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A Globally Adaptive Internal Model Regulator for MIMO Linear Systems.

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Abstract— **The problem of compensating noise and/or track some reference signal with n unknown frequencies in general linear MIMO systems is treated in this work. We derive a frequency estimator that ensures closed loop robust regulation in some neighborhood of the nominal values of the system as well.**

Keywords— **Regulation Theory, Frequency estimation, Non Linear systems.**

I. Introduction

In many industrial and defense applications, the noise and vibrations compensation is important problem. Some common class of those are the periodic and/or quasi-periodic noises which include engine noise in turboprop aircraft [2] and automobiles [3], ventilation noise in HVAC systems [14] and sea wave noise in landing systems [21].

The rejecting unknown sinusoidal noise problem was first addressed in [4], [6], [5] and recently in [7], [8] and [9] using adaptive observers scheme developed in [11]. On the other hand, in [12] the output stabilization problem with disturbance rejection was considered for a class of SISO minimum phase nonlinear systems which can be transformed in output feedback form. A local solution for stable SISO system with a single frequency sinusoidal signal satisfying the matching condition, was proposed in [4], while a global solution was given in [7]. This solution was extended in [8] for non-minimum phase systems with single frequency signal, and in [9] for the case of k frequencies that gives raise to a controller of $((n+1)(k+1)+2k(k+1))th$ order, where n is the order of the system.

The singularity problem presented in [8] was then fixed in [9] where a stable signal was included in the transformation determinant to overcome the singularity problem. Also, in [6] a supervisory control scheme was proposed for the case of k frequencies and a SISO linear system, considering that the number of frequencies is known and all frequency values lie in a pre-defined set. Using a high gain feedback technique combined with regulator theory, it was shown in [19] that the values of the frequencies must belong to a some pre-specified compact set and if the values of the frequencies leave this set, the gains of the proposed regulator must be changed in order to keep the stability property. Recently, in [13] an universal adaptive controller was proposed for minimum phase systems using K-filters and backstepping technique.

In the case of MIMO linear systems, a locally exponentially stable adaptive control law was proposed in [5] ,using Youla parameterization.

Along the same lines, in this work we propose an adaptive control scheme for the case of MIMO linear system for which the number of the frequencies is known but not necessarily belonging to a pre-defined finite set of frequencies, nor pre-specified compact set of frequencies, relaxing as well the minimum phase and matching conditions. Moreover, the proposed scheme is globally stable and robust with respect to plant parameter variations in some neighborhood of the nominal values. The order of the proposed controller is $(n+k+m(2k+1))$ where m is the number of inputs and k is the number of the frequencies.

II. Problem Statement

Consider a linear system subject to perturbation described in the form

$$\dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \quad (1)$$
$$e(t) = Cx(t) + \tilde{Q}d(t) \quad (2)$$

where $x \in \Re^n$ is the state, $u \in \Re^m$ is the input, $d \in \Re^{(k+1)}$ is a disturbance and/or reference signal, $e \in \Re^m$ represents the tracking error between the plant output $Cx(t)$ and a reference signal $-\tilde{Q}d(t)$ and $A, B, C, D, \tilde{Q}$ are matrices of appropriate dimensions, whose parameters may possibly vary in some neighborhood of the nominal values A_0, C_0, B_0, D_0 and $\tilde{Q}_0$. We consider that vector $d(t)$ consists of a constant signal with unknown magnitude d_0 and k sinusoidal signals with un-

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known magnitudes d_1, .., d_k, frequencies α_i and phases φ_i for $i = 1..k$, namely

$$d(t) = \left[\begin{array}{cccc} d_0 & d_1 \sin(\alpha_1 t + \varphi_1) & \cdots & d_k \sin(\alpha_k t + \varphi_k) \end{array} \right]^T \quad (3)$$

where $\alpha_i \neq \alpha_j$ if $i \neq j$. Assuming that these signals can be generated by an external dynamic generator or exosystem [17], the system (1)-(3) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Pw(t) && (4)\\ \dot{w}(t) &= Sw(t) && (5)\\ e(t) &= Cx(t) + Qw(t) && (6) \end{aligned}$$

where $w \in \Re^{(2k+1)}$, $P \in \Re^{n\times(2k+1)}$, $Q \in \Re^{m\times(2k+1)}$ and the matrix $S \in \Re^{(2k+1)\times(2k+1)}$ is $S = \left[\begin{array}{cccc} 0 & 0 & 0 & 0\\ 0 & S_1 & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & S_k \end{array} \right]$, with $S_i = \left[\begin{array}{cc} 0 & 1\\ -\alpha_i^2 & 0 \end{array} \right]$.

The problem we face here is that of finding a dynamic error feedback control

$$\begin{aligned} \dot{\xi}(t) &= \chi(\xi(t), e(t))\\ u(t) &= h(\xi(t)) \end{aligned}$$

with $\xi \in \Re^p$ such that the equilibrium point $(x, \xi) = (0, 0)$ of the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bh(\xi(t))\\ \dot{\xi}(t) &= \chi(\xi(t), Cx(t)) \end{aligned}$$

is asymptotically stable in the first approximation, and for any initial conditions ($x(0)$, $w(0)$, $\xi(0)$) the solution of the closed loop system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bh(\xi(t)) + Pw(t)\\ \dot{w}(t) &= Sw(t)\\ \dot{\xi}(t) &= \chi(\xi(t), Cx(t) + Qw(t)) \end{aligned}$$

satisfies that $\lim_{t\to\infty} e(t) = 0$.

