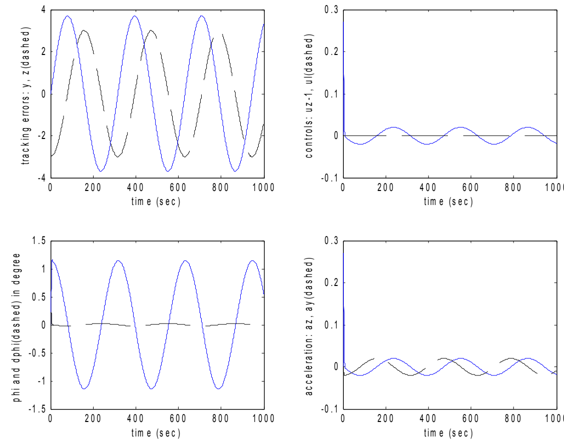

Figure 3. Nominal performance without pre-filter

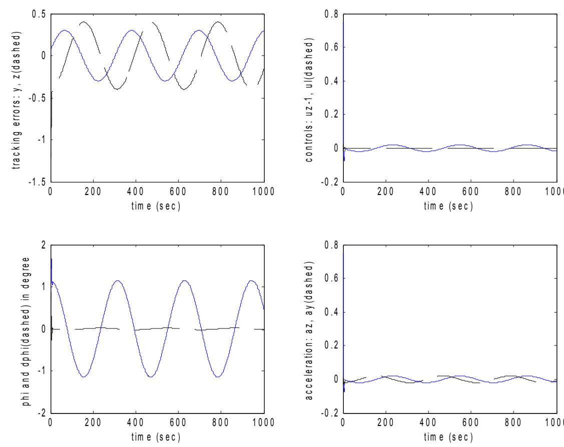

Figure 4. Nominal performance with pre-filter

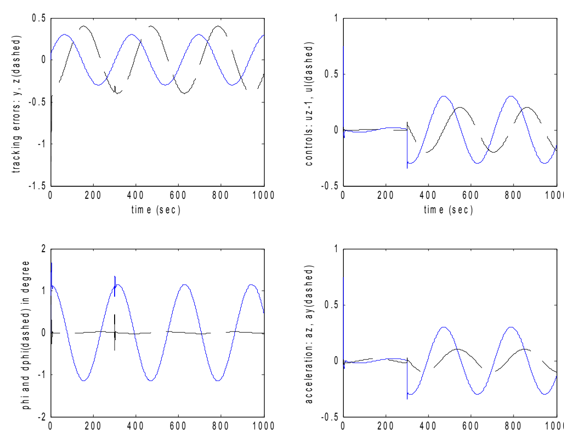

Figure 5 Maintenance of performance with disturbance

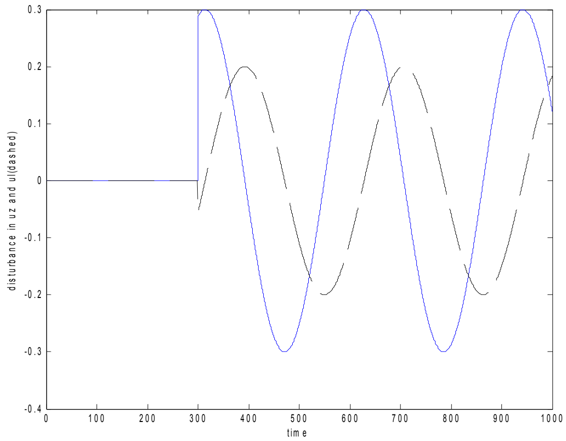

Figure 6 Disturbance

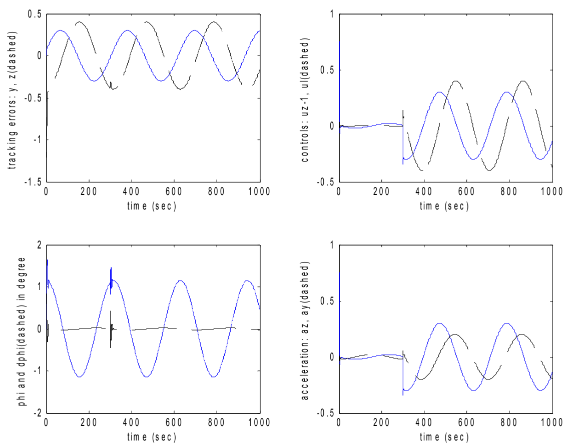