This problem is named the *regulator problem.* If in addition, the previous conditions are required to be satisfied in a certain neighborhood of the nominal values of the parameters of the system, then we refer to the *structurally stable regulator problem.* If all the frequencies of the sinusoidal signals, or equivalently, all the parameters of the exosystem are known, both problems can be solved either for linear and nonlinear systems, by directly applying the regulation theory presented for example in [15], [17] for the non robust case, or [18], [16], for the structurally stable formulation.

However, in practice, not all the frequencies are perfectly known, so the regulator theory needs to be adapted or modified in order to handle with this situation. This problem, which can be defined as the *regulator problem with uncertain exosystem*, has been recently studied in some works either for the case of linear systems [8], [9] or a particular class of nonlinear systems in normal form [12], [13], [19].

In this work we present an algorithm for solving the regulator problem with uncertain exosystem for a MIMO linear system. We show that this problem can be solved by means of an adaptation of the unknown values of the parameters of the exosystem. By using an adaptive scheme to estimate the unknown frequencies of a signal consisting of a combination of k sinusoidal signals [20], we obtain a minimal order control scheme which guarantee global robust regulation.

III. Regulation Theory Preview

From the Robust Regulation Theory [15], [16], [17], [18], it is known that if the following assumption hold:

A.1. For every P and Q there exists a solution $\Pi \in \Re^{n\times(2k+1)}$ and $\Gamma = \left[\begin{array}{ccc} \Gamma_1^T & \cdots & \Gamma_m^T \end{array} \right]^T \in \Re^{m\times(2k+1)}$, $\Gamma_i^T \in \Re^{(2k+1)}$ $i = 1..m$, to the Francis equations

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + P && (7)\\ 0 &= C\Pi + Q; && (8) \end{aligned}$$

then, taking the change of coordinates $\tilde{x}(t) = x(t) - \Pi w(t)$ and defining $z_{i,j}(t) = \Gamma_i S^{j-1} w(t) \in \Re$; $i = 1, ..., m$; $j = 1, ..., 2k+1$, $z = \left(\begin{array}{ccc} z_1^T & \cdots & z_m^T \end{array} \right)^T$, $z_i = \left(\begin{array}{ccc} z_{i,1} & \cdots & z_{i,2k+1} \end{array} \right)^T$, we have that

$$\begin{aligned} \dot{z}_{i,j}(t) &= z_{i,j+1}(t) = \Gamma_i S^j w(t), \quad j = 1, ..., 2k+1;\\ \dot{z}_{i,2k+1}(t) &= -\left(\Pi_{i=1}^k \alpha_i^2\right) z_{i,2}(t) - ....\\ &\quad -\left(\Sigma_{i=1}^k \alpha_i^2\right) z_{i,2k+1}(t),\\ u_{i,ss} &= \left(\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right) z_i := \bar{H} z_i \end{aligned}$$

and the interconnected system (4)-(5) can be rewritten as

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\,\tilde{x}(t) + Bu(t) - BHz(t) && (9a)\\ \dot{z}(t) &= \Phi(\theta) z(t) && (9b)\\ e(t) &= C\tilde{x}(t), && (9c) \end{aligned}$$

where $\Phi \in \Re^{m(2k+1)\times m(2k+1)}$ and $H \in \Re^{m(2k+1)}$ are defined as $\Phi(\theta) = diag\left(\begin{array}{ccc} \bar{\Phi} & \cdots & \bar{\Phi} \end{array} \right) \in \Re^{m(2k+1)\times m(2k+1)}$, $\bar{\Phi} = \left(\begin{array}{cc} 0_{2k\times 1} & I_{2k}\\ 0 & v(\theta) \end{array} \right)$, $v(\theta) = \left(\begin{array}{cccccc} -\theta_1 & 0 & -\theta_2 & \cdots & -\theta_k & 0 \end{array} \right)$,
$H = diag\left(\begin{array}{ccc} \bar{H} & \cdots & \bar{H} \end{array} \right) \in \Re^{m\times m(2k+1)}$. The parameters $\theta = (\theta_1, ..., \theta_k)^T$ are calculated from the characteristic polynomial of the matrix S, namely $s\left(s^{2k} + \left(\Sigma_{i=1}^k \alpha_i^2\right) s^{2k-2} + \cdots + \left(\Pi_{i=1}^k \alpha_i^2\right)\right) = 0$, from which we obtain $\theta_1 = \Pi_{i=1}^k \alpha_i^2, ..., \theta_k = \Sigma_{i=1}^k \alpha_i^2$.

If in addition, the following assumption hold:

A.2. The pair (A, B) is controllable,

then the controller $u(t) = K\tilde{x}(t) + Hz(t)$where K calculated such that the matrix $(A_0 + B_0 K)$ is Hurwitz, stabilizes the system and guarantees also the perturbation rejection.

In the case when only the output tracking error $e(t)$ is measurable, assuming that

A.3. The pair (C, A) is observable,

the control action can be provided by a dynamic system $\dot{\xi}(t) = F\xi(t) + Ge(t)$, $u(t) = \tilde{H}\xi(t)$, where the matrix F and vectors G and $\tilde{H}$ are chosen such that the matrix $\begin{pmatrix} A & B\tilde{H} \\ GC & F \end{pmatrix}$ is Hurwitz.

If the frequencies are all known then a robust controller can be constructed as an observer for the system (9a)-(9b), namely

$$
\begin{aligned}
\begin{bmatrix} \dot{\xi}_1(t) \\ \dot{\xi}_2(t) \end{bmatrix} &= \begin{bmatrix} A_0 + B_0 K - G_1 C_0 & 0 \\ -G_2 C_0 & \Phi(\theta) \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} \\
&\quad + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} e(t) \qquad (10) \\
u(t) &= \begin{bmatrix} K & H \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \tilde{H}\xi(t),
\end{aligned}
$$

where G_1 and G_2 are chosen such that the matrix $\begin{bmatrix} A_0 - G_1 C_0 & -B_0 H \\ -G_2 C_0 & \Phi(\theta) \end{bmatrix}$ is Hurwitz. This controller is robust with respect to plant parameter variations in a suitable neighborhood of the nominal values ([16]). Again, in the case when the frequencies are unknown then the parameters $\theta_1, ..., \theta_k$ of matrix $\Phi(\theta)$ are also unknown and the controller (10) does not further guarantees the regulation properties. To solve this problem, we propose the use of an adaptive scheme, by first change the form of the observer to get a desired structure.

IV. The global frequency estimator

In this section we remember briefly the result given in [20], whose with a little changes will be useful for our goal.

The estimator proposed in [20] take the form:

$$
\begin{aligned}
\dot{x}_i &= \lambda_i \, x_{i+1} \qquad i = 1...(2k-1) \qquad (11) \\
\dot{x}_{2k} &= \lambda_{2k} \, x_{2k+1} + \zeta_1(\bar{y} - \hat{y}), \quad \dot{x}_{2k+1} \\
&= -\left(\sum_{i=1}^{k} \frac{\sigma_i \, x_{2k+1+i}}{\Pi_{j=2i}^{2k} \lambda_j} x_{2i} \right) + \zeta_2(\bar{y} - \hat{y}) \\
\dot{x}_{2k+1+i} &= -\zeta_{i+2} x_{2i}(\bar{y} - \hat{y}) \qquad i = 1...k \\
\hat{y} &= \left(\sum_{i=1}^{2k} \frac{k_i}{\Pi_{j=i}^{2k} \lambda_j} \right) x_i + k_{2k+1} x_{2k+1}, \\
\bar{y}(t) &= \begin{bmatrix} 1 & 1 & ... & 1 \end{bmatrix} d(t) \\
&= d_0 + \sum_{i=1}^{k} d_i \sin(\alpha_i t + \varphi_i)
\end{aligned}
$$

where $\bar{y}(t)$ are the estimated signal and the constants satisfies $\zeta_1 = \frac{\lambda_{2k}}{k_{2k+1}}$, $\quad \zeta_{i+2} > 0$, $\sigma_i > 0$, $i = 1...k$, and $\lambda_j > 0 \quad j = 1...2k$, with the polynomial $P(\xi) = \sum_{i=0}^{2k} \frac{k_{i+1}}{k_{2k+1}} \xi^i$, chosen stable. This estimator guarantees that $\lim_{t\to\infty}(\bar{y} - \hat{y}) = 0$, $x_1 \to w_1$, $x_i \to \frac{w_i}{\Pi_{j=1}^{i-1}\lambda_j}$, $i = 2...2k+1$, and $x_{2k+1+i} \to \frac{\theta_i}{\sigma_i} \quad i = 1...k$, globally when $t \to \infty$, where

$$
\begin{aligned}
\dot{w}_i &= w_{i+1} \qquad i = 1...2k, \qquad (12) \\
\dot{w}_{2k+1} &= -\theta_1 w_2 - \theta_2 w_4 - ... - \theta_k w_{2k} \\
\bar{y}(t) &= \sum_{i=1}^{2k+1} c_i w_i \\
\theta_1 &= \left(\Pi_{i=1}^{k} \alpha_i^2 \right), \ \theta_1 = \left(\sum_{j=1}^{k} \frac{\Pi_{i=1}^{k} \alpha_i^2}{\alpha_j^2} \right), ..., \\
\theta_{k-1} &= \sum_{i=1}^{k-1} \left(\sum_{j=i+1}^{k} \alpha_i^2 \alpha_j^2 \right), \ \theta_k = \left(\sum_{i=1}^{k} \alpha_i^2 \right)
\end{aligned}
$$

In [20] is showed how the error system between (11) and (12) take the form

$$
\begin{aligned}
\dot{\bar{e}}_1 &= \bar{A} \, \bar{e}_1 \qquad (13) \\
\dot{e}_{2k+1} &= -\bar{\beta}_1^T \bar{e}_1 - \beta_2 e_{2k+1} + \bar{x}_1^T \bar{e}_3 \\
\dot{\bar{e}}_3 &= -\Lambda \bar{x}_1 \left(\bar{k}_1^T \bar{e}_1 + k_{2k+1} e_{2k+1} \right)
\end{aligned}
$$