Figure 7 With disturbance, errors in control gain and ε

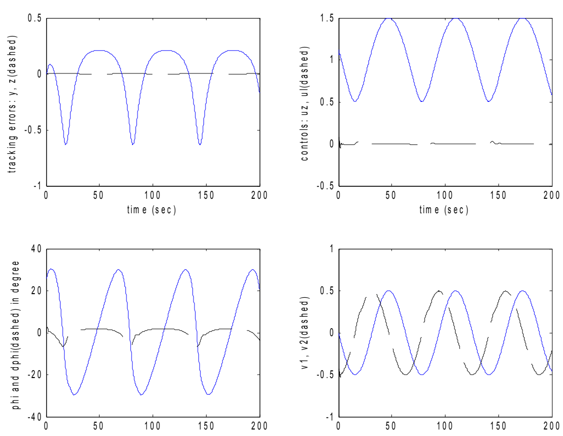

Figure 8 $R = 50$ and $\Omega = 0.1$rad/sec

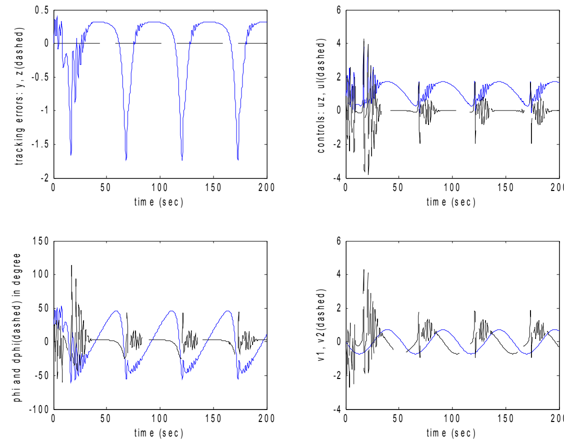

Figure 9 $R = 50$ and $\Omega = 0.12$rad/sec

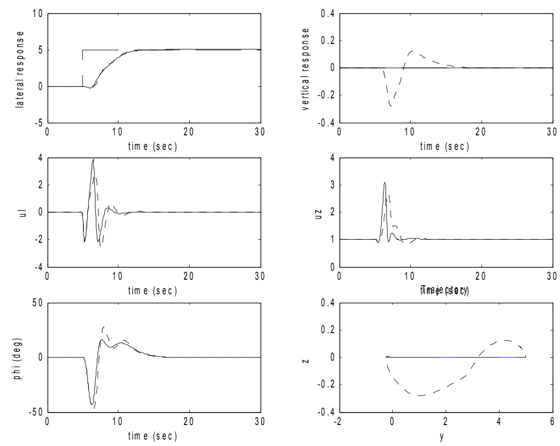

Figure 10 Step response in lateral position

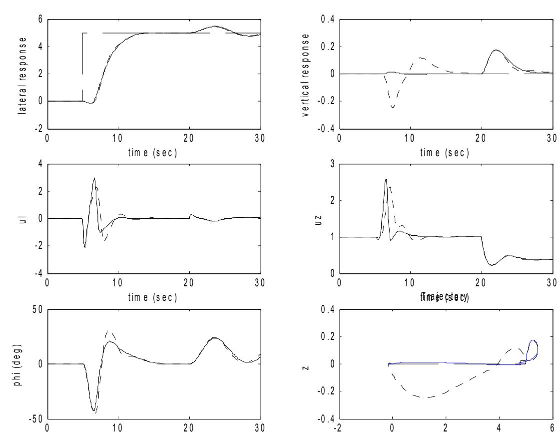

Figure 11 Robustness to model uncertainties and disturbance