where $e_1 = w_1 - x_1$, $e_i = w_i - \Pi_{j=1}^{i-1} \lambda_j x_i \quad i = 2...2k+1$, $e_{2k+1+i} = \sigma_i x_{2k+1+i} - \theta_i \quad i = 1...k$,and $\bar{A} = \begin{bmatrix} 0 & I \\ -\frac{k_1}{k_{2k+1}} & \bar{a} \end{bmatrix} \in \Re^{2k\times 2k} = Hurwitz\ Matrix$, $\bar{a} = \begin{bmatrix} -\frac{k_2}{k_{2k+1}} & \cdots & -\frac{k_{2k}}{k_{2k+1}} \end{bmatrix}$, $\bar{e}_1 = \begin{bmatrix} e_1 & \cdots & e_{2k} \end{bmatrix}^T \in \Re^{2k}$, $\bar{\beta}_1 = \begin{bmatrix} \zeta_2 k_1 & (\theta_1 + \zeta_2 k_2) & \cdots & (\theta_k + \zeta_2 k_{2k}) \end{bmatrix}^T \in \Re^{2k}$, $\beta_2 = \zeta_2 k_{2k+1} \in \Re$,
$\bar{x}_1 = \begin{bmatrix} \lambda_1 x_2 & \left(\Pi_{j=1}^{3}\lambda_j\right) x_4 & \cdots & \left(\Pi_{j=1}^{2k-1}\lambda_j\right) x_{2k} \end{bmatrix}^T \in \Re^k$, $\bar{e}_3 = \begin{bmatrix} e_{2k+2} & e_{2k+3} & \cdots & e_{3k} & e_{3k+1} \end{bmatrix}^T \in \Re^k$, $\Lambda = diag\left(\frac{\sigma_1 \zeta_3}{(\Pi_{i=1}^{2k}\lambda_i)(\lambda_1)} \ \cdots \ \frac{\sigma_i \zeta_{i+2}}{(\Pi_{j=1}^{2k}\lambda_j)(\Pi_{j=1}^{2i-1}\lambda_j)} \right)$, $i = 1..k$, $\bar{k}_1 = \begin{bmatrix} k_1 & k_2 & \cdots & k_{2k} \end{bmatrix}^T \in \Re^{2k}$. Then for persistency of excitation lemma [10] the error system (13) is globally exponentially stable.

V. The Adaptive Internal Model Regulator.

To obtain the internal model regulator we first observe that if the assumption *A*.1 is satisfied then, for given matrices (Θ, Ψ), there exist a solution $M(\theta)$ and $\tilde{\Pi}(\theta)$ to the equations

$$
\begin{aligned}
\tilde{\Pi}(\theta)\Phi(\theta) &= A_0 \tilde{\Pi}(\theta) + B_0 M(\theta) + \Theta \qquad (14) \\
0 &= C_0 \, \tilde{\Pi}(\theta) + \Psi. \qquad (15)
\end{aligned}
$$

We note also that there exist an initial condition of the system $\dot{\varsigma}(t) = \Phi(\theta) \, \varsigma(t)$, such that $Hz(t) = M(\theta) \, \varsigma(t)$.

Taking the transformation $\eta(t) = \tilde{x}(t) + \tilde{\Pi}(\theta)\varsigma(t)$ we arrive to the system

$$\begin{aligned} \dot{\eta}(t) &= A_0\eta(t) + B_0u(t) + \Theta\varsigma(t) \\ \dot{\varsigma}(t) &= \Phi(\theta)\varsigma(t) \\ e(t) &= C_0\eta(t) + \Psi\varsigma(t). \end{aligned} \tag{16}$$

Note that this system has the same structure that system (4)-(5)-(6), but with Θ and Ψ to be found.

The main result of the paper is given by the following theorem:

Theorem 1: Consider assumptions A.1, A.2 and A.3 hold. Choose the matrices Θ, Ψ *as* $\Theta = G_1\Psi$, $\Psi = diag(\Psi_1, ..., \Psi_m) \in \Re^{m\times(m(2n+1))}$, $\Psi_j = \begin{bmatrix} \psi_{(1,j)} & \psi_{(2,j)} & \cdots & \psi_{((2k+1),j)} \end{bmatrix}$, $j = 1...m$, *with* G_1 *chosen such that* $(A_0 - G_1C_0)$ *is Hurwitz. Choose also the polynomials* $P_j(\xi) = \sum_{i=0}^{2k} \frac{\psi_{((i+1),j)}}{\psi_{((2k+1),j)}}\xi^i \quad j = 1...m$, *to be stable.*
Then the adaptive regulator

$$\begin{aligned} \dot{\zeta}_1(t) &= A_0\zeta_1(t) + \Theta\zeta_2(t) + B_0u(t) + G_1(e(t) - \hat{e}(t)) \\ \dot{\zeta}_2(t) &= \Phi(\hat{\theta})\zeta_2(t) + G_2(e(t) - \hat{e}(t)) \\ \dot{\hat{\theta}}(t) &= -\Delta\ \Omega(t)(e(t) - \hat{e}(t)) \\ \hat{e}(t) &= C_0\zeta_1(t) + \Psi\zeta_2(t) \\ u(t) &= K\zeta_1(t) + \varphi(\hat{\theta})\zeta_2(t) \\ \varphi(\hat{\theta}) &= (M(\hat{\theta}) - K\tilde{\Pi}(\hat{\theta})) \end{aligned} \tag{17}$$

with $\Delta = diag(\Delta_1, ..., \Delta_k) > 0$, $\Delta_i \in \Re$, $i = 1...k$, $\zeta_2(t) = \begin{bmatrix} \bar{\zeta}_1^T & \cdots & \bar{\zeta}_m^T \end{bmatrix}^T \in \Re^{m(2k+1)}$, $\bar{\zeta}_j(t) = \begin{bmatrix} \bar{\zeta}_{(1,j)} & \bar{\zeta}_{(2,j)} & \cdots & \bar{\zeta}_{(2k+1,j)} \end{bmatrix}^T \in \Re^{(2k+1)}$, $\Omega(t) = \begin{bmatrix} \omega_1(t) & \omega_2(t) & \cdots & \omega_m(t) \end{bmatrix} \in \Re^{k\times m}$, $\omega_j(t) = \begin{bmatrix} \bar{\zeta}_{(2,j)} & \bar{\zeta}_{(4,j)} & \cdots & \bar{\zeta}_{(2k,j)} \end{bmatrix}^T \in \Re^k$, $G_2 = diag(G_{(2,1)}, ..., G_{(2,m)}) \in \Re^{((2k+1)m)\times m}$, $G_{(2,j)} = \begin{bmatrix} 0 & \cdots & 0 & g_{(1,j)} & g_{(2,j)} \end{bmatrix}^T \in \Re^{(2k+1)}$, $g_{(1,j)} = \frac{1}{\psi_{((2k+1),j)}}$, $g_{(2,j)} > 0$,*with* $j = 1...m$, *applied to* $(9a - 9c)$ *is such that the signals* $e(t)$, $\tilde{x}(t)$ *and* $\left(\hat{\theta}(t) - \theta\right)$ *tend to zero exponentially.*

Proof: First, we can see that for $\hat{\theta} = \theta$ the system (17) is an observer for the system (16).
Then consider the feedback system for the nominal values

$$\begin{aligned} \dot{\tilde{x}} &= A_0\ \tilde{x} + B_0\left(K\zeta_1 + \varphi(\hat{\theta})\zeta_2\right) - B_0M(\theta)\ \varsigma \\ \dot{\varsigma} &= \Phi(\theta)\ \varsigma \\ \dot{\zeta}_1 &= (A_0 - G_1C_0 + B_0K)\,\zeta_1 + B_0\varphi(\hat{\theta})\zeta_2 + G_1C_0\tilde{x} \\ \dot{\zeta}_2 &= \left(\Phi(\hat{\theta}) - G_2\Psi\right)\zeta_2 + G_2C_0\tilde{x} - G_2C_0\zeta_1 \\ \dot{\hat{\theta}} &= -\Delta\ \Omega(t)(C_0\tilde{x} - C_0\zeta_1 - \Psi\zeta_2) \\ e &= C_0\ \tilde{x} \end{aligned}$$

and the errors $e_1 = \tilde{x} + \tilde{\Pi}(\theta)\varsigma - \zeta_1$, $e_2 = \varsigma - \zeta_2$, $e_3 = \hat{\theta} - \theta$, and $\tilde{\varphi}(e_3) = \varphi(\hat{\theta}) - \varphi(\theta)$ where $\tilde{\varphi}(0) = 0$. It is not difficult to see that the error system take the form

$$\begin{aligned} \dot{\tilde{x}} &= (A_0 + B_0K)\,\tilde{x} - B_0Ke_1 \\ &\quad -B_0\varphi(\theta)e_2 + B_0\tilde{\varphi}(e_3)\zeta_2 \\ \dot{e}_1 &= (A_0 - G_1C_0)\,e_1 \\ \dot{e}_2 &= (\Phi(\theta) - G_2\Psi)\,e_2 + \left(\Phi(\theta) - \Phi(\hat{\theta})\right)\zeta_2 - G_2C_0e_1 \\ \dot{e}_3 &= -\Delta\ \Omega(t)\,(C_0e_1 + \Psi e_2) \end{aligned}$$

considering $e_2 = \begin{bmatrix} e_{(2,1)}^T & e_{(2,2)}^T & \cdots & e_{(2,m)}^T \end{bmatrix}^T \in \Re^{m(2k+1)}$, $e_{(2,j)} = \begin{bmatrix} e_{(2,j,1)} & \cdots & e_{(2,j,2k+1)} \end{bmatrix}^T \in \Re^{2k+1}$, $C_0 = \begin{bmatrix} c_1^T & c_2^T & \cdots & c_m^T \end{bmatrix}^T \in \Re^{m\times n}$, $c_i^T \in \Re^n$, and the special form of the matrices $\Phi(\theta)$, G_2 and Ψ, we have that the error e_2 can be viewed like m subsystems which take the forms $\dot{e}_{(2,j,i)} = e_{(2,j,i+1)} \quad i = 1...(2k-1)$ and $j = 1...m$, $\dot{e}_{(2,j,2k)} = -\sum_{i=1}^{2k} \frac{\psi_{(i,j)}}{\psi_{(2k+1,j)}} e_{(2,j,i)} - g_{(1,j)}c_je_1$, $\dot{e}_{(2,j,2k+1)} = -g_{(2,j)}\sum_{i=1}^{2k+1}\psi_{(i,j)}e_{(2,j,i)} - g_{(2,j)}c_je_1 - \sum_{i=1}^{k}\theta_ie_{(2,j,2i)} + e_3^T\omega_j$, defining $\bar{e}_{2\rho} = \begin{bmatrix} \bar{e}_{(2,1)}^T & \bar{e}_{(2,2)}^T & \cdots & \bar{e}_{(2,m)}^T \end{bmatrix}^T \in \Re^{2km}$, $\bar{e}_{(2,j)} = \begin{bmatrix} e_{(2,j,1)} & \cdots & e_{(2,j,2k)} \end{bmatrix}^T \in \Re^{2k}$, $\bar{e}_1 = \begin{bmatrix} e_1^T & \bar{e}_{2\rho}^T \end{bmatrix}^T \in \Re^{n+2km}$,
$\bar{e}_2 = \begin{bmatrix} e_{(2,1,2k+1)} & e_{(2,2,2k+1)} & \cdots & e_{(2,m,2k+1)} \end{bmatrix}^T \in \Re^m$, we have the system

$$\begin{aligned} \dot{\bar{e}}_1 &= \bar{A}\ \bar{e}_1 \\ \dot{\bar{e}}_2 &= -\beta_1\bar{e}_1 - \beta_2\bar{e}_2 + \Omega^T(t)e_3 \\ \dot{e}_3 &= -\Delta\ \Omega(t)\left(\bar{q}_1\bar{e}_1 + \bar{\Psi}_2\ \bar{e}_2\right) \end{aligned} \tag{18}$$

where $\bar{A} = \begin{bmatrix} (A_0 - G_1C_0) & 0 \\ -\upsilon_2C_0 & diag(\Phi_{21}, ..., \Phi_{2m}) \end{bmatrix} \in \Re^{(n+2km)\times(n+2km)}$, $\upsilon_2 = diag(\bar{g}_1, ..., \bar{g}_m) \in \Re^{2km\times m}$, $\bar{g}_j = \begin{bmatrix} 0 & \cdots & 0 & g_{(1,j)} \end{bmatrix}^T \in \Re^{2k}$, $\Phi_{2j} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\frac{\psi_{(1,j)}}{\psi_{(2k+1,j)}} & -\frac{\psi_{(2,j)}}{\psi_{(2k+1,j)}} & \cdots & -\frac{\psi_{(2k,j)}}{\psi_{(2k+1,j)}} \end{bmatrix} \in \Re^{2k\times 2k}$,
$\beta_1 = diag(g_{(2,1)}, ..., g_{(2,m)})\begin{bmatrix} C_0 & \vartheta(\theta) \end{bmatrix} \in \Re^{m\times(n+2km)}$, $\vartheta(\theta) = diag(v_1(\theta), ..., v_m(\theta))$,
$v_j(\theta)^T = \begin{bmatrix} \psi_{(1,j)} & \cdots & (\frac{\theta_k}{g_{(2,j)}} + \psi_{(2k,j)}) \end{bmatrix}^T \in \Re^{2k}$, $\beta_2 = diag(g_{(2,1)}\psi_{(2k+1,1)}, ..., g_{(2,m)}\psi_{(2k+1,m)})$, $\beta_2 \in \Re^{m\times m} > 0$, $\bar{q}_1 = \begin{bmatrix} C_0 & \bar{\Psi}_1 \end{bmatrix} \in \Re^{m\times(n+2km)}$,
$\bar{\Psi}_1 = diag(\bar{\psi}_1, ..., \bar{\psi}_m) \in \Re^{m\times 2km}$,
$\bar{\psi}_j = \begin{bmatrix} \psi_{(1,j)} & \psi_{(2,j)} & \cdots & \psi_{(2k,j)} \end{bmatrix}$,
$\bar{\Psi}_2 = diag(\psi_{(2k+1,1)}, ..., \psi_{(2k+1,m)}) \in \Re^{m\times m} > 0$, observe that the form (18) is similar to (13) and have the same properties, for see that, from the equations (18) we can see that exist some Lyapunov function such that $V(e_1) = \frac{1}{2}\bar{e}_1^TP\bar{e}_1 > 0$ and $\dot{V}(e_1) = -\|\bar{e}_1\|^2$, because $(A_0 - G_1C_0)$ are Hurwitz matrix and $P_j(\xi)$ are stables polynomials. Choosing the Lyapunov function like

$V(\bar{e}_1, \bar{e}_2, e_3) = \kappa\frac{1}{2}\bar{e}_1^T P\bar{e}_1 + \frac{1}{2}\bar{e}_2^T\bar{\Psi}_2\bar{e}_2 + \bar{e}_2^T\bar{q}_1\bar{e}_1 + \frac{1}{2}e_3^T\Delta^{-1}e_3$ with $\kappa \in \Re > 0$, exist some constant κ_{1*} such that for every constant $\kappa > \kappa_{1*}$ the function is positive definite. The derivative of $V(\bar{e}_1, \bar{e}_2, e_3)$ is $\dot{V}(\bar{e}_1, \bar{e}_2, e_3) = -\kappa\|\bar{e}_1\|^2 + \bar{e}_2^T\bar{\Psi}_2\left(-\beta_1\bar{e}_1 - \beta_2\bar{e}_2 + \Omega^T(t)e_3\right)$ $+\left(-\beta_1\bar{e}_1 - \beta_2\bar{e}_2 + \Omega^T(t)e_3\right)^T\bar{q}_1\bar{e}_1 + \bar{e}_2^T\bar{q}_1\left(\bar{A}\ \bar{e}_1\right)$ $-e_3^T\Omega(t)\left(\bar{q}_1\bar{e}_1 + \bar{\Psi}_2\ \bar{e}_2\right) = -\kappa\|\bar{e}_1\|^2 - \bar{e}_1^T\beta_1^T\bar{q}_1\bar{e}_1$ $-\bar{e}_2^T\bar{\Psi}_2\beta_2\bar{e}_2 + \bar{e}_2^T\left(\bar{q}_1\bar{A} - \bar{\Psi}_2\beta_1 - \beta_2^T\bar{q}_1\right)\bar{e}_1$ $\leq \left(-\kappa + \left\|\beta_1^T\bar{q}_1\right\|\right)\|\bar{e}_1\|^2 - \lambda_{\min}(\bar{\Psi}_2\beta_2)\|\bar{e}_2\|^2 + \nu_0\|\bar{e}_1\|\|\bar{e}_2\|$, then exist some constant κ_{2*} such that for every constant $\kappa > \kappa_{2*}$ and given $\nu_0 = \left\|\bar{q}_1\bar{A} - \bar{\Psi}_2\beta_1 - \beta_2^T\bar{q}_1\right\|$ the derivative is negative then we choose $\kappa = \max(\kappa_{1*}, \kappa_{2*})$. The invariant set is $\mho = \{(\bar{e}_1, \bar{e}_2, e_3) \mid \bar{e}_1 = 0, \bar{e}_2 = 0\}$, but in this set we have $\Omega^T(t)e_3 = 0$, which imply that $\omega_j^T e_3 = 0$, $j = 1...m$, and $\digamma_\omega^T(t)e_3 = \left(\sum_{j=k_1}^{k_2}\omega_j^T(t)\right)e_3 = 0$, $1 \leq k_1 \leq k_2 \leq m$,then if $\digamma_\omega^T(t)$ have linear independent components is a persistence excitation signal and e_3 must be zero, moreover, the convergence is exponentially because the persistency excitation lemma [10], and we have that $\tilde{x} \to 0$ because $(A_0 + B_0K)$ is Hurwitz and there for $e(t) \to 0$. This completes the proof. ∎

Remark 2: Observe that the condition of linear independence of the $\digamma_\omega^T(t)$ vector components always is possible because the polynomials $P_j(\xi)$ and the constant k_1, k_2 are freely chosen.

Remark 3: Let us note that the algebraic equations (14)-(15) may be rewritten as a linear equations system $A_e(\theta)X = b_e$, where X is a vector containing the unknowns terms $\tilde{\Pi}(\theta)$ and $M(\theta)$. Since the solution of this system depends on the values of θ, if the system presents transmission zeros, the determinant $\delta(\hat{\theta})$ of $A_e(\hat{\theta})$ could eventually pass by zero. In this situation, a slightly change on the structure of the controller may be introduce, by selecting the function $\varphi(\hat{\theta})$ as

$$\begin{aligned} \varphi(\hat{\theta}) &= \frac{\delta(\hat{\theta})(M_n(\hat{\theta}) - K\tilde{\Pi}_n(\hat{\theta}))}{\max(\delta(\hat{\theta})^2, \epsilon)},\ \epsilon > 0 \\ \epsilon &< \delta(\hat{\theta}_{ss})^2 \end{aligned}$$

where $\hat{\theta}_{ss}$ is the steady state of $\hat{\theta}$ The number ϵ is estimated using $\delta(\hat{\theta}_{ss})$ and considering that the eigenvalues of $\Phi(\hat{\theta}_{ss})$ are imaginary. Then we can apply the Lagrange multiplier method to obtain the minimum of $\delta(\hat{\theta}_{ss})$.

For one application of this algorithm to the case $m = 1$ refer to [24]. With respect to the robustness we have the next Theorem.

Theorem 4: The regulator (17) is robust for some neighborhood of the nominal values (A_0, B_0, C_0).

Proof: The proof of stability property is follows from the exponential convergence and the converse Lyapunov theorem (see [22]). The robustness of the error property (**??**) is follows because the signal $\Gamma w(t)$ is obtained for every matrix Γ. (see [17]). ∎

VI. Simulation results

Let us consider the system $\dot{x}_1 = (1 + \mu_1)x_2 + 15\sin(t)$, $\dot{x}_2 = x_3 + x_4 + u_2$, $\dot{x}_3 = -(1 + \mu_2)x_4 + u_1 + 13\cos\left(\sqrt{3}t + 1\right)$, $\dot{x}_4 = x_1 - x_2 + x_3$, $y_1 = x_1$, $y_2 = x_4$, which do not have transmission zeros because $\left|\left[\begin{array}{cc} \lambda I - A & B \\ C & 0 \end{array}\right]\right| = -1$, then there exists no vector $\theta \in \Re^k$ such that $\delta(\theta) = 0$. The frequencies $\alpha_1 = 1$, $\alpha_2 = \sqrt{3}$ are suppose unknown frequencies. Calculating the robust adaptive internal model controller with $\sigma(A + BK) = \{-1, -2, -3, -4\}$; $\sigma(A - G_1C) = \{-3, -4, -5, -6\}$ and the matrices $K = \left[\begin{array}{cccc} -5.9161 & -0.5010 & -4.0578 & -2.1685 \\ -7.0110 & -5.9422 & -0.7902 & -0.3969 \end{array}\right]$, $G_1 = \left[\begin{array}{cccc} 9.6435 & 25.9098 & 15.1321 & -0.6244 \\ -0.4095 & 2.4989 & 13.6684 & 8.3565 \end{array}\right]^T$, $G_2 = \left[\begin{array}{cc} G_{(2,1)} & 0 \\ 0 & G_{(2,2)} \end{array}\right]$, $G_{(2,1)} = G_{(2,2)} = \left[\begin{array}{cccc} 0 & 0 & 1 & 2 \end{array}\right]^T$, $\Delta = \left[\begin{array}{cc} 2000 & 0 \\ 0 & 5000 \end{array}\right]$, $\Psi_1 = \left[\begin{array}{cccc} 6 & 11 & 6 & 1 \end{array}\right]$, $\Psi_2 = \left[\begin{array}{cccc} 8 & 14 & 7 & 1 \end{array}\right]$, $\Theta = G_1\Psi = G_1\left[\begin{array}{cc} \Psi_1 & 0 \\ 0 & \Psi_2 \end{array}\right]$, $\bar{\Phi}(\hat{\theta}) = \left[\begin{array}{cc} 0 & I \\ -\hat{\theta}_1 & v_\Phi \end{array}\right]$, $v_\Phi = \left[\begin{array}{ccc} 0 & -\hat{\theta}_2 & 0 \end{array}\right]$, $\Phi(\hat{\theta}) = diag(\bar{\Phi}(\hat{\theta}), \bar{\Phi}(\hat{\theta}))$, $\varphi(\hat{\theta}) = \left[\begin{array}{cccccccc} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} & f_{17} & f_{18} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} & f_{26} & f_{27} & f_{28} \end{array}\right]$,where the components for the matrix $\varphi(\hat{\theta}) = (M(\hat{\theta}) - K\tilde{\Pi}(\hat{\theta}))$ are given by $f_{11} = -350.5169 + 18.5779\ \hat{\theta}_1$, $f_{12} = -718.0818 + \hat{\theta}_1$, $f_{13} = -494.8738 + 18.5779\ \hat{\theta}_2$, $f_{14} = -144.8868 + \hat{\theta}_2$, $f_{15} = -391.0326 + 19.0048\ \hat{\theta}_1$, $f_{16} = -780.3454 + \hat{\theta}_1$, $f_{17} = -518.2207 + 19.0048\ \hat{\theta}_2$, $f_{18} = -146.9126 + \hat{\theta}_2$, $f_{21} = -531.2519 + 21.3759\ \hat{\theta}_1$, $f_{22} = -1066.2173 + \hat{\theta}_1$, $f_{23} = -706.3868 + 21.3759\ \hat{\theta}_2$, $f_{24} = -191.7973 + \hat{\theta}_2$, $f_{25} = 17.63849 - 0.6193\ \hat{\theta}_1$, $f_{26} = 35.821761$, $f_{27} = 24.10388 - 0.6193\ \hat{\theta}_2$, $f_{28} = 6.5399115$, the results are presented in the figures 1 and 2, where in the figure 2 we are changed the parameters μ_1 and μ_2.

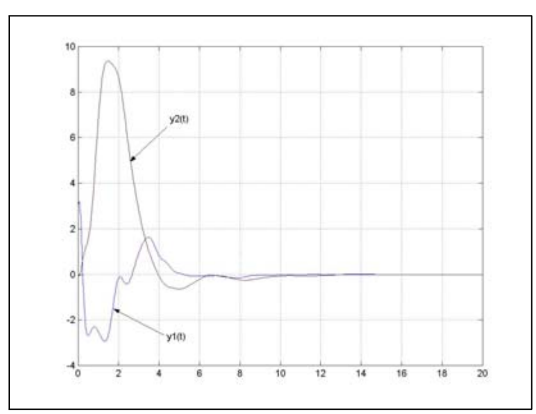

Fig. 1. Output signals y_1, y_2.

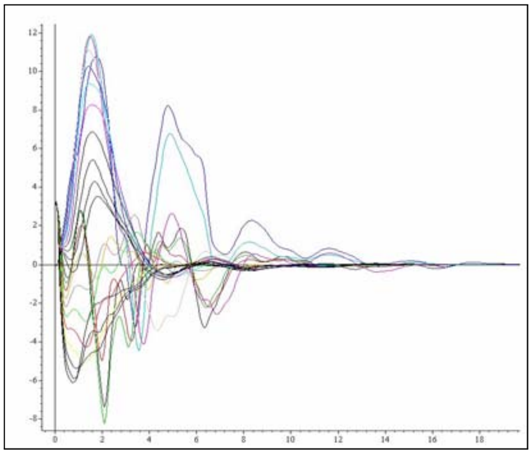

Fig. 2. Outputs signals for $-0.5 \leq \mu_1 = \mu_2 \leq 0.5$.

In the figure 3 we impose the reference $r_1 = \sin(t)$, $r_2 = 2\sin(t)$, and in $t = 25$ *seg* we change the reference r_2 to $r_2 = \cos(t)$.

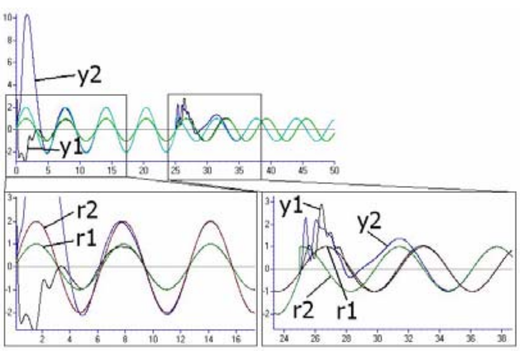

Fig. 3. Outputs y_1, y_2 and reference signals r_1, r_2.

VII. Conclusion.

In this paper, we present a new compensator with adaptive internal model, for minimum and non-minimum phase linear MIMO systems. It is shown that the proposed controller ensures global convergence of the estimator and preserves the output regulation properties despite variations of the external signal generator parameters. The property of robustness of the closed-loop system in presence of the plant parameters variations in a neighborhood of nominal values, is also guaranteed.

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